Working Paper No. 68/03

Pricing in Iterative Combinatorial Auctions

by

Mette Bjørndal
Kurt Jørnsten

THE ECONOMICS OF TELECOMMUNICATIONS
This report is one of a series of papers and reports on telecommunication economics published by the Institute for Research in Economics and Business Administration (SNF) as part of its telecommunication economics program. The main focus of the research program is to study the deregulation process of the telecommunication industry, and the economic and organizational consequences of changes in markets, technology and regulation. Being started in 1992, the program is now in its fourth period ending in 2005/2006. The program is financed by Telenor AS.

SNF- project No. 7880: "Supply chain management in the telecom sector"
The project is financed by Telenor AS

INSTITUTE FOR RESEARCH IN ECONOMICS AND BUSINESS ADMINISTRATION
BERGEN, DECEMBER 2003
ISSN 1503-2140

SNF Working Paper No. 68/2003

Abstract. Our objective is to find prices on individual items in a combinatorial auction that support the optimal allocation of bundles of items, i.e. the solution to the winner determination problem of the combinatorial auction. The item-prices should price the winning bundles according to the corresponding winning bids, whereas the bundles that do not belong to the winning set should have strictly positive reduced cost. I.e. the bid on a non-winning bundle is strictly less than the sum of prices of the individual items that belong to the bundle, thus providing information to the bidders why they are not in the winning set. Since the winner determination problem is an integer program, in general we cannot find a linear price-structure with these characteristics. However, integer programming duality can be used to obtain this kind of price-information. Normally, it is computationally to expensive to derive the integer programming dual function, but in an iterative combinatorial auction it might be worth to do it since the information provided to the bidders from the non-linear dual function is of great importance for the bidders. Throughout, the ideas are illustrated by means of numerical examples.

Introduction

In some auctions/markets, a participant's valuation of an object depends significantly on which other objects the participant acquires. Objects can be substitutes or complements, and the valuation of a particular bundle of items may not be equal to the sum of the valuations of the individual items, i.e. valuations are not additive. This may be represented by letting bidders of the auction have preferences not just for particular items, but for sets or bundles of items as well. In this setting, economic efficiency is increased by allowing bidders to bid on combinations of objects, which is exactly what a combinatorial auction does.

A recent survey of combinatorial auctions is provided by de Vries and Vohra (2000), also an excellent overview is given by Parkes (2001). In the literature, there are a number of examples of combinatorial auctions, ranging from the allocation of rights to radio frequencies (FCC (1994)), auctions for airport time slots (Rassenti et al. (1982)), railroad segments (Brewer (1999)) and delivery routes (Caplice (1996)). Bundle pricing (Hanson and Martin (1990)) and the effects of discounts on vendor selection (Moore et al. (1991)) can also be analyzed within this framework.

1 The Winner Determination Problem

Given a set of bids for subsets of objects, selecting the winning set of bids is denoted "the winner determination problem". This problem can be formulated as an integer programming problem. Let N be the set of bidders, M the set of m distinct objects, and S a subset of M. Agent j's ($j \in N$) bid for bundle S is denoted by $b^j(S)$, and we let

$$b(S) = \max_{j \in N} b^j(S)$$

The binary variable x_S is equal to 1 if the highest bid on S is accepted, and 0 otherwise. The winner determination problem can then be formulated as

$$\begin{array}{lll} \text{(IP1)} \quad \max & \sum_{S \subset M} b(S) \cdot x_S & \\ \text{s.t.} & \sum_{S \ni i} x_S \leq 1 & \forall i \in M \\ & x_S = 0/1 & \forall S \subset M \end{array}$$

In some formulations of the winner determination problem, there is also a restriction stipulating that every agent/bidder can only receive at most one bundle. If we let binary variable $x^j(S)$ be equal to 1 if agent j receives bundle S and 0 otherwise, the corresponding formulation of the winner determination problem is the following

$$\begin{array}{lll} \text{(IP2)} \quad \max & \sum_{S \subset M} \sum_{j \in N} b^j(S) \cdot x^j(S) & \\ \text{s.t.} & \sum_{S \ni i} \sum_{j \in N} x^j(S) \leq 1 & \forall i \in M \\ & \sum_{S \subset M} x^j(S) \leq 1 & \forall j \in N \\ & x^j(S) = 0/1 & \forall j \in N,\, S \subset M \end{array}$$

In both formulations the objective function maximizes the "revenue", i.e. the value of the bids, whereas the first set of constraints requires that no object can be assigned to more than one bidder. The second set of restrictions in (IP2) guarantees that no agent is assigned more than one bundle. An alternative interpretation of the maximization problems is the following: If bidders submit their true values, i.e. bid their reservation prices on different bundles, implying that $b^j(S) = v^j(S)$, for all $j \in N$ and $S \subset M$, the solution to the winner determination problem represents the efficient allocation of indivisible objects in an exchange economy.

Formulation (IP1) is valid for the winner determination problem in case of superadditive bids, i.e. if

$$b^j(A) + b^j(B) \leq b^j(A \cup B) \qquad \forall j \in N,\; A, B \subset M \text{ and } A \cap B = \emptyset$$

In case of substitutes, as shown in de Vries and Vohra (2000) dummy goods can be introduced to make the formulation valid, or the more general formulation (IP2) can be used. In any case, the formulation of the winner determination problem is an instance of the set packing problem (SPP). The linear programming relaxation of the SPP produces integer solutions in a number of cases (ref. de Vries and Vohra (2000)). We will however, focus on instances where the LP-relaxation gives fractional solutions. In general, the SPP belongs to the class of NP-hard problems, and is closely related to set partitioning and set covering problems.

Since in general the LP-relaxation produces fractional solutions, it is obvious that a set of market clearing linear prices need not exist for a combinatorial auction. This has led to the development of a number of stronger formulations of the winner determination problem. Bikchandani and Ostroy (1999, 2000) have presented two stronger formulations of (IP2). The first one is obtained by introducing artificial variables $y(k)$, and replacing the set of constraints requiring that each agent can obtain at most one bundle with the alternative set of constraints

$$\text{(LP1)} \qquad \sum_{j \in N} x^j(S) \leq \sum_{k \ni S} y(k) \qquad \forall S \subset M$$

$$\sum_{k \in K} y(k) \leq 1$$

where K is the set of all possible partitions, or "bundlings", of the items in M, and $k \ni S$ indicates that bundle S is represented in partition k.

This lead to a stronger problem formulation, in the sense that some of the fractional solutions that are feasible in the LP-relaxation of the weaker formulation, are cut off. However, the linear programming relaxation of this problem can still produce fractional optimal solutions. Another problem with this formulation is that the value of the dual is the sum of the maximal utility to each agent with bundle prices $p(S)$, plus the auctioneers maximal revenue. The use of *bundle* prices makes the price mechanism more complicated, and we are in this paper looking for a simpler evaluation scheme, avoiding bundle prices.

In the strongest formulation of the winner determination problem, the disaggregation goes even further, by replacing the constraints discussed above, with the constraints

$$\text{(LP2)} \qquad x^j(S) \leq \sum_{k \ni [j,S]} y(k) \qquad \forall j \in N, S \subset M$$

$$\sum_{k \in K'} y(k) \leq 1$$

where K' is the set of all possible *agent*-partitions, i.e. all possible combinations of "bundlings" and their allocation to different agents, and $k \ni [j, S]$ indicates that agent-partition k contains bundle S, which is allocated to agent j.

This formulation possesses the integrality property and hence, the linear programming relaxation is integer. However, the value of the dual becomes even more complicated since it is the maximal utility to each agent with bundle prices $p^j(S)$ plus the auctioneers maximal revenue over all feasible allocations at the prices. Note that the bundle prices $p^j(S)$ are non-anonymous or discriminatory bundle prices, i.e. every agent may face a unique vector of bundle-prices, making the evaluation even more complicated.

The problem as we see it with the two stronger formulations and their duals, is that they lead to non-linear price structures, with prices of objects and prices for bundles, that make it difficult to use them in a market mechanism design. In this paper we will present an alternative set of non-linear prices, that can be used to evaluate bids and give information back to the bidders/agents that can be used to determine the prospects of a bid-increase, or explain easily why a particular bid did not win. What we suggest is the use of integer programming duality to derive a non-linear price function. Although complicated to derive, the price function derived from integer programming duality has the desired properties, i.e. prices only the original constraints in the winner determination problem. Also in an iterative combinatorial auction we will normally solve a sequence of related problems which makes the computational effort of deriving the dual function worthwhile. This is so since the alternative is to either calculate the true deficits for all non-winning bids, which requires the solution of a large number of integer programming problems or to use pseudo-dual price information which might be misleading.

2 Pricing in Combinatorial Auctions

The objective of this paper is not to focus on solution methods for the winner determination problem, but rather to find prices on individual items that support the optimal allocation of bundles of items. By “support”, we mean that the prices on the individual objects should price the winning bundles according to the winner bids, whereas the bundles that do not belong to the winning set, should have strictly positive reduced cost, i.e.

bid on non-winning bundle	$<$	Σ prices of individual objects that belong to bundle

Prices with these characteristics will provide information to the bidders why they are not in the winning set, and this information may be used in a specific market design. Since the winner determination problem is an integer problem, in general, we will have to consider non-linear price structures.

It is only possible to find a single price-vector that excludes all non-winning bids if 1) the LP-relaxation of the winner determination problem has an integer solution, and 2) the LP-relaxation has a unique dual solution such that every non-winning bundle has reduced cost (RC) > 0. As will be illustrated in the next sections by means of a simple example, it seems to be difficult to find a unique price-vector with the characteristics searched for. Therefore, in this article, we suggest the use of integer programming duality to obtain this kind of price-information in terms of a non-linear price function.

There have been many suggestions for calculating price information in combinatorial auctions. Rassenti et al suggest the use of two pseudo-dual pprograms in order to define bid rejection prices. De Martini et al and Milgrom present alternative ways of calculating pseudo-diual prices. Bjørndal and Jørnsten suggest the use of a restricted linear program and the corresponding set of dual sulutions as an alternative. The convex set of price vectors, multiple linear prices, in this approach has the property that the reduced costs for all non-winning bids are positive for at least one of the price vectors. The set of multiple linear prices can be viewed as anon-linear pricing system.

All these alternatives for pricing are used as a proxy for calculating the true shortfall. Since shortfall calculations involves solving a hugh number of integer programs this is an extremely costly method. However, since this is the case it might be a feasible alternative to solve the winner determination problem in such a way that we also get information that yields the non-linear price function associated with the integer programming dual.

In the illustrative example we will use the first formulation of the winner determination problem presented (IP1), i.e. without the restriction that an agent can receive at most one bundle. However, we give other examples using the alternative formulation (IP2), taken from Parkes (2001) and extensions thereoff, and illustrate the applicability of the non-linear pricing scheme given by the integer programming dual price function.

3 Integer Programming Duality

In the paper by Wolsey, it has been shown how to formulate the dual of an integer linear programming problem. The basic idea is to use dual price functions instead of dual prices. Consider the primal integer linear program z=sup cx

s.t Ax•b
x•0, integer

Let F_+^m be the set of nondecreasing functions $F : R^m \rightarrow R^* = R \cup \{-\infty. + \infty\}$. A function $F \in F_+^m$ is called superadditive if $F(b_1 + b_2) \geq F(b_1) + F(b_2) \; \forall \; b_1, b_2 \in R^m$.
Let **F** = $\{F \in F_+^m \,|\, F \; is \sup erad ditive \; and \; F(0) \geq 0\}$. We call F the set of dual price functions. We can then define the dual of the integer linear program as

w=inf $F(b)$

s.t $F(a_j) \geq c_j$ j=1,...,n

$F \in$ **F**

Wolsey shows how an integer programming dual function can be derived by solving the integer progrram either with a cutting plane or a Branch and Bound technique. In the following we will use price functions derived from solving the primal integer program using a cutting plane technique**.**

Note that the winner determination problem is a binary integer program. However, the binary constraints can be replaced by rephrasing the problem and replacing the binary constraints $x^j(S) = 0/1$ with $x^j(S) \geq 0$ and integer . This is so since the requirement that the variables has to be less than one is already stated in the other constraints. Hence the reformulated winner determination problem is

(IP2`) max $\sum_{S \subset M} \sum_{j \in N} b^j(S) \cdot x^j(S)$

s.t. $\sum_{S \ni i} \sum_{j \in N} x^j(S) \leq 1 \qquad \forall i \in M$

$\sum_{S \subset M} x^j(S) \leq 1 \qquad \forall j \in N$

$x^j(S) \geq 0 \qquad \forall j \in N, S \subset M$

Which has a corresponding dual with dual price functions F .

4 Examples

In the first example, we assume that the following 9 bids have been handed in for different combinations of 7 objects, A-G:

Bid	17	10	10	9	20	12	4	15	26
Object									
A	1	1	1		1		1	1	
B		1	1					1	
C		1			1	1		1	1
D				1					
E	1		1		1	1			1
F		1	1				1		1
G	1			1	1			1	1

The interpretation of the table is as follows: the bid of 17 includes objects A, E and G, the next bid of 10 is on objects A, B, C and F, etc.

The winner determination problem of the combinatorial auction can be formulated as the following set packing problem, which is an instance of (IP1):

$$
\begin{array}{llllllllllll}
\max & 17x_1 + & 10x_2 + & 10x_3 + & 9x_4 + & 20x_5 + & 12x_6 + & 4x_7 + & 15x_8 + & 26x_9 & \\
\text{s.t.} & x_1 + & x_2 + & x_3 + & & x_5 + & & x_7 + & x_8 & & \le 1 \\
 & & x_2 + & x_3 + & & & & & x_8 & & \le 1 \\
 & & x_2 + & & & x_5 + & x_6 + & & x_8 + & x_9 & \le 1 \\
 & & & & x_4 & & & & & & \le 1 \\
 & x_1 + & & x_3 + & & x_5 + & x_6 + & & & x_9 & \le 1 \\
 & & x_2 + & x_3 + & & & & x_7 + & & x_9 & \le 1 \\
 & x_1 + & & & x_4 + & x_5 + & & & x_8 + & x_9 & \le 1 \\
 & x_i & \text{binary} & & & & & & & &
\end{array}
$$

The optimal integer solution has a value of 26, $x_9 = 1$ and all other variables are zero. In the following, we will consider various potential price-structures, based on 1) the LP-relaxation and 2) using sensitivity analysis together with linear programming. In the next section we will consider the use of IP marginal values.

1) LP-relaxation

If we relax the integer restrictions on the variables and solve the corresponding linear program, we obtain a fractional solution with value 26.5, where $x_1 = x_2 = x_9 = 0.5$ and all other variables are equal to zero. The shadow prices for the seven constraints are given by the vector (0.5, 0, 6, 0, 7.5, 3.5, 9) implying reduced costs for the 9 bundles that have been bid on equal to (0, 0, 1.5, 0, 3, 1.5, 0, 0.5, 0). However, this dual solution is not very useful in combinatorial auction terms, since it produces reduced cost equal to zero for a number of inferior bids. This is so for bids 1 and 2, that is part of the fractional solution, but it is also so for bids 4 and 7, which are inferior even in the LP-relaxation.

Note that there exist multiple dual solutions to the linear program. The alternative dual solutions are (0.5, 0, 5, 0, 7, 4.5, 9.5) and (0.5, 0, 6, 0, 6, 3.5, 10.5), with reduced costs for the nine bundles equal to (0, 0, 2, 0.5, 2, 0, 1, 0, 0) and (0, 0, 0, 1.5, 3, 0, 0, 2, 0), respectively. We notice that for all the alternative dual solutions, several inferior bids have reduced cost equal to zero, but not necessarily in all the alternative solutions. Only the inferior alternatives consisting of bids 1 and 2 do not get any indications of the inferiority of the value of their bids, which is reasonable since they are part of the fractional LP-solution.

2) Cutting Planes and Dual Functions

We know that a price-system that works for a combinatorial auction must in general be non-linear. Such a non-linear price-system can be derived using a cutting plane or branch-and-bound technique when solving the winner determination problem. For our example we will use a cutting plane approach to generate a non-linear price-system. By adding constraints 1, 3 and 5, dividing by two, and rounding down, the following cutting plane is derived:

$$x_1 + x_2 + x_3 + x_5 + x_6 + x_8 + x_9 \leq 1$$

If we append this cutting plane to the winner determination problem and solve the new linear programming relaxation, we get the solution $x_9 = 1$ with value 26, shadow prices (0, 0, 0, 0, 0, 4, 9, 13), and reduced costs (5, 7, 7, 0, 2, 1, 0, 7, 0). I.e. bundles 1 and 2 are priced out, but not bundles 4 and 7.

There are however, multiple dual solutions, as shown in the table below:

Dual Solutions (π,μ)

Constraint	1	2	3	4	5	6	7
A	0	0	0	0	0	0	0
B	0	0	0	0	0	0	0
C	0	5	0	4	5	0	4
D	0	0	0	0	0	0	0
E	0	7	0	7	6	0	6
F	4	4	4	4	4	5	5
G	9	9	10	10	10	9	9
Cut	13	1	12	1	1	12	2

Each of these dual solutions gives rise to a nonlinear price-function of the form

$$\mathbf{F}(\mathbf{a}_j) = \boldsymbol{\pi}\mathbf{a}_j + \mu \cdot \lfloor (a_{1j}+a_{3j}+a_{5j})/2 \rfloor$$

where $\mathbf{a}_j$ is the coefficient vector of the jth bundle, $\boldsymbol{\pi}$ is the vector of shadow prices for the original constraints A-G, μ is the shadow price of the cut, a_{ij} is the coefficient in the ith row of bundle j, and $\lfloor u \rfloor$ represents the greatest integer less than or equal to u. However, none of the above dual non-linear price-functions alone will produce a positive reduced cost for all the unsuccessful bids. Nevertheless, if we take the maximum reduced costs produced by the dual solutions we get the reduced costs (5,7,7,1,2,1,1,7,0). This should be compared with the true shortfall costs which are (9,7,7,1,6,1,1,11,0). Hence the dual price functions give us underestimates of the true shortfall costs. The estimates are exact for 5 out of the eight bids in the example. Also note that not all dual solution stated above are needed.

In his Ph.D. thesis, Parkes (2001) presents a set of illustrative examples. One of the examples is as follows:

	Bundles						
	A	B	C	AB	BC	AC	ABC
Bidders							
Agent 1	60	50	50	200	100	110	250
Agent 2	50	60	50	110	200	100	255
Agent 3	50	50	75	100	125	200	250

The numbers in the table give the bids for the various bundles, from each agent or bidder. In this example it turns out that the linear programming relaxation of this instance of (IP2) has the optimal solution x^1(AB) = x^2(BC) = x^3(AC) = 0.5 with value 300, whereas the integer solution and hence optimal solution to the winner determination problem (IP2) is x^1(AB) = x^3(C) = 1, with value 275.
Also in this example only one cutting plane is needed to derive the super additive price function. However, the dual linear program obtained from the primal with the cutting plane added possesses massive dual degeneracy. Several of these solutions are needed to obtain the desired reduced costs result. It should be noted that in order to obtain the desired result giving the maximum reduced costs only lone linear program has to be solved for each non-winning bid.

A second example from Parkes (2001) is given by the following table:

	Bundles		
	A	B	AB
Bidders			
Agent 1	0	0	3
Agent 2	2	2	2

In this example, the optimal solution to (IP2) is given by x^1(AB) = 1, while the LP-relaxation gives x^1(AB) = x^2(A) = x^2(B) = 0.5. Only the strongest formulation (LP2) possesses the integrality property, and the non-anonymous / discriminatory vectors of bundle-prices resulting from the LP-relaxation of the strong formulation is p^1 = (0, 0, 2.5) and p^2 = (2, 2, 2). These prices should be compared to price function obtained by adding one cutting plane including bids AB for bidder one and bids A,B and AB for player two are included. The dual function which gives us an anonymous price function of the form
$\mathbf{F}(\mathbf{a}_j) = \boldsymbol{\pi}\mathbf{a}_j + \mu \cdot \lfloor (a_{1j}+a_{2j}+a_{4j})/2 \rfloor$ In order to get the reduced cost result (1,1,0,1,1,1) all the 4 potential dual price functions are needed.

The final example is derived from the first Parkes example above by adding one more object and one more bidder. As in the ordinary Parkes example only a few of the combinations have more than additive value for the bidders.

The bids are for

	A	B	C	D	AB	ABC	ABCD
Agent 1	60	50	50	50	200	285	336

.

	A	B	C	D	AC	ABD
Agent 2	50	60	50	50	200	285

	A	B	C	D	AD	ACD
Agent 3	50	50	75	50	200	286

	A	B	C	D	CD	BCD
Agent 4	50	50	50	75	149	250

Only the non-additive bids are shown in the table. In this example four cutting planes are needed in order to derive the price function. The dual linear program possesses also here massive dual degeneracy. However only a very small number of them are needed to give us the maximal reduced costs and less than 20% of the reduced costs differ from the true shortfall costs. It should also be noted that the three cutting planes added first are redundant when the last cutting plane is appended.

The dual function derived is of the same form as the one shown above although more complicated.

$$\mathbf{F}(\mathbf{a}_j) = \boldsymbol{\pi}\mathbf{a}_j + \pi_9 \left\lfloor \frac{a_{1j} + a_{2j} + a_{3j} + a_{4j}}{3} \right\rfloor +$$

$$\pi_{10} \left\lfloor \frac{a_{1j} + a_{2j} + \left\lfloor \frac{a_{1j} + a_{2j} + a_{3j} + a_{4j}}{3} \right\rfloor}{2} \right\rfloor +$$

$$\pi_{11} \left\lfloor \frac{a_{1j} + a_{3j} + \left\lfloor \frac{a_{1j} + a_{2j} + \left\lfloor \frac{a_{1j} + a_{2j} + a_{3j} + a_{4j}}{3} \right\rfloor}{2} \right\rfloor}{2} \right\rfloor +$$

$$\pi_{12} \left\lfloor \frac{a_{1j} + a_{4j} + \left\lfloor \frac{a_{1j} + a_{3j} + \left\lfloor \frac{a_{1j} + a_{2j} + \left\lfloor \frac{a_{1j} + a_{2j} + a_{3j} + a_{4j}}{3} \right\rfloor}{2} \right\rfloor}{2} \right\rfloor}{2} \right\rfloor$$

The examples given above are shows that there might be a potential for using integer programming duality and non-linear price function in market design for multi-unit auctions

5 Conclusions and suggestions for future research

In this paper, we have suggested the use of non-linear price functions derived from the use of integer programming duality to handle the pricing problem in combinatorial auctions. Although complicated and computationally hard to obtain, we argue that in combinatorial auctions the price functions might be beneficial to use since the alternative, to calculate the true deficit for a loosing bid is computationally expensive. In an iterative auction market design the use and calculation of the non-linear price functions can be the best alternative.

Concerning future research, we are currently designing an experiment in which students act as bidders in a combinatorial auction setting and are presented various kind of price information in each round, pseudo-dual information and/or price functions. We are interested to see how the students handle different price information and how this information affect their bidding behavior.

Another line of research is to investigate how price functions derived by using Branch and Bound or Branch and Cut approaches that yield other types of super additive price functions are met by the bidders. Are there observable differences in the bidding behavior given different types of price function?

6 References

Bikchandani, Sushil / Ostroy, Joseph M. (1999): The Package Assignment Model. Technical Report, Anderson Graduate School of Management and Department of Economics, U.C.L.A.

Bikchandani, Sushil / Ostroy, Joseph M. (2000): Ascending Price Vickrey Auctions. Technical Report, Anderson Graduate School of Management and Department of Economics, U.C.L.A.

Bjørndal M., Jørnsten K. (2001): An Analysis of a Combinatorial Auction in Klose A. et al Quantitative Approaches to Distribution Logistics and Supply Management Springer Lecture Notes in Economics and Mathematical Systems Vol. 519

Brewer, P. J. (1999): Decentralized Computation Procurement and Computational Robustness in a Smart Market. *Economic Theory*. 13;41-92.

Caplice, C. G. (1996): An Optimization Based Bidding Process: A New Framework for Shipper-Carrier Relationships. Thesis, Department of Civil and Environmental Engineering, School of Engineering, MIT.

DeMartini C., Kwasinca A.M., Ledyard J.O., Portes D. (1999): A New Improved Design for Multi-Object Iterative Auctions Social Science Working Paper 1054 California Institute of Technology

de Vries, Sven / Vohra, Rakesh (2000): Combinatorial Auctions: A Survey. Technical Report, Kellogg Graduate School of Management, Northwestern University.

Dunford M., Durbin M., Hoffman K., Menon D., Sultana R., Wilson T. (2001): Issues in Scaling Up the 700 MHz Auction Design Wye River Conference II October 27, 2001

FCC Auction, Broadband Personal Communication Services, Bidder Information Package (1994): Washington DC.

Hanson, W. / Martin, R. K. (1990): Optimal Bundle Pricing. *Management Science*, 36;155-174.

Milgrom P. (2001): Putting Auction Theory to Work: Ascending Auctions with Pacakge Bidding Drafft January 8, 2001

Moore, E. W. / Warmke, J. M. / Gorban, L. R. (1991): The Indispensable Role of Management Science in Centralizing Freight Operations at Reynolds Metals Company. *Interfaces*, 21;107-129.

Parkes, David C. (2001): Iterative Combinatorial Auctions: Achieving Economic and Computational Efficiency. Doctoral Dissertation, Computer and Information Science, University of Pennsylvania.

Rassenti, S. J. / Smith, V. J. / Bulfin, R. L. (1982): A Combinatorial Auction Mechanism for Airport Time Slot Allocation. *Bell Journal of Economics*, 13;402-417.

Wolsy L. (1981): Integer Programming Duality: Price Functions and Sensitivity Analysis
Mathematical Programming 20 (2)

PUBLIKASJONER INNEN TELEØKONOMI 1998 –

Mette Bjørndal Kurt Jørnsten	Pricing in Iterative Combinatorial Auctions SNF-Working paper No. 68/2003, Bergen
Tommy S. Gabrielsen Steinar Vagstad	Why is on-net traffic cheaper than off-nett traffic? Access markup as a collusive device and a barrier to entry SNF-Working paper No. 63/2003, Bergen
Herbjørn Nysveen Helge Thorbjørnsen Per E. Pedersen	*Effects of WebTV on Consumer-Brand Relationships* SNF-Working paper No. 45/2003, Bergen
Leif Jarle Gressgård Leif B. Methlie Inger Stensaker	*Mobile Internet Services:* *Integration Models and Structural Determinants* SNF-Report No. 36/2003, Bergen
Hagen, Kåre P. Ola Nafstad	*Telepolitikk i Europa* SNF-rapport nr. 27/2003, Bergen
Per E. Pedersen Herbjørn Nysveen Helge Thorbjørnsen	*Identity expression in the adoption of mobile services:* *The case of multimedia messaging services* SNF-Working paper No. 26/2003, Bergen
Herbjørn Nysveen Per E. Pedersen Helge Thorbjørnsen	*Using mobile services to strengthen brand relationships:* *The effects of SMS and MMS channel additions on brand knowledge, satisfaction, loyalty and main channel use.* SNF-Report No. 22/2003, Bergen
Stemsrudhagen, Jan Ivar	*The structure of balanced scorecard:* *Empirical evidence from Norway.* SNF-Working paper No. 24/2003, Bergen
Stemsrudhagen, Jan Ivar	*The balanced scorecard and leadership:* *The system's (ir)relevance in an uncertain world.* SNF-Working paper No. 23/2003, Bergen
Spiller, Pablo T. Ulset, Svein	*Why Local Loop Unbundling Fails?* SNF-Working paper No. 87/2002, Bergen
Ulset, Svein	*The Rise and Fall of Global Network Alliances.* *Success or Failure?* SNF-Working paper No. 86/2002, Bergen
Ulset, Svein	*The Disintegration of Network Externalities Industries. The Computer and the Telecommunications Equipement Industries.* SNF-Working paper No. 85/2002, Bergen
Ulset, Svein	*Restructuring Diversified Telecom Operators.*

	SNF-Report No. 57/2002, Bergen
Ulset, Svein	*Corporate versus Contractual Knowledge Transfer to Foreign Operations. The Case of Mobile Network Operators.* SNF-Report No. 56/2002, Bergen
Andersson, Kjetil Fjell, Kenneth Foros, Øystein	*Are TV-viewers and surfers different breeds?* *Broadband demand and asymmetric cross-price effects.* SNF-Working paper No. 14/2003, Bergen
Stemsrudhagen, Jan Ivar	*The causality of strategic control:* *three-level construction of causality.* SNF-Working paper No. 77/2002, Bergen
Barros, Pedro Kind, Hans Jarle Nilssen, Tore Sørgard, Lars	*Media competition when the audience dislikes advertising:* *A theory of vertical alliance on the Internet.* SNF-Working paper No. 79/2002, Bergen.
Pedersen, Per E. Nysveen, Herbjørn	*The adoption of a mobile parking service: Instrumentality and expressiveness.* SNF-Working paper No. 76/2002, Bergen.
Foros, Øystein Kind, Hans Jarle	*Økonomiske aspekter ved Internetts markedsstruktur.* SNF-arbeidsnotat nr. 74/2002, Bergen.
Nysveen, Herbjørn Pedersen, Per E.	*Individual and cross media communication in converging media environments: A review of research on the antecedents and effects of communication using various media in marketing contexts.* SNF-Working paper No. 65/2002, Bergen
Pedersen, Per E. Nysveen, Herbjørn Thorbjørnsen, Helge	*The adoption of mobile services: A cross service study.* SNF-Report No. 31/2002, Bergen
Jensen, Sissel Sannarnes, Jan Gaute	*Prisregulering av et telenettverk: Prinsipiell analyse med særlig fokus på avkastnings- og pristaksregulering.* SNF-Report No. 37/2002, Bergen
Stemsrudhagen, Jan Ivar	*Creative SMC in a Norwegian ICT company: managerial interaction with sources of realised strategies.* SNF-Report No. 26/2002, Bergen
Stemsrudhagen, Jan Ivar	*Strategic control systems in action: managers' use of information devices in creational and institutionalising work.* SNF-Working paper No. 38/2002, Bergen
Pedersen, Per E.	*The adoption of text messaging services among Norwegian teens: development and test of an extended adoption model.*

	SNF-Report No. 23/2002, Bergen.
Foros, Øystein Kind, Hans Jarle Sand, Jan Yngve	*Do Incumbents Have Incentives to Degrade Interconnection Quality in the Internet?* SNF-Working paper No. 22/2002, Bergen.
Fjell, Kenneth	*Elasticity based pricing rules in telecommunications – a cautionary note* SNF-Working paper No. 19/2002, Bergen.
Kristoffersen, Siri Hege	*Prising og uforstand – ein analyse av prissetjing med ufullstendig informerte konsumentar i den norske marknaden for mobiltelefoni* SNF-rapport nr. 54/2001, Bergen.
Pedersen, Per E.	*Adoption of mobile commerce: An exploratory analysis* SNF-Report No. 51/2001, Bergen.
Gabrielsen, Tommy Stahl Vagstad, Steinar	*On how size and composition of customer bases affect equilibrium in a duopoly with switching cost* SNF-Working paper No. 26/2001, Bergen.
Berge, Laila Kristin	*Prisdiskriminering i oligopol -en teoretisk gjennomgang og eksempler fra telemarkedet* SNF-rapport nr. 13/2001, Bergen.
Bjørndal, Mette Jørnsten, Kurt	*An Analysis of a Combinatorial Auction.* SNF-Working paper No. 19/2001, Bergen.
Vagstad, Steinar	*Price discrimination with uncertain consumers.* SNF-Working Paper No. 12/2001, Bergen.
Gooderham, Paul N. Ulset, Svein	*Knowledge transfer between parent and developing country subsidiaries. A Conceptual Framework.* SNF-Working Paper No. 08/2001, Bergen.
Staahl Gabrielsen, Tommy Vagstad, Steinar	*Second-period pricing in a duopoly with switching costs: the effect of size and composition of customer bases.* SNF-Working Paper No. 09/2001, Bergen.
Pedersen, Per E.	*Adopsjon av mobil handel (m-handel) -en forstudie* SNF-rapport nr. 07/2001, Bergen.
Hundsnes, Tore	*The Management of Complexity* SNF-Report No. 58/2000, Bergen.
Knivsflå, Kjell Henry Rud, Linda Sættem, Frode	*Kapitalnettverk for små og mellomstore bedrifter* SNF-rapport nr. 72/2000, Bergen.

Foros, Øystein
Strategic Investments with Spillovers, Vertical Integration and Foreclosure in the Broadband Access Market
SNF-Working Paper No 75/2000, Bergen

Lommerud, Kjell Erik
Sørgard, Lars
Entry in Telecommunication: Customer Loyalty, Price Sensitivity and Access Prices.
SNF-Working Paper No. 62/2000, Bergen.

Munkeby, Trygve Osvoll
Konkurransen mellom norske internetleverandørar.
SNF-rapport nr. 48/2000, Bergen.

Pedersen, Per E.
Methlie, Leif B.
Tjenesteintegrering i elektronisk handel.
SNF-rapport nr. 21/2000, Bergen.

Methlie, Leif B.
Pedersen, Per E.
MAP-IT: A Model of intermediary Integration Strategies in online Markets.
SNF-Working Paper No. 26/2000, Bergen.

Gabrielsen, Tommy Staahl
Vagstad, Steinar
Consumer heterogeneity and pricing in a duopoly with switching costs.
SNF-Working Paper No. 25/2000, Bergen.

Ulset, Svein
Gooderham, Paul
Internasjonalisering av telesektoren. Generelle lærdommer og spesielle utfordringer for Telenor.
SNF-arbeidsnotat nr. 16/2000, Bergen.

Ulset, Svein
Virtual Network Operation. Strategy, Structure and Profitability.
SNF-Working Paper No. 15/2000, Bergen.

Foros, Øystein
Kind, Hans Jarle
Sørgard, Lars
Access Pricing, Quality Degradation and Foreclosure in the Internet.
SNF arbeidsnotat nr. 12/2000, Bergen.

Foros, Øystein
Sand, Jan Yngve
Asymmetrisk regulering innen telekommunikasjon.
SNF særtrykk nr. 03/2000, Bergen.

Ulset, Svein
Ekspansive teleselskap. Finansiering, organisering og styring.
SNF-rapport nr. 64/1999, Bergen.

Sannarnes, Jan Gaute
Ulike reguleringsregimer i telesektoren sett ut fra et dynamisk perspektiv.
SNF-rapport nr. 58/1999, Bergen.

Seime, Gunn Randi
Konkurransen i det norske mobiltelefonimarkedet.
SNF-rapport nr. 49/1999, Bergen.

Methlie, Leif B.
Multimedia Banking

Pedersen, Per E.
Bankenes strategiske situasjon. Ny teknologi – ny konkurransearena – ny struktur.
SNF-rapport nr. 41/1999, Bergen.

Pedersen, Per E.
Multimedia Banking
Programvareagenter i elektronisk handel. En kartlegging med vekt på agentbaserte tjenester og finanstjenestesektoren.
SNF-rapport nr. 40/1999, Bergen.

Pedersen, Per E.
Multimedia Banking
En agentbasert tjeneste for produkt- og leverandør-sammenlikning av finanstjenester.
SNF-rapport nr. 39/1999, Bergen.

Pedersen, Per E.
Nysveen, Herbjørn
Jensen, Leif Magnus
Multimedia Banking
En eksperimentell studie av atferdskonsekvenser ved bruken av agentbaserte tjenester i finanssektoren.
SNF-rapport nr. 38/1999, Bergen.

Fjell, Kenneth
Foros, Øystein
Gabrielsen, Tommy S.
Hagen, Kåre P.
Sørgard, Lars
Vagstad, Steinar
Problemstillinger for videre forskning på prising av tele-tjenester.
SNF-rapport nr. 27/1999, Bergen.

Fjell, Kenneth
Hagen, Kåre P.
Oversikt over forskningsprogrammet i teleøkonomi ved SNF: 1996-1998.
SNF-rapport nr. 26/1999, Bergen.

Fjell, Kenneth
Foros, Øystein
Hagen, Kåre P.
Sørgard, Lars
Telenor – bare lave priser ?
Drøfting av Telenors rabattstruktur utfra et bedriftsøkonomisk og samfunnsøkonomisk perspektiv.
SNF-rapport nr. 23/1999, Bergen.

Staahl Gabrielsen, Tommy
Vagstad, Steinar
Konkurranseform i telesektoren: Hvordan rasjonalisere observert atferd?
SNF-rapport nr. 65/1998, Bergen.

Altenborg, Ellen
Koordinering og insentiver i samarbeid om produktutvikling mellom forretningsområder i Telenor.
SNF-rapport nr. 39/98, Bergen

Methlie, Leif
Multimedia Banking
Strukturendring i bank. Distribusjon – grovanalyse.
SNF-arbeidsnotat nr. 31/1998, Bergen.

Methlie, Leif
Multimedia Banking

Strukturendring i bank. Strategisk posisjonering – grovanalyse.
SNF-arbeidsnotat nr. 30/1998, Bergen.

Foros, Øystein
Ulset, Svein
Naturlige grenser for teleselskapene.
SNF populærvitenskapelig særtrykk nr. 10/1998, Bergen.

Ulset, Svein
Spiller, Pablo
Organizing Global Seamless Networks: Contracts, Alliances and Hierarchies.
SNF Reprint No. 26/1998, Bergen.

Ulset, Svein
Infrastruktur og konkurranse i telesektoren.
SNF særtrykk nr. 27/1998, Bergen.

Ulset, Svein
Value-Creating Interconnect
International Competition in Global Network Services. Technology, Organization and Performances.
SNF-report No. 28/1998, Bergen.

Ulset, Svein
Value-Creating Interconnect
Optimal Organization of the Converging Information and Communication Technology (ICT) Industries.
Theoretical analysis and some illustrative data from the Norwegian market.
SNF-report No. 27/1998, Bergen.

Methlie, Leif B.
Nysveen, Herbjørn
Multimedia Banking
Kundeatferd ved bruk av Internett og andre kanaler.
SNF-rapport nr. 29/1998, Bergen.

Ulset, Svein
Verdiskapende samtrafikkavtaler.
Hvordan kan organisering av infrastruktur bidra til utvikling av bedre og billigere teletjenester. En analyse av betingelsene for konkurranse og samarbeid i telesektoren.
SNF-rapport nr. 25/1998, Bergen.

Spiller, Pablo T.
Value-Creating Interconnect.
Unbundling and the Promotion of Local Telephone Competition: Is Unbundling Necessary in Norway?
SNF-Report No. 03/1998, Bergen.

Bjørnenak, Trond
Gabrielsen, Tommy Staahl
Vagstad, Steinar
Verdiskapende samtrafikkavtaler.
Prising av samtrafikk.
SNF- rapport nr. 02/1998, Bergen.

Andersen, Christian
Sannarnes, Jan Gaute
Analyse af tilgangsafgifter ved irreversible investeringer under usikkerhed.
SNF-rapport nr. 07/1998, Bergen.

Oversikt over publikasjoner innen teleøkonomi i perioden 1992-1997 fås ved henvendelse til tonje.skaar@snf.no, telefon 55 95 95 00.