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Endogenous multihoming and network effects: PlayStation, Xbox, or both?

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Endogenous multihoming and network effects: PlayStation, Xbox, or both?

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Abstract: Competition between firms that sell incompatible varieties of network products might be fierce, because it is important for each of them to attract a large number of users. The literature therefore predicts that stronger network effects decrease prices and profits. We show that this prediction hinges critically on an implicit or explicit assumption that each consumer buys only one of the varieties offered in the market (singlehoming consumers). We show that multihoming (some consumers buy more than one variety) may arise endogenously if the number of exclusive features that each variety offers is sufficiently high. In sharp contrast to the conventional prediction under consumer singlehoming, we further show that both prices and profits could increase in the strength of the network effects if (some) consumers multihome. However, this does not necessarily imply that profits are higher under multihoming than under singlehoming. On the contrary, multihoming might constitute a prisoner's dilemma for the firms, in the sense that they could make higher profits if each consumer bought only one of the varieties.

Keywords: multihoming, incremental pricing, network effects.

JEL Classification: L13, L 14, L82.

1 Introduction

A good exhibits positive network effects if the utility a user derives from it increases with the number of other consumers using the same or a compatible good. We consider competition between two firms that provide incompatible varieties of a network product or service. Each consumer may choose to buy only one of the varieties (singlehome) or both (multihome), depending on what generates the higher net utility. Opening up the possibility that consumers buy more than one variety, endogenous multihoming, has surprising effects on market equilibrium and performance in network markets. Assuming that all consumers singlehome, as is common in the vast majority of the literature, may generate highly misleading policy advices and business strategy predictions.

The archetypical example of a network effect is one where consumers directly benefit from communicating with each other, as is the case for telephony and many other telecommunications services. Since gamers now play online, we also have direct network effects in the market for video game consoles, where Microsoft (Xbox) and Sony (PlayStation) are the main providers. A user of PlayStation, for instance, benefits if the pool of PlayStation users becomes larger. Furthermore, network effects arise if individual users of a good generate data that e.g., machine learning algorithms can utilize to increase the perceived quality of the good for other users.[1] One example of such data network effects is driver assistance systems (autonomous cars). Another example is recommendation systems used extensively by e.g., streaming services like Netflix and HBO (movies/serials) and Spotify and Apple Music (music).[2] It requires a lot of user data to build good recommendation systems, but they might become highly efficient. In 2017, around 70 % of the videos watched on YouTube were recommended by the company's algorithms.[3] Consumer reviews are considered as one of Amazon's most important features. Allowing consumers to post their reviews was introduced as early as in 1995 by Amazon (Chen et al., 2008).

Even if network effects increase the willingness to pay for successful goods, a main

[1] Strong network effects were present in the video game market even before online gaming. Shanker and Bayus (2003) find strong network effects when Nintendo and Sega were the dominant gaming consoles in the 1990's.

[2] Hagiu and Wright (2020) provide a large number of examples of data network effects, or what they label data-enabled learning.

[3] See CNET, January 13, 2018.

https://www.cnet.com/news/youtube-ces-2018-neal-mohan/

result in the literature is that they intensify competition if different firms offer different incompatible varieties. A typical prediction is that prices in a competitive equilibrium are decreasing in the strength of the network effects. Much of the literature analyses the interaction between network effects and pricing decisions within spatial competition frameworks (like the Hotelling model), and presupposes that each consumer has unit demand and thus buys at most one of the goods (Farrell and Saloner, 1992, among others; see also Shy, 2001, for a textbook presentation). Also economics of business strategy best-sellers, like Shapiro and Varian (1998), predict that in absence of compatibility among the providers, network effects intensify competition.

For a lot of products and services we observe that at least a fraction of the consumers multihome. Digitization has made multihoming easier for the consumers in industries like media and entertainment services. The motivation for buying more than one variety of a product is that there exists an incremental value of doing so. For instance, consumers need to subscribe to both HBO and Netflix to have access to Game of Thrones (HBO) as well as The Crown (Netflix). In 2020, 20 % of Netflix subscribers in the US also subscribed to HBO Max, and 80 % of HBO Max customers subscribed to Netflix.[4] Multihoming is quite common also in the gaming industry; at the household level, almost half of Xbox One users owns a PlayStation 4.[5] Both Microsoft and Sony provide exclusive games on their consoles. Such exclusives give rise to an incremental value for buying Xbox in addition to PlayStation, and vice versa. Additionally, the presence of network effects gives rise to an incremental value of becoming a multihomer, since the scope for users of different game consoles to play together is limited.[6] This is understood by the gaming community. In a post on Reddit.com, for instance, both exclusives and network effects are highlighted when

[4]See Statista, 2021. https://www.statista.com/statistics/778912/video-streaming-service-multiple-subscriptions/-

[5]More specifically, Ampere Analysis (2020) shows that 47.5% of Xbox One households also have a PlayStation 4, while 27.4% of households that own an Xbox One also have a PlayStation 4 Pro. Conversely, 32.2% of PlayStation 4 households also own an Xbox One, while 18.5 % of PlayStation 4 owners have an Xbox One X too. The report by Ampere Analysis. (2020) is behind paywall, but the results are summarized by gamedeveloper.com in "Data suggests almost half of Xbox One users also own a PlayStation 4".

[6]The degree of compatibility, i.e., whether Xbox and PlayStation users can play together differs between games (cross-platform play is the jargon used in the online gaming community). Among the most popular games, such cross-platform play is possible for Fortnite, but not for Fifa, for instance. See https://en.wikipedia.org/wiki/Cross-platform_play. Accessed Dec. 1, 2021.

stressing the benefit of having Xbox as well as PlayStation:[7]

"I own both an Xbox One X and a PS4 Pro. The benefit is being able to play any console exclusives you want. As for downsides, aside from the initial cost, there are none. It's also nice to be able to play various games with different friends who have picked one or the other."

This quote summarizes the driving forces in the present paper.[8]

Applying the framework of Hotelling (1929), we show that if some consumers choose to multihome, the presence of network effects may increase both prices and profits. Think of HBO and Netflix, where a fraction of the consumers multihomes. Assume that the services at the outset exhibit no network effects, and that both HBO and Netflix then introduce a feature that gives rise to small, but positive, network effects (e.g., a recommendation system). Then demand for both varieties increase, and we show that this results in higher prices and profits.

However, the relationship between the strength of the network effect and prices (and profits) is not monotonically increasing under multihoming. If the network effects are sufficiently strong, a further increase in the intensity of the network effects reduces prices and profits. Prices and profits are consequently hump-shaped functions of the strength of the network effects. This sharply contrasts the prediction from the standard network economics literature, where it is presupposed that all consumers are singlehomers. In a singlehoming equilibrium, there is a monotonic negative relationship between the strength of the network effects and the level of the prices and profits. If competing firms introduce

[7] See https://www.reddit.com/r/AskReddit/comments/91zuwh/for_those_who_own_both_a_ps4_and_xbox_one_what/.Accessed Dec 1, 2021.

[8] Another illustrative example is the following, related to the introduction of a new generation of Xbox and Playstation in 2020. At the same time, Microsoft acquired the video game producer ZeniMax Media, and the Forbes' columnist on video games writes (Thier, 2020): *"In one moment, the entire exclusive content conversation about the Xbox Series X and PS5 generation shifted... When we talk about the Microsoft vs. Sony console war we don't tend to talk about the large number of players that get both, behavior that a more aggressive exclusive strategy would encourage. And Microsoft has the perfect console for those PS5 owners that will want to try out exclusives from across the aisle: the lower-priced Xbox Series S. The thing is debuting at $299 right now, and it's easy to imagine that it could be down to$249 or $199 by the time Elder Scrolls 6 or even Starfield comes out [games from ZeniMax Media]. I know plenty of people that would have paid $199 to play Skyrim."*

features that exhibit network effects, such as recommendation systems by streaming service providers, prices and profits will fall.

The number of exclusive features (attributes) is crucial for the equilibrium outcome. If the fraction of exclusive features is sufficiently high, multihoming is a unique equilibrium. The reason is that more exclusives increase the consumers' incremental value of multihoming. To ensure that singlehoming is a unique equilibrium, which is implicitly assumed in most of the literature, the level of exclusives needs to be low.

The number of exclusives further determines whether firms' profits are higher under multihoming compared to an outcome where all consumers singlehome. If firms have many exclusives, profits are greater under multihoming than under singlehoming. For an intermediate level of exclusive features, multihoming is a unique equilibrium even though the firms' profits would have been higher in a counterfactual outcome where all consumers singlehome.

In an equilibrium where all consumers singlehome, we find - in line with previous literature - that prices are strategic complements. However, a main prediction from the scarce literature that allows for multihoming, is that prices are strategically independent if some consumers choose to multihome in equilibrium (Anderson et al., 2017; Kim and Serfes, 2006). As an illustration, take the market for streaming services. Without network effects, the multihoming prediction is that the demand for HBO depends only on HBO's own price, and not on the price Netflix charges. We show that this independency result breaks down even if there is only a small network effect; prices will then be strategic complements under multihoming as well. Hence, our results may have relevance also for markets where the network effect is relatively weak (as might be the case for e.g., streaming services).

The rest of the paper is organized as follows. In Section 2 we review related literature. Section 3 develops the model of network effects when allowing for multihoming. In section 4, we provide an application where show how multihoming and network effects may have impact on merger incentives and effects. Section 5 concludes.

2 Related literature

The focus on consumer multihoming has mainly taken place within the literature on two-sided markets (Ambrus et al., 2016; Anderson et al., 2018: Athey et al., 2018; Bakos and Halaburda, 2020; Belleflamme and Peitz, 2019, among others). This literature has shown that multihoming on one side of the market typically reduces the profits that can be extracted from the other side of the market.[9] Ad financed media platforms are an example. If all consumers singlehome, each media platform can offer exclusive eyeballs to advertisers. They can use this monopoly power to charge high advertising prices. In contrast, when consumers multihome, competition for advertisers arises between the media platforms. The platforms could therefore be better off if all consumers singlehomed, in which case they would have an incentive to discourage multihoming (Athey and Scott Morton, 2021).

In contrast to the literature on two-sided markets, we consider direct network effects, and combine elements from the substantial network economics literature and the more limited literature that allows for multihoming consumers (Anderson et al., 2017; Kim and Serfres, 2006). The literature on network effects focuses on (i) technology adoption decisions (Farrell and Saloner, 1985; Katz and Shapiro, 1986, are early contributions), (ii) compatibility decisions (Katz and Shapiro, 1985; Farrell and Saloner, 1992, and subsequent papers) and (iii) competitive pricing strategy decisions among providers of incompatible products or services.[10] Our focus is on the latter, i.e., pricing decisions of firms that provide incompatible products, where we allow that consumers select themselves into singlehomers and multihomers.

Within the Hotelling framework, Farrell and Saloner (1992), Shy (2001), and Grilo et al. (2001), among others, show that network effects intensify price competition between providers of incompatible products.[11] Analogously, Katz and Shapiro (1985) show how network effects intensify competition under Cournot competition. A crucial presumption in all these articles is that all consumers singlehome. However, as noted above, singlehoming does not constitute an equilibrium if the firms have a sufficiently large number of exclusives.

[9] We used the video game console market as an illustrative example. Landsman and Stremersch (2011) empirically analyze multihoming in the sense that content providers (game developers) offer their content on several platforms. However, Landsman and Stremersch do not analyze consumer multihoming that is our focus.

[10] There are dynamic aspects of network effects. For a model of dynamic pricing with network effects, see Cabral (2011). Hagiu and Wright (2020) show, in a dynamic framework, how data-enabled learning (such as driving assistance systems discussed in the introduction) may create network effects. The dynamic effects are outside the scope of the present paper.

[11] See also Tolotti and Yepez (2020), Xie and Sirbu (1995), and Baake and Boom (2001).

Consumer reviews may be an important source of network effects, and such reviews have become important in several digital platform markets. There is a literature analyzing the importance of word-of-mouth information (e.g., consumer reviews) in consumers' purchase decisions (Chen et al., 2008, among others), but network effects are not formally modelled.

In the literature that allows for multihoming, Anderson et al. (2017) show the condition for endogenous multihoming to arise, while Kim and Serfes (2006) focus on the location incentives under consumer multihoming. The key takeaway for our analysis is that it is the incremental value that matters under multihoming. This gives rise to the distinction between exclusive and overlapping features (in the Xbox-PlayStation example, an overlapping feature could be a game that is accessible on both platforms). For simplicity, we assume that there is no gain from accessing any given feature more than once. Hence, only exclusives matter for the incremental value of buying more than one variety. Anderson et al. (2017) also allow for overlapping features contribute to the incremental value. However, this is not qualitatively important for our results. More important, in our model the network effects contribute to the incremental value, and this is what breaks the strategic independency result from Anderson et al. (2017) and Kim and Serfes (2006), such that prices become strategic complements.

Our model has some similarities with Doganoglu and Wright (2006), who employ a Hotelling framework where consumers may multihome in order to gain maximum benefit from the network effects. In contrast to our model, they do not endogenize multihoming. In their model, each consumer perceives the marginal network benefit as being either small or large, and they assume that all consumers with large marginal network effects prefer multihoming, while those with small marginal network effects prefer singlehoming.

Our model is also related to the merger literature. Anderson et al. (2017), for instance, show that due to the independence result explained above, a merger has no impact on firms' prices if some consumers multihome. Again, this result hinges critically on the assumption that network effects are absent. In an extension we consider the pricing incentives for a multiproduct monopoly, which we interpret as a merger between two firms. We show that the merger leads to higher prices and profits if there are network effects, independent of whether consumers singlehome or multihome. We also show that under some circumstances the merger might shift the market equilibrium from one with multihoming to one where all consumers singlehome. This raises a cautionary tale for competition authorities.

3 The model

We consider a Hotelling model with two firms that sell each their good, $i = 0, 1$. There are N consumers, uniformly distributed on a Hotelling line with unit length. The mismatch cost (transportation cost) for a consumer located at x of buying good i is given by $t\,|X_i - x|$, where X_i is the location of firm i, and $t > 0$ is the size of the mismatch cost. We open up for the possibility that there are positive network effects, such that the value of buying good i is increasing in the number of users. The value of this network effect is equal to μz_i, where $\mu \geq 0$ measures the strength of the network effect and z_i is the expected number of consumers buying good i. The gross utility of buying only good i (singlehoming) for a consumer located at x can consequently be expressed as

$$u_i = v_i - t\,|X_i - x| + \mu z_i, \tag{1}$$

where v_i is the intrinsic value of buying good i. By splitting the parameter v_i in two, we can take into account that consumers who buy good i may benefit from some features that are exclusive for that good and some features that the two goods share. We denote the exclusive features for good i by e_i and the overlapping (shared) features by o, such that $v_i = e_i + o$.

A consumer who buys both goods (a multihomer) can communicate with everyone, and will therefore enjoy a network benefit equal to μN. She will further enjoy the full range of exclusive and overlapping features of the two goods, but we assume that she will not gain any extra value of accessing the overlapping features more than once. The gross utility of buying both goods for a consumer located at x is consequently equal to

$$u_b = \mu N + (v_0 - t\,|X_0 - x|) + (v_1 - t\,|X_1 - x|) - o \tag{2}$$

Firm 0 is located to the far left on the Hotelling line ($X_0 = 0$) and firm 1 to the far right ($X_1 = 1$), and without loss of generality we normalize the number of consumers to 1 ($N \equiv 1$). Inserting this into (2), we have

$$u_b = o + e_0 + e_1 - (t - \mu)\,. \tag{3}$$

The net utility of buying only good i is $u_i - p_i$, where p_i denotes the price of good i, while the net utility of buying both goods equals $u_b - p_0 - p_1$. Clearly, a consumer will buy both goods if this yields a higher net utility than buying only one of the goods.

Throughout, we assume that the intrinsic values of the goods (v_0 and v_1) are sufficiently large to ensure market coverage in equilibrium such that each consumer buys at least one of the varieties. We shall further restrict attention to cases where each firm has a positive market share. Below, we shall see that these assumptions are fulfilled if:

Assumption 1: *(i)* $t > \mu$, *(ii)* $o + (e_0 + e_1)/2 > 3(t - \mu)/2$, *(iii)* $e_i > e_j - 3(t - \mu)$.

In the next two sections we characterize both an outcome with pure singlehoming and an outcome with multihoming. Subsequently, we determine which of the outcomes might constitute an equilibrium.

3.1 Benchmark: All consumers assumed to singlehome

As a benchmark, let us scrutinize network effects in the standard Hotelling model in which each consumer by assumption can buy only one of the varieties. Consumers for whom $u_0(x) - p_0 \geq u_1(x) - p_1$ will then buy good 0, while the rest will buy good 1.[12] Firms and consumers have rational expectations, such that $z_0 = x$ and $z_1 = 1 - x$. Using equation (1), we find that demand for good i equals

$$x_i(p_i, p_j) = \frac{1}{2} + \frac{(e_i - p_i) - (e_j - p_j)}{2(t - \mu)} \; (i \neq j). \tag{4}$$

From (4) we see that stronger network effects ($d\mu > 0$) change the demand similarly to a reduction in the mismatch cost ($dt < 0$); they make demand more responsive to changes in both own and the rival's price.

We normalize marginal production costs to zero, such that firm i's profits equal

$$\pi_i = p_i x_i(p_i, p_j). \tag{5}$$

Maximizing profits with respect to the own price yields the reaction function

$$p_i^S(p_j) = \frac{t - \mu + e_i}{2} + \frac{p_j - e_j}{2}, \tag{6}$$

where the superscript S denotes that we are considering a pure singlehoming outcome. As expected, prices are strategic complements; $dp_i/dp_j > 0$.

[12] Assumption 1 ensures that firms share the market.

Solving the profits maximization problem for both firms simultaneously, we find that prices in a possible singlehoming equilibrium are given by

$$p_i^S = t - \mu + \frac{1}{3}(e_i - e_j), \tag{7}$$

Network effects increase the importance for each firm of having a large market share, and this intensifies price competition. Indeed, from (7) we observe that $dp_i/d\mu = -1$, which means that the beneficial network effect is passed on in full to consumers.[13] This result resembles the findings by Farrell and Saloner (1992) and subsequent papers.

Inserting (7) into (4) we find that output and profits are respectively equal to

$$x_i^s = \frac{1}{2} + \frac{e_i - e_j}{6(t-\mu)} \text{ and } \pi_i = \left[t - \mu + \frac{1}{3}(e_i - e_j)\right]\left[\frac{1}{2} + \frac{e_i - e_j}{6(t-\mu)}\right]. \tag{8}$$

It can be shown that the utility of the consumer who us indifferent between buying good 0 and 1 is equal to $o + (e_0 + e_1 - 3(t-\mu))/2$, which is positive given Assumption 1 (ii). From (8) we find that $d\pi_i/d\mu < 0$ whenever both firms have positive profits margins (as Assumption 1 (iii) ensures). Stronger network effects consequently harm the firms in a competitive environment where all consumers singlehome.

3.2 Some consumers assumed to multihome

We now consider an outcome where some - but not all - consumers choose to multihome (and will later analyze whether such partial multihoming constitutes an equilibrium). In this context, the incremental value of good i for a consumer - i.e., the benefit of buying good i in addition to good j - is important. In particular, we define the (gross) incremental value of good i, denoted by I_i, as the difference between the utility of buying both goods and the utility of only buying good j :

$$I_i = u_b - u_j. \tag{9}$$

All consumers for whom $I_i \geq p_i$ will buy good i, either as the only good (if $I_j < p_j$) or together with good j (if $I_j \geq p_j$). The incremental value of a good is smaller than its

[13]Expression (7) assumes $p_i^* > 0$ such that $3(t-\mu) > (e_i - e_j)$. If this condition is not fulfilled, $p_i^* = 0$ if $3(t-\mu) \leq (e_i - e_j)$ and $p_j^* = (t - \mu + e_j - e_i)/2$, making p_i^* insensitive to changes in μ since it cannot decline further, but $dp_j^*/d\mu < 0$ continues to hold.

standalone value ($I_i < u_i$), and the difference is increasing in the number of overlapping features (o). This follows because $I_i - u_i = -\left[o + \mu\left(z_0 + z_1 - 1\right)\right]$, where $z_0 + z_1 > 1$ if there are some multihomers.

Inserting for (1) and (3) into (9), we can write the incremental value of good 1 as

$$I_1(x) = e_1 + \mu\left(1 - z_0\right) - t(1 - x). \tag{10}$$

Solving $I_1 \geq p_1$, we find that the total number of consumers who buy good 1 is equal to

$$x_1 = \frac{e_1 - p_1}{t} + \frac{\mu\left(1 - z_0\right)}{t}. \tag{11}$$

An advantage for a consumers of buying both goods is that she can then enjoy full network effects (given $\mu > 0$). Stronger network effects generate a positive demand shift for good 1 as long as not everyone buys good 0 ($\partial x_1/\partial\mu = (1 - z_0)/t) > 0$). As a corollary, it follows that the incremental value of buying good 1 is lower the larger the number of consumers who buy good 0 (if there are positive network effects). Equation (11) thus shows that demand for good 1 is decreasing in expected demand of the other good if $\mu > 0$; $dx_1/dz_0 = -\mu/t < 0$.

The consumers who do not buy good 1 will singlehome at good 0. The number of consumers who only buys good 0 is consequently equal to $s_0 = 1 - x_1$. This is illustrated in Figure 1.

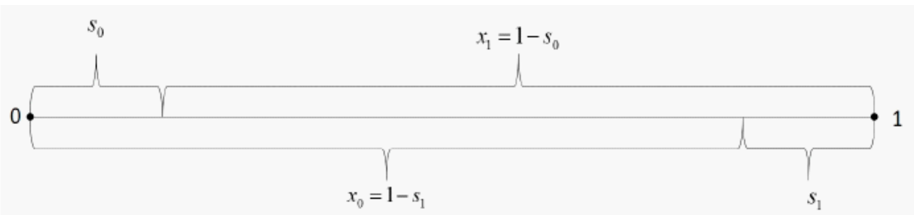

Figure 1: *Single- and multihoming consumers.*

The incremental value of good 0 is likewise equal to

$$I_0 = e_0 + \mu(1 - z_1) - tx, \tag{12}$$

from which we can deduce that demand for good 0 equals

$$x_0 = \frac{e_0 - p_0}{t} + \frac{\mu(1 - z_1)}{t}, \tag{13}$$

and that the number of singlehomers for good 1 is to $s_1 = 1 - x_0$.

In absence of network effects ($\mu = 0$), we see from equations (11) and (13) that demand for good i is proportional to the difference between the consumers' valuation of the good's exclusives and the price they have to pay ($e_i - p_i$). This implies that demand for either good is *independent* of the price of the other good ($dx_i/dp_j|_{\mu=0} = 0$). The intuition is simply that the price of good j is irrelevant when a consumer considers the *incremental* utility of buying good i in addition to good j when $\mu = 0$. However, above we found that the incremental value of each good is decreasing in output of the rival good if there are network effects. This indicates that the independency result breaks down if $\mu > 0$. To verify this, we solve equations (11) and (13) simultaneously, taking into account that $z_i = x_i$ in equilibrium. We can then write demand for good i in the multihoming case (denoted by superscript M) as

$$x_i^M(p_i, p_j) = z_i^M(p_i, p_j) = \frac{\mu}{t + \mu} + t\frac{e_i - p_i}{t^2 - \mu^2} - \mu\frac{e_j - p_j}{t^2 - \mu^2}, \tag{14}$$

from which it follows that $\partial x_i^M(p_i, p_j)/\partial p_j = \mu/(t^2 - \mu^2) > 0$ if $\mu > 0$.

As under singlehoming, we assume that the firms maximize profits with respect to prices. Solving $\max_{p_i} p_i x_i(p_1, p_2)$ we arrive at the reaction function

$$p_i^M(p_j) = \frac{t\mu - \mu^2 + te_i}{2t} + \mu\frac{p_j - e_j}{2t}. \tag{15}$$

Under singlehoming, we found that prices are always strategic complements. Kim and Serfes (2006) and Anderson et al. (2017) argue that the principle of incremental pricing implies that prices are strategically independent under multihoming. However, equation (15) shows that this does not hold if there are positive network effects. Indeed, we have $\partial p_i^M(p_j)/\partial p_j = \mu$, so that prices are strategic complements also under multihoming if $\mu > 0$.[14] This reflects the fact that own demand is increasing in the rival's price if, and only if, $\mu > 0$. This also explains why an increase in e_j induces a negative shift in firm i's reaction function if $\mu > 0$, and that the shift is greater the stronger the network effects;

[14]We could also accommodate negative network effects ($\mu < 0$), for example congestion costs. Prices would then be strategic substitutes. The focus of our paper is, however, on positive network effects.

$dp_i^M(p_j)/de_j = -\mu/(2t)$. In this sense the competitive pressure is increasing in the size of the network effects also under multihoming.

Solving the reaction functions in equation (15) simultaneously for the two firms, and inserting into (14), we find that the candidate equilibrium prices and outputs under multihoming are given by

$$p_i^M = \frac{t-\mu}{2t-\mu}(\mu + e_i) + \frac{\mu t}{4t^2-\mu^2}(e_i - e_j) \text{ and } x_i^M = \frac{\mu}{t+\mu} + t\frac{e_i - p_i}{t^2-\mu^2} - \mu\frac{e_j - p_j}{t^2-\mu^2}. \quad (16)$$

The number of consumers who buys both goods is equal to the total number of consumers minus the number of consumers who buys either only good 0 or good 1; $m \equiv 1 - s_1 - s_2$. This implies

$$s_i = 1 - x_j = \frac{t}{t+\mu} - t\frac{e_j - p_j}{t^2-\mu^2} + \mu\frac{e_i - p_i}{t^2-\mu^2} \quad (17)$$

$$m = 1 - s_0 - s_1 = \frac{(e_0 - p_0) + (e_1 - p_1) - (t-\mu)}{t+\mu}. \quad (18)$$

Inserting from (16) we thus find

$$s_i = \frac{(2t^2-\mu^2)\left((t-\mu)(\mu+2t) - e_j t\right) + e_i \mu t^2}{\mu^4 - 5\mu^2 t^2 + 4t^4}, \quad (19)$$

$$m = \frac{(e_0+e_1)\,t + \mu^2 - 2t^2 + \mu t}{(2t-\mu)(\mu+t)} \text{ and } x_i = \frac{\mu\,(2t+\mu)\,(t-\mu) + (2t^2-\mu^2)\,e_i - t\mu e_j}{(t^2-\mu^2)\,(4t^2-\mu^2)}t. \quad (20)$$

In subsection 3.1 we showed that stronger network effects reduced equilibrium prices and profits if the all consumers singlehome. We will now show that we do not have an equally clear relationship when some consumers multihome. To this end, let us start out by considering the effects of introducing some weak network effects. More precisely, we examine how the firms are affected by a small increase in network effects in the neighborhood of $\mu = 0$ under partial multihoming. With respect to pricing incentives, we find that

$$\frac{\partial}{\partial\mu}\left(\frac{\partial\pi_i}{\partial p_i}\right)\bigg|_{\mu=0} = \frac{\partial x_i^*}{\partial\mu} = \frac{\partial m}{\partial\mu} + \frac{\partial s_i}{\partial\mu}. \quad (21)$$

If stronger network effects increase demand for good i, the firm will be induced to charge a higher price ($\frac{\partial}{\partial\mu}\left(\frac{\partial\pi_i}{\partial p_i}\right)\Big|_{\mu=0} > 0$). To see whether demand increases, we differentiate (17) and (18) with respect to μ and then insert for p_i from (16). This yields

$$\frac{\partial m}{\partial\mu}\bigg|_{\mu=0} = \frac{(2t-e_0)+(2t-e_1)}{4t^2} > 0 \text{ and } \frac{s_i}{\partial\mu}\bigg|_{\mu=0} = -\frac{2t-e_i}{2t^2} < 0. \quad (22)$$

The signs in (22) follow because we can derive from from (19) that a necessary requirement for partial multihoming to exist is that $(2t - e_i) > 0$.

Stronger network effects in the neighborhood of $\mu = 0$ thus increase the number of multihomers and reduces the number of singlehomers. However, we also see that total output increases, such that (21) is positive.[15] We can thus state:

Proposition 1: *Assume partial multihoming. Stronger network effects in the neighborhood of $\mu = 0$ increase total demand for each good (but reduce the number of singlehomers), and lead to higher prices and profits.*

The results in Proposition 1 are fundamentally different from the ones we arrived at in the singlehoming regime, where prices and profits are strictly decreasing in μ. Network effects might benefit the firms under multihoming, but not under singlehoming. This is true even if the firms differ in size (given that both firms have strictly positive market shares).[16]

Another implication from the results above is that even a small network effect qualitatively changes firms' pricing behavior as long as at least some consumers multihome. In absence of network effects, we have strategic independency; the rival's price and number of exclusives do not affect the own price, since it does not affect the total demand a firm faces (Kim and Serfes, 2006; Anderson et al., 2017). This changes as soon as there is a small network effect, and prices then become strategic complements.

Symmetric firms

We shall now derive properties of a possible symmetric equilibrium, and henceforth set $e_1 = e_2 \equiv e$. Equations (19) and (20) then imply that the number of multihomers and singlehomers, respectively, are given by

$$m = 2\frac{e - \underline{e}}{(2t-\mu)(t+\mu)}t \text{ and } s_i = \frac{\overline{e} - e}{(2t-\mu)(t+\mu)}t, \tag{23}$$

where

$$\underline{e} = \underline{e}(\mu, t) = (t-\mu)(2t+\mu)/(2t) \text{ and } \overline{e} = \overline{e}(\mu, t) = \left(2t^2 - \mu^2\right)/t. \tag{24}$$

[15] It is equal to $\partial\pi_i/d\mu|_{\mu=0} = (2t - e_j)\, e_i/(4t^2) > 0$.

[16] If the network effects are so strong that one firm captures the whole market, the winner might benefit from network effects also under singlehoming.

Equation (23) shows that there will be no multihomers if $e < \underline{e}$, while everyone will multihome if $e > \overline{e}$, where $\underline{e} < \overline{e}$. This relationship is intuitively appealing; the more exclusive features the goods have, the greater the benefit of consuming both, other things equal. Partial multihoming is a possible equilibrium outcome if $e \in [\underline{e}, \overline{e}]$.

Adding up the number of singlehomers and multihomers yields demand

$$x_i^M = t\frac{\mu + e}{(2t - \mu)(t + \mu)}. \tag{25}$$

Using (25), we find that prices and profits equal

$$p_i^M = \frac{(t-\mu)(\mu+e)}{2t-\mu} \text{ and } \pi_i^M = \frac{t(t-\mu)(e+\mu)^2}{(t+\mu)(2t-\mu)^2}. \tag{26}$$

From (26), it is straightforward to verify that consumer prices are lower with than without network externalities if $e > e_p \equiv 2(t - \mu)$. This reflects the fact that a larger number of exclusive features tends to increase the competitive pressure, as revealed by reaction function (15). For this reason there also exists a critical value e_π such that profits are lower with than without network externalities if $e < e_\pi$. We can thus state:

Proposition 2: *Assume partial multihoming. Compared to the case without network effects ($\mu = 0$),*

(i) prices are higher with network externalities if $e < e_p$ and lower if $e > e_p$;

(ii) profits are larger with network externalities if $e < e_\pi$ and lower if $e > e_\pi$, where $e_\pi > e_p$.

Proposition 2 shows that network externalities drive up consumer prices and profits unless the firms have sufficiently many exclusives. Note that even though prices are lower with than without network effects if $e > e_p$, network effects nonetheless have a positive impact on profits also if $e \in [e_\pi, e_p]$. The reason is that lower prices are overcompensated by larger sales in this range.

In Appendix A1 we further show:

Proposition 3: *Assume partial multihoming. Prices and profits are then hump-shaped functions of the strength of network effects.*

The hump-shaped forms of the price and profits functions are illustrated by the solid curves in Figure 2, where we have set $t = 1$.[17] To recapitulate, the firms' prices and profits are increasing in network effects in the neighborhood of $\mu = 0$ because stronger network effects generate a positive demand shift (c.f. equation (11)). To see intuitively why the firms are nonetheless harmed if the network effects become sufficiently strong, it is useful to rewrite the equilibrium price in equation (26) as

$$p^M = \frac{t-\mu}{2t-\mu}\left(\mu + e\right).$$

As μ approaches t, competition to attract consumers becomes so strong that it is *as if* the firms are (almost) undifferentiated. Similar to the Bertrand paradox, we therefore find that $p^M \to 0$ as $\mu \to t$. If the strength of the network effects is beyond a critical level, the firms would thus be better off in a counterfactual scenario with no network effects.[18]

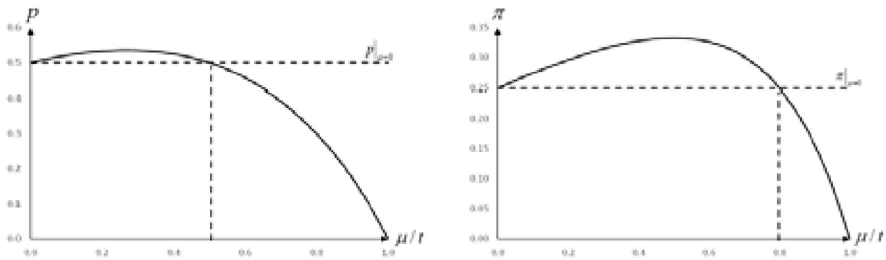

Figure 2: *Multihoming and network effects.*

3.3 Nash equilibrium with symmetric firms: singlehoming or multihoming?

An advantage of the multihoming regime from the firms' point of view is that demand will be greater than under the singlehoming regime. However, a possible disadvantage is that prices might be lower. More precisely, from equations (7) and (26) we find that the difference between the multihoming and the singlehoming price with symmetric firms is

$$p^M - p^S = -\left(t-\mu\right)\frac{2\left(t-\mu\right)-e}{2t-\mu}.$$

[17] In Figures 2-4 we have set $t = 1$.

[18] This can be seen in Figure 2 by comparing the dashed, straight lines, which shows the outcome if there are no network effects ($\mu = 0$), with the solid curves. Prices are higher with than without network effects if $\mu < 0.5$, while the same is true for maximized profits if $\mu < 0.8$.

If the goods do not have a sufficiently large number of exclusives (i.e., if $e < 2(t - \mu)$), the firms are unable to persuade any of the consumers to buy both goods unless they charge less than the singlehoming price. This leaves us with the question of the existence or even coexistence of a singlehoming and a multihoming equilibrium. We continue to limit our attention to the case with symmetric firms. It is now useful to define

$$e^*(\mu, t) = \sqrt{2t(t + \mu)} - 2\mu. \tag{27}$$

In Appendix A2 we prove the following:

Proposition 4: *Suppose that*

a) If $e(\mu, t) > \bar{e}(\mu, t)$ *a unique Nash equilibrium with complete multihoming exists.*

b) If $e(\mu, t) \in [e^*(\mu, t), \bar{e}(\mu, t)]$, *a unique Nash equilibrium with partial multihoming exists.*

c) If $e(\mu, t) \in [\underline{e}(\mu, t), e^*(\mu, t)]$, *multiple Nash equilibria exist; one with singlehoming and one with partial multihoming.*

d) If $e(\mu, t) < \underline{e}(\mu, t)$, *a unique Nash equilibrium with singlehoming exists..*

Hence, singlehoming equilibria only arise if the level of exclusive features is sufficiently low. If the level of exclusives becomes high enough, on the other hand, singlehoming equilibria will not exist. In such a case, business strategy predictions and policy recommendations based on presupposed singlehoming will be misleading.

Proposition 4 is illustrated by Figure 3, which measures the strength of the network effects (μ) on the horizontal axis and the number of exclusives (e) on the vertical axis (the red dashed curve $e^{crit}(\cdot)$ will be explained below). The upper and lower solid curves are given by equation (24); all consumers will multihome in equilibrium if $e > \bar{e}(\cdot)$, whereas all consumers will singlehome in equilibrium if $e < \underline{e}(\cdot)$. The equilibrium outcome is unique also if $e \in (e^*, \bar{e})$, where the dashed black curve represents $e^*(\cdot)$; in this area there will be partial multihoming, such that some consumers singlehome and others multihome (the number of singlehomers and multihomers are given by equation (23)). If $e \in (\underline{e}, e^*)$, on the other hand, we have multiple equilibria: none of the firms will have incentives to unilaterally deviate from an outcome where both firms charge the (relatively high) singlehoming price, and neither will any of the firms have incentives to deviate from an outcome where both firm charge the (relatively low) multihoming price.

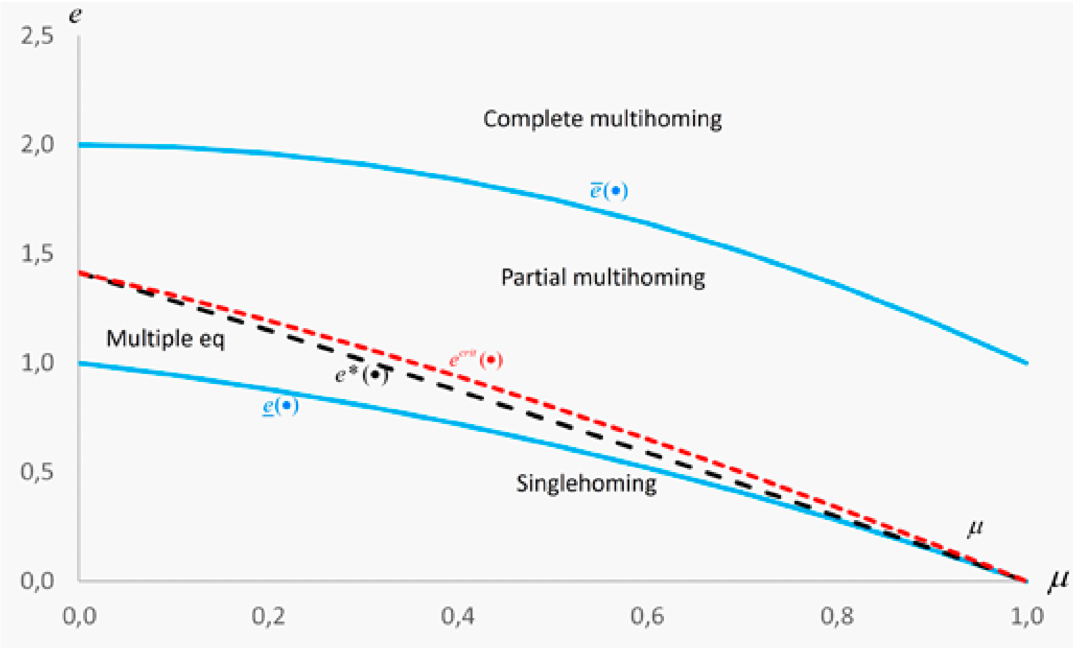

Figure 3: *Equilibrium constellations.*

Proposition 4 characterizes the possible equilibrium constellations, but does not tell us how profits under singlehoming compare to profits under multihoming. To investigate this question, we use equations (8) and (26) to find that the difference between multihoming and singlehoming profits equal

$$\pi_i^M - \pi_i^S = (t-\mu)\,\frac{2te^2 + 4t\mu e - (t-\mu)\left(4t\left(t+\mu\right)-\mu^2\right)}{2\left(t+\mu\right)\left(2t-\mu\right)^2}. \tag{28}$$

From (28) we find that $\pi_i^M - \pi_i^S > 0$ if $e > e^{crit}$, where

$$e^{crit}(\mu, t) \equiv \frac{(2t-\mu)\sqrt{2t\left(t+\mu\right)} - 2t\mu}{2t}. \tag{29}$$

Recall that profits under symmetric singlehoming are independent of the size of e, while they are increasing in e under multihoming. This explains why profits under multihoming are greater than under singlehoming if the firms have a large number of exclusives. The red dashed curve in Figure 3 draws the function $e^{crit}(\cdot)$. Profit is higher with singlehoming than with multihoming below this curve, while the opposite is true above it. In the area between $e^{*}(\cdot)$ and $e^{crit}(\cdot)$ the unique Nash equilibrium is partial multihoming, as noted in Proposition 4, even though the firms would have been better off with singlehoming, while both high-profit singlehoming and low-profit multihoming are possible in the area between $\underline{e}(\cdot)$ and $e^*(\cdot)$. More generally, it can be verified from (24), (27) and (29) that $\bar{e}(\cdot) > e^{crit}(\cdot) > \underline{e}(\cdot)$ for $\mu \in (0, t)$.

We can state:

Proposition 5: *Singlehoming would yield higher profits than partial multihoming if* $e \in (e^*(\cdot), e^{crit}(\cdot))$, *but will not arise in equilibrium. Both the high-profit singlehoming equilibrium and the low-profit partial multihoming equilibrium might arise if* $e \in (\underline{e}(\cdot), e^*(\cdot))$.

Figure 4, where $t = 1$ and $e = 1$, provides an illustration of Proposition 5. With the chosen parameter values, we find from equation (28) that profits are higher in the singlehoming regime than in the multihoming regime if $\mu < \mu' \approx 0.354$. However, from equation (27) it is clear that there is a unique equilibrium with partial multihoming if $\mu > \mu'' \approx 0.309$. From (24) we further find that partial multihoming is a possible equilibrium for all values of $\mu < t$ (there are multiple equilibria for $\mu \in [0, 0.309]$ with $e = t = 1$). Multihoming constitutes a bad equilibrium for the firms unless the network effects are sufficiently strong.

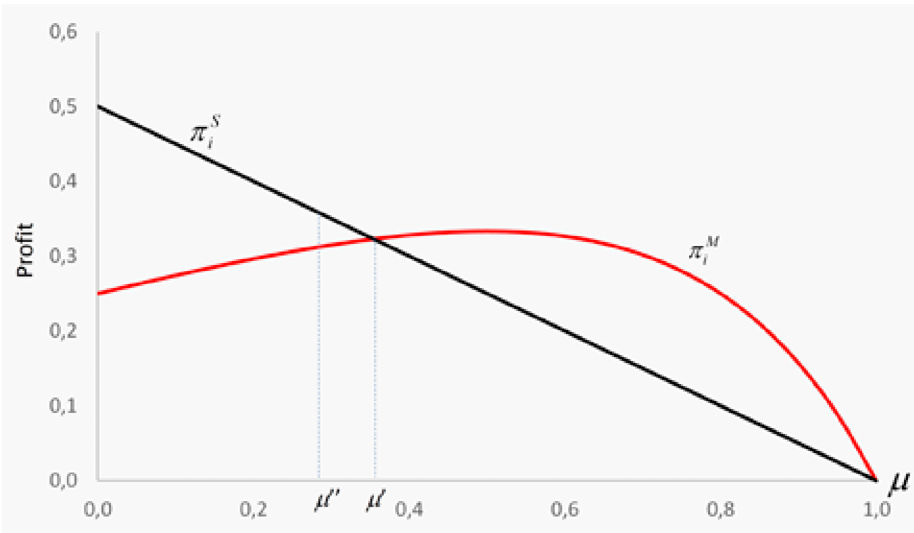

Figure 4: *Profit under SH and MH.*

3.4 Consequences of a merger between symmetric firms

Above, we have seen that competitive forces imply that firm profits are independent of the number of both overlapping and exclusive features in a symmetric singlehoming equilibrium. In the multihoming regime, on the other hand, the competing firms get paid for exclusive features, but not for overlapping. Now, suppose that the firms merge, and that the merged unit offers both goods in equilibrium and covers the market (see Appendix A3 for an analysis of when this is profitable). We restrict attention to the case of symmetric goods ($e_0 = e_1 = e$). In a singlehoming regime the merged firm will then charge a price such that the consumer at the midpoint of the Hotelling line ends up with zero net utility ($u_i = p_i$). Skipping subscripts, we can then use equations (1) and (5) to find that the price and the profit level for each good equal

$$p^S_{merged} = o + e - \frac{t}{2} + \frac{\mu}{2} \text{ and } \pi^S_{merged} = \frac{1}{2}\left(o + e - \frac{t}{2} + \frac{\mu}{2}\right). \tag{30}$$

Let us now consider an outcome with partial multihoming. The range of overlapping features the goods offer is irrelevant for the incremental utility of buying a second good. It thus follows that profits in a multihoming regime are independent of the number of overlapping features even if the firms merge and become a monopoly. Formally, this can be verified by maximizing joint profits for the two goods, $(p_0 x_0 + p_1 x_1)$, where x_i is given by equation (14). This yields

$$p^M_{merged} = \frac{e + \mu}{2} \text{ and } \pi^M_{merged} = \frac{(e + \mu)^2}{4(t + \mu)}. \tag{31}$$

We can state:

Proposition 6: *A larger number of overlapping features will increase singlehoming prices and profits if the firms have merged, but will have no effect on singlehoming prices and profits if the firms are competing. Multihoming prices and profits are independent of the number of overlapping features under partial multihoming, both with and without merger.*

Since a larger number of overlapping features has no effect on multihoming profits but strictly increases singlehoming profits for the merged unit, it is clear that the firms prefer singlehoming if the range of overlapping features is sufficiently large. More precisely, from (30) and (31) we find that

$$\pi^S_{merged} - \pi^M_{merged} = \frac{1}{2}\left[o - o^{SH}\right],$$

where

$$o^{SH} \equiv \frac{(e - t)^2}{2\,(t + \mu)}.$$

Singlehoming is consequently more profitable than multihoming for the merged unit if $o > o^{SH}$ (and none of the consumers will multihome if the merged unit sets the price equal to p^S_{merged}). Otherwise, the multihoming regime is the most profitable.

Now, let us assume that $o < o^{SH}$, such that we might have multihoming also if the firms merge. Restricting attention to the cases with partial multihoming both with and without merger, we then find that

$$p^M_{merged} - p^M = \mu \frac{e+\mu}{2(2t-\mu)} \geq 0 \text{ and } \pi^M_{merged} - \pi^M = \mu^2 \frac{(e+\mu)^2}{4(t+\mu)(2t-\mu)^2} \geq 0. \qquad (32)$$

Anderson et al. (2017) argue that the principle of incremental pricing implies that a merger will neither affect prices nor profits if some consumers multihome. Equation (32) shows that this fails to hold of there are positive network effects. The reason is that incremental pricing does not imply strategic independence if $\mu > 0$. On the contrary, with network effects there will be competition between the firms under both singlehoming and multihoming.

We now summarize the following results:

Proposition 7: *Suppose that*

(i) $o > o^{SH}$. *Then the merged unit sets* $p = p^S_{merged}$, *and all consumers singlehome for any* $e \geq 0$. *In contrast, some or all consumers would multihome if there were no merger and* $e > \underline{e}$.

(ii) $o < o^{SH}$. *With network effects, a merger leads to higher prices and higher profits even if consumers would multihome both in the merger regime and in the competitive regime.*

Note from (32) that a merger has a greater negative effect on the consumers (and a greater positive effect on the firms) the larger the number of exclusives ($d\left(p^M_{merged} - p^M\right)/de > 0$ and $d\left(\pi^M_{merged} - \pi^M\right)/de > 0$). In this sense, a merger between two network providers is more problematic from the consumers' point of view the more different the network goods are.

4 Concluding remarks

This paper shows that network effects have completely different impacts on market performance in the case of partial multihoming compared to in the case of singlehoming. Our model determines the extent of multihoming endogenously, as consumers select themselves into those who buy only one of the two goods and those who buy both. A large strand of the literature simply assumes complete singlehoming. This can generate highly misleading predictions of how competition works in network industries and how mergers might affect the market outcome.

In our model, the strength of the network effect is exogenous and identical for both firms. Given our results above, it is clear that firms may have incentives to choose technologies that allow for some, but not too strong, network effects. Thus, an interesting question is how and to which degree firms can influence the size of the network benefit for consumers. Another interesting question is whether they can implement strategies that can prevent multihoming to arise in equilibrium in cases where singlehoming would yield higher profits (see also Athey and Scott Morton, 2021, for a similar discussion in the context of two-sided markets). A third question is whether firms might find it profitable to modify their products such that network effects work at the industry level rather than at the product level (e.g., make PlayStation completely compatible with Xbox). As far as we know, this issue has not been analyzed in a setting where consumers might multihome. We leave these questions to future research.

5 Appendix

Appendix A1: Proof of Proposition 2 and Proposition 3

We use equation (26) to find that

$$\pi^M(\mu) > \pi^M(\mu = 0) \text{ if } e < e_\pi \equiv \frac{4t^2(t-\mu) + 2t(2t-\mu)\sqrt{t^2-\mu^2}}{4t^2 - 3t\mu + \mu^2}.$$

We must further show that $\overline{e} > e_\pi > e_p > \underline{e}$. $e_\pi < \overline{e}$ because

$$\overline{e} - e_\pi = (2t-\mu)\frac{\mu^3 - t\mu^2 + 2t^3 - 2t^2\sqrt{t^2-\mu^2}}{t(4t^2 - 3t\mu + \mu^2)}$$

We have $\overline{e} - e_\pi > 0$ if $\mu^3 - t\mu^2 + 2t^3 > 2t^2\sqrt{t^2-\mu^2}$, or $(\mu^3 - t\mu^2 + 2t^3)^2 > 4t^4(-\mu^2 + t^2)$. This holds if $\mu^3(t+\mu)(4t^2 - 3t\mu + \mu^2) > 0$, which is always true.[19] $e_\pi > e_p$ because

[19] The bracket $(4t^2 - 3t\mu + \mu^2)$ is positive, because $(4t^2 - 3t\mu + \mu^2) > (4t^2 - 3t^2 + \mu^2) > 0$.

$$e_\pi - e_p = 2\left(2t-\mu\right)\frac{t\sqrt{t^2-\mu^2}-\left(t-\mu\right)^2}{4t^2-3t\mu+\mu^2}$$

We thus see that $e_\pi - e_p > 0$ if $t\sqrt{t^2-\mu^2} > \left(t-\mu\right)^2$. This is equivalent to requiring $\mu\left(t-\mu\right)\left(4t^2-3t\mu+\mu^2\right) > 0,$ which is always true. $e_p > \underline{e}$ because

$$e_p - \underline{e} = \frac{\left(t-\mu\right)\left(2t-\mu\right)}{2t} > 0.$$

We now show that both prices p^M and maximized profits π^M are hump-shaped in μ. Writing both as functions of μ, we find from (26):

$$\frac{d\pi^M}{d\mu} = \frac{2t\left(e+\mu\right)\left(e+t\right)}{\left(t+\mu\right)^2\left(2t-\mu\right)^3}\left(r_1-\mu\right)\left(r_2+\mu\right),$$

where

$$r_1 = \frac{t\sqrt{3\left(4t^2-e^2\right)}-2t^2+et}{2\left(e+t\right)} \text{ and } r_2 = \frac{t\left(2t-e+\sqrt{3\left(4t^2-e^2\right)}\right)}{2\left(e+t\right)}$$

It is straight forward to show that both r_1 and r_2 are positive as long as $2t-e>0$, which is a necessary condition for partial multihoming. It follows that

$$\left.\frac{d\pi^M}{d\mu}\right|_{\mu=0} > 0.$$

At the other extreme, $\mu = t$, we have

$$r_1 - t = -t\frac{e+4t-\sqrt{3\left(4t^2-e^2\right)}}{2\left(e+t\right)},$$

where it can be shown that $e+4t-\sqrt{3\left(4t^2-e^2\right)}$ is always positive. We consequently observe that

$$\left.\frac{d\pi^M}{d\mu}\right|_{\mu=t} < 0.$$

Since there is only one value of μ for which $d\pi^M/d\mu = 0$, that is, that maximizes π^M, we can conclude that π^M is a humped-shaped function of μ. It is also immediately clear from (26) that $\pi^M(\mu=0) > \pi^M(\mu=t)$, such that the maximized profits are larger without network effects than in an otherwise similar market with network effects if the network effects are sufficiently strong. For the behavior of the price p^M with μ, we observe that

$$\frac{dp^M}{d\mu} = \frac{\left(2t + \sqrt{t\,(2t+e)} - \mu\right)\left(2t - \sqrt{t\,(2t+e)} - \mu\right)}{(2t-\mu)^2}.$$

In the same manner as above it can be shown that

$$\left.\frac{dp^M}{d\mu}\right|_{\mu=0} > 0 \text{ and } \left.\frac{dp^M}{d\mu}\right|_{\mu=t} < 0$$

in case of partial multihoming. Since $dp^M/d\mu = 0$ can hold for only one value of μ, we can also conclude for p^M that it is a humped-shaped function of μ.

Appendix A2: Proof of Proposition 4

In case of singlehoming $p_i^S = p_j^S = t - \mu, \pi_i^S = \pi_j^S = (t-\mu)/2$. Suppose that firm i deviates and multi-homes with a price $p_i' < p_i^S$:

$$p_i' = \frac{(t-\mu)(\mu+e) + \mu p_j^*}{2t} = \frac{(t-\mu)(2\mu+e)}{2t}.$$

$p_i' < p_i^S$ requires $2\mu + e < 2t$. The market share of firm i is given by

$$x_i' = \frac{\mu}{t+\mu} + t\frac{e-p_i'}{t^2-\mu^2} - \mu\frac{e-p_j^S}{t^2-\mu^2} = \frac{2\mu+e}{2(t+\mu)},$$

leading to profits of deviating to multihoming of size

$$\pi_i' = p_i' x_i' = \frac{(2\mu+e)^2(t-\mu)}{4t(t+\mu)}.$$

Firm i does not want to deviate if

$$\pi_i^S = \frac{t-\mu}{2} \geq \pi_i' = \frac{(2\mu+e)^2(t-\mu)}{4t(t+\mu)},$$

which requires

$$e \leq e^* = \sqrt{2t(t+\mu)} - 2\mu.$$

In case of multihoming

$$p_i^M = p_j^M = \frac{e}{2} + \mu\frac{2(t-\mu)-e}{2(2t-\mu)}, \pi_i^M = \pi_j^M = \frac{t(t-\mu)(\mu+e)^2}{(t+\mu)(2t-\mu)^2}.$$

Suppose that firm i deviates and single-homes with a price $p_i'' > p_i^M$:

$$p_i'' = \frac{t-\mu+p_j^M}{2} = \frac{t-\mu}{2} + \frac{e}{4} + \mu\frac{2(t-\mu)-e}{4(2t-\mu)}.$$

$p_i'' > p_i^M$ requires

$$\frac{t-\mu}{2} + \frac{e}{4} + \mu\frac{2(t-\mu)-e}{4(2t-\mu)} > \frac{e}{2} + \mu\frac{2(t-\mu)-e}{2(2t-\mu)} \Leftrightarrow$$

$$e < 4t - 3\mu.$$

The market share of firm i is given by

$$x_i'' = \frac{1}{2} + \frac{p_j^M - p_i''}{2(t-\mu)} = \frac{e+\mu}{4(2t-\mu)},$$

leading to profits of deviating to singlehoming of size

$$\pi_i'' = p_i'' x_i'' = \frac{(e+\mu)(t-\mu)(e-\mu+4t)}{8(2t-\mu)^2}.$$

Firm i does not want to deviate if

$$\pi_i^M = \frac{t(t-\mu)(\mu+e)^2}{(t+\mu)(2t-\mu)^2} \geq \pi_i'' = \frac{(e+\mu)(t-\mu)(e-\mu+4t)}{8(2t-\mu)^2},$$

which requires

$$e \geq e^{**} = \frac{(4t-\mu)(\mu+t) - 8\mu t}{7t-\mu}.$$

$\overline{e} > e^*$ warrants

$$\frac{2t^2-\mu^2}{t} > \sqrt{2t(t+\mu)} - 2\mu \Leftrightarrow 2\mu - \frac{\mu^2}{t} - \sqrt{2}\sqrt{t(\mu+t)} + 2t > 0$$

which is true as $\mu^2/t \leq \mu$, $\sqrt{t(\mu+t)} < t$ and $\sqrt{2} < 2$. $e^* \geq \underline{e}$ warrants

$$\sqrt{2t(t+\mu)} - 2\mu \geq \frac{(t-\mu)(2t+\mu)}{2t} \Leftrightarrow \frac{1}{2}\underbrace{\left(\frac{\mu^2}{t} - 3\mu + 2\sqrt{2}\sqrt{t(\mu+t)} - 2t\right)}_{\equiv f(\mu)} \geq 0.$$

Note that $f(\mu = t) = f(\mu = \mu_1) = f(\mu = \mu_2) = 0$ for $\mu_1 = -2\left(\sqrt{2}-1\right)t < 0$, $\mu_2\left(\sqrt{2}+1\right)t > t$ such that only $\mu = t$ implies a zero $f(\cdot)$ in the relevant range. Furthermore, $f'(\mu = t) = 0$ and $f''(\mu = t) = 7/(4t)$ such that $\mu = t$ is also a local minimum and $f(\mu) > 0$ must hold for all $\mu \in [0, t[$. $\underline{e} > e^{**}$ warrants

$$\frac{(t-\mu)(2t+\mu)}{2t} > \frac{(4t-\mu)(\mu+t) - 8\mu t}{7t-\mu} \Leftrightarrow \frac{\mu^3 + \mu t^2 + 6t^3 - 4\mu^2 t}{2t(7t-\mu} > 0$$

which is true as $6t^3 > 4\mu^2 t$.

Appendix A3: Conditions ensuring that both goods will be produced also under merger

Since there are no fixed costs involved in producing the goods, it can be shown that the merged unit will produce both goods as long as $t > \mu$. We have assumed that this inequality holds in Assumption 1 (i) - if it did not hold, a stable equilibrium where both goods are produced in a competitive equilibrium would not exits either. The question is therefore whether the merged unit will cover the market under singlehoming, or whether it will choose not to serve some consumers around the midpoint of the Hotelling line. Since the goods are symmetric, we can answer this question by noting that consumer $x's$ willingness to pay for good 0 is equal to $p = (v - (t-\mu)\,x)$. Maximizing profits per variety, $\pi = px$, with respect to x, we find $x = \frac{v}{2(t-\mu)}$. This yields $p = v/2$ and $\pi = v^2/\left[4\,(t-\mu)\right]$. If $x < 1/2$, which is true if $v < t - \mu$, the merged unit will thus not cover the market. Propositions 7 and 8 consequently hold for $v \geq t - \mu$.

6 Literature

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Competition between firms that sell incompatible varieties of network products might be fierce, because it is important for each of them to attract a large number of users. The literature therefore predicts that stronger network effects decrease prices and profits. We show that this prediction hinges critically on an implicit or explicit assumption that each consumer buys only one of the varieties offered in the market (singlehoming consumers). We show that multihoming (some consumers buy more than one variety) may arise endogenously if the number of exclusive features that each variety offers is sufficiently high. In sharp contrast to the conventional prediction under consumer singlehoming, we further show that both prices and profits could increase in the strength of the network effects if (some) consumers multihome. However, this does not necessarily imply that profits are higher under multihoming than under singlehoming. On the contrary, multihoming might constitute a prisoner's dilemma for the firms, in the sense that they could make higher profits if each consumer bought only one of the varieties.

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