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The impact of targeting technologies and consumer multi-homing on digital platform competition

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Abstract

In this paper, we address how targeting and consumer multi-homing impact platform competition and market equilibria in two-sided markets. We analyze platforms that are financed by both advertising and subscription fees, and let them adopt a targeting technology with increasing performance in audience size: a larger audience generates more consumer data, which improves the platforms' targeting ability and allows them to extract more ad revenues. Targeting therefore increases the importance of attracting consumers. Previous literature has shown that this could result in fierce price competition if consumers subscribe to only one platform (i.e. single-home). Surprisingly, we find that pure single-homing possibly does not constitute a Nash equilibrium. Instead, platforms might rationally set prices that induce consumers to subscribe to more than one platform (i.e. multi-home). With multi-homing, a platform's audience size is not restricted by the number of subscribers on rival platforms. Hence, multi-homing softens the competition over consumers. We show that this might imply that equilibrium profit is higher with than without targeting, in sharp contrast to what previous literature predicts.

Keywords: Two-sided markets, digital platforms, targeted advertising, incremental pricing, consumer multi-homing.
JEL classifications: D11, D21, L13, L82.

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1 Introduction

Media platforms compete for consumer attention. While some consumers are devoted to a single media provider, others spread their attention across multiple platforms.[2] The emergence of digital technologies has facilitated the latter, which we refer to as consumer multi-homing. All it takes to read an additional newspaper online, is a few extra clicks. In contrast, to access more print news, one has to go out and buy another newspaper.[3]

For ad-financed platforms, the distinction between exclusive (single-homing) and shared (multi-homing) consumers is utterly important. Having exclusive access to certain consumers implies that advertisers cannot reach them elsewhere, allowing the platforms to price their ad space accordingly. Consumers that are shared with other platforms, on the other hand, are typically worth less in the ad market. Since the advertisers can reach these consumers on other platforms as well, the platforms can only charge advertisers the incremental value of an additional impression. This is known as the incremental pricing principle (see e.g., Ambrus *et al.*, 2016; Athey *et al.*, 2018 and Anderson *et al.*, 2018).

In the digital era, platforms increasingly adopt advanced advertising technologies such as targeting. This can help identify the consumers that are most likely to buy the advertisers' product and make sure that impressions are directed towards the most promising candidates. Our model incorporates a targeting technology with increasing returns to scale in the audience size. One could, for instance, think of a machine-learning algorithm that improves as it is exposed to more consumer data. Platforms that collect more user data could therefore be better able to connect advertisers with the target audience. The upshot is that advertisers might be willing to pay more per impression on platforms with a large audience and higher targeting ability.[4]

Previous studies have shown that targeting might increase competition and benefit consumers through lower subscription prices (see e.g. Kox *et al.*, 2017; Crampes *et al.*, 2009).[5] Moreover, the studies emphasize that the additional revenues from the ad side of

[2]This has caught the attention of a number of researchers, such as Ambrus *et al.* (2016), Athey *et al.* (2018), Anderson *et al.* (2017; 2018; 2019).

[3]Gentzkow and Shapiro (2011) and Affeldt *et al.* (2019) argue why digital technologies make multi-homing more compelling.

[4]Goettler (2012) studies broadcast networks and provides empirical evidence that the ad price per viewer might increase in audience size.

[5]Kox *et al.* (2017) explicitly examine targeting, while Crampes *et al.* (2009) consider a more general

the market tend to get competed away on the consumer side of the market, leaving the platforms worse off. Kox *et al.* (2017) also point out that even though it would be in the platforms' common interest not to target ads, each platform might have individual incentives to do so. These findings are, however, based on the assumption that consumers single-home (join a single platform). Despite the relevance to modern media markets, the literature that combines consumer multi-homing and targeting is scarce. The purpose of this paper is to enrich the theoretical understanding of this phenomenon.

We scrutinize the outcomes under both single-homing and multi-homing, and investigate whether they constitute Nash equilibria. In line with existing literature, we find that targeting generates a prisoner's dilemma situation under the assumption of single-homing. But remarkably, we find that platforms may not want to set subscription prices that makes consumers prefer single-homing. Indeed, setting prices that incentivize consumers to multi-home could be a unique equilibrium. Combining elements from Crampes *et al.* (2009), Ambrus *et al.* (2016) and Anderson *et al.* (2017), we show that things turn out quite differently if consumers multi-home. In the absence of targeting, consumer multi-homing makes subscription prices strategically independent: if one platform changes its price, this has no impact on rival platforms' optimal price setting. To put it more concretely: suppose that you are going to purchase The Washington Post and consider to buy a copy of The New York Times (NYT) as well. When deciding whether to purchase NYT as an additional newspaper, what matters is the price of NYT (and not The Post).

This does not change if we introduce targeting. However, in that case, the platforms must take into account that the price setting of rival platforms will affect the profitability of targeting. If we revert to our previous example: By reducing its subscription price, The NYT could improve its targeting ability and charge advertisers extra. Since advertisers are not willing to pay the full extra for shared consumers (recall the incremental pricing principle), this would be more attractive the larger the number of exclusive consumers. A price reduction by The Post would, however, increase the number of consumers that buy The Post in addition to NYT. The NYT's gain from increased targeting ability would therefore be counteracted by a greater fraction of shared consumers. Hence, targeting has surprising consequences. In contrast to what is usually observed, subscription prices become strategic substitutes: it is less profitable for a platform to reduce its subscription

advertising technology.

price if rival platforms do the same.

Although targeting still makes it optimal to reduce subscription prices when consumers multi-home, it does not trigger an aggressive response from rival platforms. As a result, it is more imperative to implement targeting. Yet, softer competition alone cannot ensure that targeting is profitable. We show that this can only be guaranteed if multi-homing consumers are sufficiently valuable to advertisers.

Outline. The paper proceeds as follows. In section 2, we review related literature. In sections 3 and 4, we present a basic model and introduce a targeting technology. In section 5, we compare our results to disclose when targeting is profitable, and in section 6, we investigate potential equilibria. We conclude in section 7.

2 Related literature

This paper draws on two strands of media literature that are not usually brought together. One strand investigates the importance of consumer multi-homing, and the other examines the impact of targeted advertising.

Athey and Gans (2010) and Bergemann and Bonatti (2011) were among the first to address the impact of targeting on media platform competition. The former paper considers competition between a local platform that is tailored to the local audience (which is the local advertiser's intended audience) and a general platform that depends on targeting technologies to identify the advertiser's relevant consumer base. Targeting helps the general platform allocate constrained ad space more efficiently and allows it to accommodate a larger number of advertisers.

Bergemann and Bonatti (2011) model competition between online and offline media under the assumption that online media has higher targeting ability. Absent targeting, each advertiser places ads on several platforms to ensure that it reaches enough consumers. In this model, consumers' interests are correlated with their presence on a specific platform. Increased targeting ability thus allows advertisers to concentrate on just the most relevant platforms, reducing the overall demand for ads.

More recently, Gong *et al.* (2019) propose a different approach in which competition for consumers plays a prominent role. In their model, differences in the platforms' ability to target ads are exogenously given.[6] Assuming that consumers dislike irrelevant ads, Gong *et al.* suggest that improved targeting reduces the consumers' nuisance costs. At the same time, greater targeting ability attracts more advertisers. Hence, platforms with superior targeting abilities attract more consumers and advertisers, and they are more profitable.

A common feature of these papers is that platform differences are exogenously given. This gives rise to significant effects on the supply and demand of ads, which would be less prominent in a model with symmetric platforms (like ours). We disregard the allocative effects, and allow targeting ability to be determined within the model: by reducing its subscription price, a platform can increase its audience size and improve its targeting ability. Since none of the mentioned papers regards subscription fees, a similar interplay between the two sides of the market does not occur in these papers. This is one explanation of why we arrive at quite different results. Another reason is that we use a different targeting technology. As demonstrated by Crampes *et al.* (2009), the nature of the advertising technology is decisive for platform behavior and market outcomes.

Regarding the targeting specification, we find the contribution by Hagiu and Wright (2020) interesting. The paper pays attention to how technologies may improve based on learning from consumer data. This insight is useful for the understanding of how learning-based targeting technologies function. A general form of our targeting specification can be recognized in Crampes *et al.* (2009), who model the impact of advertising technologies with constant, decreasing and increasing returns to scale in the audience size, and point at the limitation of assuming linearity. Although the authors do not accentuate increasing returns to scale, we argue that the current focus on first-party data and technology makes this particular specification highly relevant. We therefore use a variant of this technology in our set-up. Like most previous research on targeting and media platform competition, Crampes *et al.* (2009) assume consumer single-homing. We relax this assumption, and show that this provides entirely different outcomes.

The use of consumer data to target ads has raised privacy concerns. Johnson (2013) stresses that targeting might be harmful when consumers value privacy. He investigates the impact of targeting when consumers have access to ad-avoidance tools, and shows that consumers tend to block too few ads in equilibrium. Kox *et al.* (2017) incorporate privacy

[6]In an extension, Gong *et al.* (2019) allow the platforms to invest in targeting ability, and show that under-investment is most likely to occur.

considerations in a work that is closer to ours. In a similar framework, the authors show that targeting reduces consumer welfare if the disutility of sharing personal information is greater than the advantage of lower subscription prices. Recall from the Introduction that Kox *et al.* also find that platform profits decrease in targeting. As a result, their model suggests that stricter privacy regulations benefit both consumers and platforms. An important difference between Kox *et al.* (2017) and this paper is that the former assumes a linear advertising technology and consumer single-homing.

This paper adds to the growing literature that covers consumer multi-homing. A key take away from existing research is the incremental pricing principle that we describe in the Introduction. Ambrus *et al.* (2016) emphasize that an implication of advertisers' lower valuation of shared consumers is that it is not only the overall demand that counts; the *composition* of the demand also matters. When advertisers place ads on platforms with multi-homing consumers, there is a risk that some consumers have seen the ad before. As pointed out by Athey *et al.* (2018), impressing the same consumer twice is less efficient than impressing two different consumers. We combine this insight of ad-financed platform markets with elements from the user-financed platform market in Anderson *et al.* (2017) to derive a two-sided model with dual source financing.

Although various papers assess different aspects of consumer multi-homing, the literature that integrates multi-homing with targeted advertising is scarce. There are, however, a few exceptions. Taylor (2012) investigates how targeting affects platforms' incentives to improve content in order to increase their share of consumer attention. The paper focuses on how the platforms can retain consumer attention. In contrast, we disregard the attention span of the audience and rather focus on its size. Another exception is D'Annunzio and Russo (2020), who study the role of ad networks and how tracking technologies affect market outcomes. However, since they focus on a different part of the industry (ad networks), they address other and complementary questions.

3 The model

We consider two media platforms that offer subscriptions to consumers and advertising space (eyeballs) to advertisers. We employ a simple Hotelling (1929) model, with a line of length *one*, and assign one platform to each endpoint, i.e., platform 1 is located at

$x_1 = 0$ and platform 2 is located at $x_2 = 1$. Along the line, there is a unit mass of uniformly distributed consumers. The distribution represents the consumers' taste: the greater distance to a platform, the greater mismatch between the consumer preferences and the platform characteristics.

We consider two different regimes (which we later analyze whether constitute Nash equilibria). One of them is a pure single-homing regime (hereafter referred to as the single-homing regime) where all consumers subscribe to only one platform. The other is a multi-homing regime where some (but not all) consumers use more than one platform.[7] The outcomes for both regimes are presented.

3.1 Consumer demand

Single-homing consumers join only the platform they prefer the most. Let u_i represent the utility a consumer located at x obtains from subscribing to platform $i = 1, 2$:

$$u_i = \ v - t|x - x_i| - p_i. \tag{1}$$

The parameter $v > 0$ is the intrinsic utility of joining a platform, $t > 0$ represents the disutility of the mismatch between the consumer's preferences and the platform's characteristics, and p_i is the subscription price.

The consumer that is indifferent between only subscribing to platform 1 and only subscribing to platform 2 is located at $\widetilde{x}$, where $u_1 = u_2$. Consumers to the left of $\widetilde{x}$ subscribe to platform 1 and consumers to the right subscribe to platform 2. Hence, the demand function (superscript 'S' for single-homing regime) equals:

$$D_i^S = \frac{1}{2} + \frac{p_j - p_i}{2t}. \tag{2}$$

We do, however, allow consumers to subscribe to both platforms. The utility from dual subscription equals the sum of the individual utilities:

$$u_{1+2} = 2v - t - p_1 - p_2. \tag{3}$$

If the incremental utility of multi-homing is positive for some consumers, $u_{1+2}(x) \geq u_i(x)$, those consumers will subscribe to both platforms. Hence, each platform potentially

[7] With complete multi-homing, targeting would neither affect demand nor subscription prices. In this case, the analysis simply boils down to the change in ad prices.

serves two groups of consumers: exclusive subscribers and subscribers who are shared with the rival platform.

Let x_{12} represent the location of the consumer who is indifferent between subscribing to just platform 1 and subscribing to both platform 1 and platform 2.[8] Since platform 2 does not provide any additional utility to the indifferent consumer, $u_{1+2} = u_1$. Platform 1's exclusive demand then arises from the consumers who are located to the left of x_{12}. It follows that the platforms' shared demand is made up by the consumers located between x_{12} and x_{21}. This is illustrated in Figure 1.

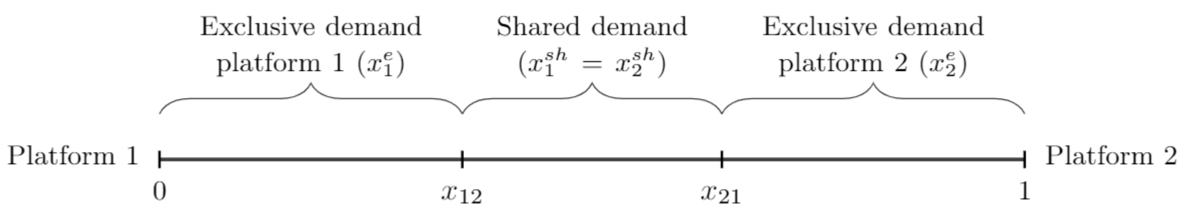

Figure 1: Demand platform $i = 1, 2$.

We solve $u_{1+2} = u_1$ and $u_{1+2} = u_2$ and find $x_{12} = \frac{1}{t}(-v + t + p_2)$ and $x_{21} = \frac{1}{t}(v - p_1)$, respectively. With symmetric platforms, we get that platform i's exclusive demand is given by

$$x_i^e = \frac{-v + t + p_j}{t}, \tag{4}$$

whereas its shared demand equals

$$x_i^{sh} = \frac{2v - t - p_i - p_j}{t}. \tag{5}$$

Total demand is the sum of exclusive demand and shared demand:

$$D_i^M = x_i^e + x_i^{sh} = \frac{v - p_i}{t}. \tag{6}$$

Equation (6) tells us that total demand for platform i is independent of the rival platform's subscription price (p_j). A change in p_j will, however, affect the composition of

[8]Vice versa, x_{21} represents the location of the consumer that is indifferent between subscribing to only platform 2 and both platform 2 and platform 1.

platform i's demand. From equation (4), we see that the number of exclusive subscribers is increasing in p_j, while equation (5) shows an inverse relationship between the number of shared subscribers and p_j.

3.2 Advertisers and Platforms

Turning to the advertising side,we normalize the number of advertisers to *one*. The demand for ads is perfectly elastic, and we assume that each advertiser purchase space for one ad per platform. In line with the incremental pricing principle, we assume that the advertisers are willing to pay α_i to reach an exclusive consumer, but only a fraction $\sigma\alpha_i$ to reach a shared consumer, where $\sigma \in (0, 1)$.[9] It follows that platform i's ad revenue can be defined as

$$A_i^k = \alpha_i x_i^e + \sigma\alpha_i x_i^{sh}, \tag{7}$$

where superscripts $k = S$ and $k = M$ represent the single-homing regime and the multi-homing regime, respectively.

Total profit is given by[10]

$$\pi_i^k = p_i^k D_i^k + \alpha_i \left(x_i^e + \sigma x_i^{sh} \right). \tag{8}$$

Notice that if all consumers single-home, then $x_i^{mh} = 0$ and $x_i^e = D_i^S$.

3.3 No targeting

Consider first a model without targeting. In this situation, we assume that the advertiser value of reaching a consumer is not platform dependent, such that $\alpha_i = \alpha_j = \alpha$. We differentiate equation (8) and find the first-order condition

$$\frac{\partial \pi_i^k}{\partial p_i^k} = \left[D_i^k + \frac{\partial D_i^k}{\partial p_i^k} p_i^k \right] + \left[\alpha \left(\frac{\partial x_i^e}{\partial p_i^k} + \sigma \frac{\partial x_i^{sh}}{\partial p_i^k} \right) \right] = 0. \tag{9}$$

The first square bracket on the right-hand side of equation (9) deals with the consumer side of the market, corresponding to a standard one-sided model. If we consider an increase in p_i, this implies that each consumer pays more, but it also means a lower number of

[9] This corresponds to Anderson *et al.* (2018).

[10] We set all costs to zero to simplify the model.

subscribers. In our two-sided model, the price increase has an impact on the ad side of the market as well: platform i displays fewer ads and thereby loses ad revenues. This is captured by the second square bracket. Because of the negative effect a price increase has on ad revenues, the optimal subscription price is lower in a two-sided model.

Solving equation (9) for p_i^k gives the best-response functions:

$$p_i^M(p_j) = \frac{v - \sigma\alpha}{2} \text{ and } p_i^S(p_j) = \frac{t + p_j - \alpha}{2}. \tag{10}$$

We note that in the multi-homing regime, subscription prices are strategically independent.[11] In other words, platform i's subscription price is not responsive to changes in platform j's subscription price. To see why, suppose that platform j adjusts p_j. From section 3.1, we know that even though it alters the number of exclusive and shared consumers, the price change has no effect on platform i's total demand. This is because the location of platform i's marginal consumer stays the same. Keep in mind that the marginal consumer is located where her incremental value of subscribing to platform i is zero. Hence, platform i's subscription price still extracts the marginal consumer's incremental benefit. Besides, platform i's price setting does not affect the advertisers' valuation of the marginal consumer. Consequently, platform i has no incentive to change its subscription price in response to an adjustment in p_j. In the single-homing regime, we get the standard result that prices are strategic complements.

Lemma 1 *(No targeting) Subscription prices are*

(i) strategic complements in the single-homing regime

(ii) strategically independent in the multi-homing regime

4 Introducing targeting

Next, we introduce targeting to our model. We recognize that advertisers may not only care about the reach of ads, but also about the quality of the match with the audience. Suppose that the platforms implement targeting technologies that enable them to create better matches between advertisers and viewers. Moreover, the technology becomes more

[11] This is in line with Anderson *et al.* (2017).

accurate as the platforms increase their audience size and thereby generate more data.[12] We assume that advertisers are willing to pay for improvements in the platforms' targeting ability, and formulate the ad price as follows:

$$\alpha_i^k = \alpha(1 + \varphi D_i^k) \tag{11}$$

where φ is a dummy that takes on the value *one* when targeting is included in the model and *zero* otherwise. Notice that in the latter case, equation (11) reverts to the non-targeting ad price (α). For φ equal to *one*, the definition implies that the ad price is increasing in the platform's audience size ($\frac{\partial \alpha_i^k}{\partial D_i^k} > 0$), capturing the benefit of having more consumer data and improved targeting ability.

In the targeting model, α is interpreted as measurement of how efficiently the platforms are using consumer data to improve their targeting ability. As we proceed, we will see how this adjustment of the model can change the results drastically.

Inserting equation (11) into equation (8), and differentiating with respect to own price, we find the new first-order condition:

$$\frac{\partial \pi_i^k}{\partial p_i^k} = D_i^k + \frac{\partial D_i^k}{\partial p_i^k} p_i^k + \alpha(1 + \varphi D_i^k) \left(\frac{\partial x_i^e}{\partial p_i^k} + \sigma \frac{\partial x_i^{sh}}{\partial p_i^k} \right) + \varphi \frac{\partial \alpha_i^k}{\partial p_i^k} \left(x_i^e + \sigma x_i^{sh} \right) = 0. \tag{12}$$

When φ equals *zero*, we recognize equation (12) as the first-order condition in the model without targeting (cf. equation (9)). The two additional terms that appear when φ equals *one* represent the effects that emerge when we incorporate targeting. First, consider the third term on the right hand side. It tells us that ad revenues are more sensitive to changes in the number of ad impressions (in response to a change in the subscription price) than without targeting.[13] The explanation is that the ad price, which corresponds to the first part of the third term (cf. equation (11)), is higher with targeting ($\varphi = 1$). Second, we evaluate the fourth term. This expression captures a property that is not present in the model without targeting, namely that a platform's ad price responds to changes in its own subscription price. An increase in p_i^k causes a reduction in α_i^k, and vice versa.

[12] We assume that each consumer delivers one data point, such that we measure the amount of data by the number of consumers.

[13] Since each consumer is impressed once, the number of subscribers is equivalent to the number of ad impressions.

Solving equation (12) for p_i^k, we find the best-response functions:

$$p_i^M(p_j) = \frac{v\,(t+\alpha) - \alpha\,(t + 3v\sigma) - \alpha p_j\,(1-\sigma)}{2(t-\alpha\sigma)} \text{ and } p_i^S(p_j) = \frac{t\,(t-2\alpha) + p_j\,(t-\alpha)}{2t-\alpha}. \quad (13)$$

The best-response functions reveal a striking difference between the single-homing regime and the multi-homing regime. If all consumers single-home, subscription prices are strategic complements ($dp_i^S/dp_j > 0$). In contrast, if at least some consumers multi-home, subscription prices are strategic substitutes ($dp_i^M/dp_j < 0$). This means that the optimal response to changes in the rival platform's subscription price depends on whether consumers only single-home or if some of them multi-home.

We can state:

Proposition 1 *(Targeting) When platforms target ads, subscription prices are*

(i) strategic complements in the single-homing regime

(ii) strategic substitutes in the multi-homing regime

The first result in Proposition 1 is well known in the literature: in a single-homing regime, the best response to a change in the rival subscription price is to adjust own price in the same direction.

The second result in Proposition 1, however, is quite surprising. While platform i's best response to a change in the rival subscription price is to do nothing in the multi-homing model without targeting (cf. Lemma 1), the best response in the targeting model is to adjust p_i^M in the opposite direction. Since targeting does not change the property of total demand being independent of the rival subscription price, the difference between the models may not be intuitive. After all, this property implies that p_i^M extracts the marginal consumer's incremental benefit regardless of any changes in p_j^M. The key to understanding why a change in p_j^M still induces a response, is that targeting enables platform i to affect the advertisers' willingness to pay. To see why, suppose that platform j increases p_j^M. This creates a shift from shared to exclusive subscribers for platform i, which implies a smaller share of discounted ad impressions. Platform i would therefore gain from increasing its ad price. Targeting enables the platform to do so by reducing p_i^M and improving its targeting ability. Conversely, a reduction in p_j^M provides incentives to increase p_i^M.

5 When is targeting profitable?

In this section, we compare the outcomes with and without targeting, and reveal when targeting is profitable. First, we find the symmetric non-targeting equilibrium prices. Solving the best-response functions in equation (10) simultaneously, we have

$$p^M = \frac{v - \sigma\alpha}{2} \text{ and } p^S = t - \alpha. \tag{14}$$

We then find the symmetric targeting equilibrium prices (the asterisk superscript denotes targeting) by solving the best-response functions in equation (13) simultaneously:

$$p^{M^*} = \frac{v\,(t+\alpha) - \alpha\,(t + 3v\sigma)}{2t + \alpha(1 - 3\sigma)} \text{ and } p^{S^*} = t - 2\alpha. \tag{15}$$

Comparing equations (14) and (15), we observe that subscription prices are lower when platforms target ads, irrespective of whether all consumers single-home or if some multi-home.

Targeting provides greater incentives to attract a larger audience, and to do so, the platforms lower their subscription prices.

We can state:

Lemma 2 *Subscription prices will be lower when platforms use targeting technologies.*

Proof. See Appendix. ■

In the following, we first analyze the single-homing regime, then proceed to the multi-homing regime.

5.1 Single-homing

We restrict our attention to markets with full coverage and endogenously non-negative prices. This, as well as fulfillment of the stability and second-order conditions, is ensured by Condition 1:

Condition 1 *(Single-homing)* $\frac{5}{2}\alpha < t < \frac{2}{3}(v + \alpha)$.

It follows from Lemma 2 and Proposition 1 that targeting leads to fiercer price competition.

The symmetric equilibrium demand is equivalent with and without targeting:

$$D^{S^*} = D^S = \frac{1}{2}. \tag{16}$$

It follows that subscription revenues are lower with targeting. Even though ad revenues are higher, they do not fully compensate for the lost subscription revenues. Inserting (16), (15) and (14) into (8), we find the equilibrium profits with and without targeting, respectively:

$$\pi^{S^*} = \frac{1}{4}(2t - \alpha) \text{ and } \pi^s = \frac{1}{2}t. \tag{17}$$

Equation (17) shows clearly that the targeting profit is lower than the non-targeting profit and decreasing in the technology's sensitivity to more data. The reason is that the higher α, the greater the incentive to reduce the subscription price, which significantly reduces subscription revenues. This raises the question of whether the platforms at all wish to adopt targeting technologies. Although it is in the platforms' common interest not to target, each platform has incentives to deviate from the mutually beneficial strategy. The platforms might therefore end up in a prisoner's dilemma situation where all platforms target (see also Kox *et al.*, 2017).

We state:

Lemma 3 *(Prisoner's dilemma) When all consumers single-home, targeting is a dominant strategy and the platforms end up in a prisoner's dilemma.*

Proof. See Appendix. ■

As we demonstrate in the equilibrium analysis, the platforms could, however, be better off by setting the multi-homing price and also attract consumers who already subscribe to the rival platform.

5.2 Multi-homing

Assume now consumer multi-homing. We consider partial multi-homing, i.e. situations where some, but not all, consumers use both platforms. Note that $t > \frac{1}{2}(v + 3\sigma\alpha)$ and $t < v + \sigma\alpha$ ensure the existence of exclusive and shared consumers, respectively. Moreover, we confine the analysis to situations with endogenously non-negative subscription prices and parameter values that satisfy all second-order and stability constraints. The conditions are given in the Appendix.

From Lemma 2 and Proposition 1 it follows that targeting provides incentives to reduce the subscription price, and that the rival platform will respond favorably. Moreover, the incentive to lower the price increases with advertisers' willingness to pay for shared consumers. This is captured in our model by the σ-parameter, where $\partial p^{M^*}/\partial\sigma < 0$. The price reduction contributes to greater overall demand, and the increase is reinforced by the rival platform's response. Nonetheless, we find that equilibrium subscription revenues are lower with targeting ($p^{M^*}D^{M^*} < p^M D^M$).

For targeting to be profitable, two conditions must therefore be satisfied: (i) Ad revenues must increase with targeting; and (ii) the increase in ad revenues must be greater than the loss in subscription revenues. Comparing ad revenues with and without targeting, we find that ad revenues are greater with targeting if $\sigma > \frac{1}{3}$. However, if $\sigma \leq \frac{1}{3}$, that is not necessarily true. The smaller σ, the lower is the ad price the platforms can charge for impressing shared consumers. This is particularly harmful in combination with weak platform preferences (low t), because targeting then creates a greater shift from exclusive consumers to shared consumers. A larger proportion of less valuable shared consumers could, in this case, offset the advantage of an increased ad price.

Finally, whether targeting is profitable or not thus depends on σ. We find it useful to consider $\sigma > \frac{1}{3}$ and $\sigma \leq \frac{1}{3}$ separately. The exact calculations are found in the Appendix.

First, we look at the case where $\sigma > \frac{1}{3}$. We find that $v > \alpha\left(\sigma + \sqrt{\sigma(\sigma+1)}\right)$ is required to satisfy the multi-homing conditions. The more responsive the ad price is to the audience size (α) and the more advertisers value shared consumers (σ), the stronger incentives the platforms have to set lower subscription prices, and the greater must the intrinsic utility (v) be to ensure non-negative subscription prices.

As σ goes towards $\frac{1}{3}$, the minimum value of v is given by $v_{\min} = \alpha + \varepsilon$. By definition, $\sigma_{\min} = \frac{1}{3} + \varepsilon$. For both $v_{\min}$ and $\sigma_{\min}$, we have that targeting provides greater profits:

$$(\pi^{M^*} - \pi^M)|_{v_{\min}} > 0 \text{ and } (\pi^{M^*} - \pi^M)|_{\sigma_{\min}} > 0.$$

Moreover, we have that the difference between profits with and without targeting is increasing in σ evaluated at $v = v_{\min}$ ($\frac{d(\pi^{M^*}-\pi^M)}{d\sigma}|_{v_{\min}} > 0$), and the difference is increasing in v evaluated at $\sigma = \sigma_{\min}$ ($\frac{d(\pi^{M^*}-\pi^M)}{dv}|_{\sigma_{\min}} > 0$). Finally, higher v-values enhance the increase in $(\pi^{M^*} - \pi^M)$ that follows from a higher σ:

$$\frac{d\left(\frac{d(\pi^{M^*}-\pi^M)}{d\sigma}\right)}{dv} > 0.$$

In sum, this means that targeting is profitable for all $\sigma > \frac{1}{3}$. We then consider $\sigma \leq \frac{1}{3}$. Because shared consumers have lower value in the ad market for small $\sigma-$values, the incentives to increase the audience size are weaker, and positive subscription prices can be achieved even for $v < \alpha$. For $\sigma \leq \frac{1}{3}$, targeting does not necessarily increase ad revenues. Since targeting also reduces subscription revenues, it might lead to lower profits.

We summarize the results in the following proposition:

Proposition 2 *(Multi-homing). Suppose that the multi-homing conditions hold. Targeting is profitable if advertisers place a high enough value on shared consumers. A sufficient condition is* $\sigma > \frac{1}{3}$.

Proof. See Appendix. ■

Combining Lemma 3 and Proposition 2, gives us the following corollary:

Corollary 1 *Targeting can only be profitable in the multi-homing regime*

6 Equilibrium analysis

We now proceed to comparing the market outcomes with pure single-homing and multi-homing and examining the existence of Nash equilibria. In this part, we restrict our attention to parameter values that fulfill the conditions for both the single-homing model and the multi-homing model. From Condition 1, we have that this requires that $v > \frac{11}{4}\alpha$. To illustrate the key point, we set $v = 3\alpha$, which is close to the minimum $v-$value. In the Robustness section in the Appendix, we show that the results we arrive at are valid also for $v > 3\alpha$, at least if shared consumers are not virtually worthless to advertisers.

Condition 2 ensures partial multi-homing in the multi-homing regime, non-negative prices and full market coverage in the single-homing regime, in addition to satisfying second-order and stability conditions.

Condition 2 *(Equilibrium)* $\max\{\frac{5}{2}\alpha, \frac{3}{2}\alpha(\sigma+1)\} < t < \frac{10}{3}\alpha$.

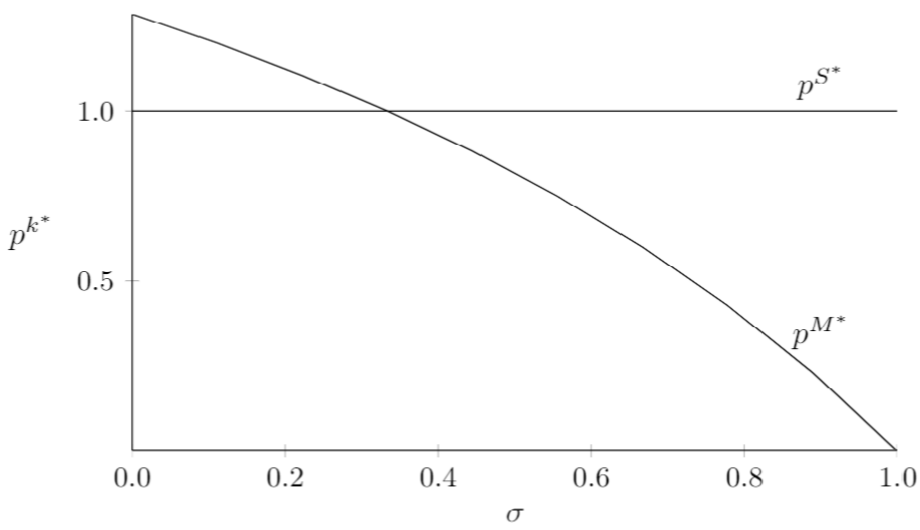

Figure 2: Equilibrium prices.

6.1 Comparison of equilibrium outcomes

Comparing the subscription prices in equation (15), we find that $p^{S^*} \geq p^{M^*}$ for $\sigma > \frac{2}{3}$. For lower σ, the single-homing price may be both greater and smaller than the multi-homing price, as illustrated in Figure 2 (parameter values: $t = 3\alpha$ and $\alpha = 1$).

When σ is low, the platforms have weaker incentives to reduce the multi-homing price. However, the higher t, the greater price reduction is required to persuade consumers to multi-home. Hence, if t is sufficiently high (the condition is given in the Appendix), the multi-homing price could still be lower than the single-homing price. Conversely, a higher σ (corresponding to shared consumers being more valuable) provides stronger incentives to reduce subscription prices in the multi-homing regime. This is why we observe that p^{M^*} decreases in σ, both in absolute value and relative to p^{S^*}.

Turning to advertising prices, we find that these are always lower with single-homing ($\alpha^{S^*} < \alpha^{M^*}$). Finally, we consider profits. We find that if $\sigma \geq 0.65$, single-homing profits cannot be greater than multi-homing profits ($\pi^{S^*} < \pi^{M^*}$). For $\sigma < 0.65$, however, profits may or may not be greater with single-homing. A sufficiently high t can ensure that single-homing makes the platforms better off. This is illustrated by Figure 3 (parameter values: $t = 3.3\alpha$ and $\alpha = 1$).[14]

[14]We use different sets of parameter values in the two figures because it enables us to demonstrate that prices and profits can be both higher and lower with single-homing compared to multi-homing.

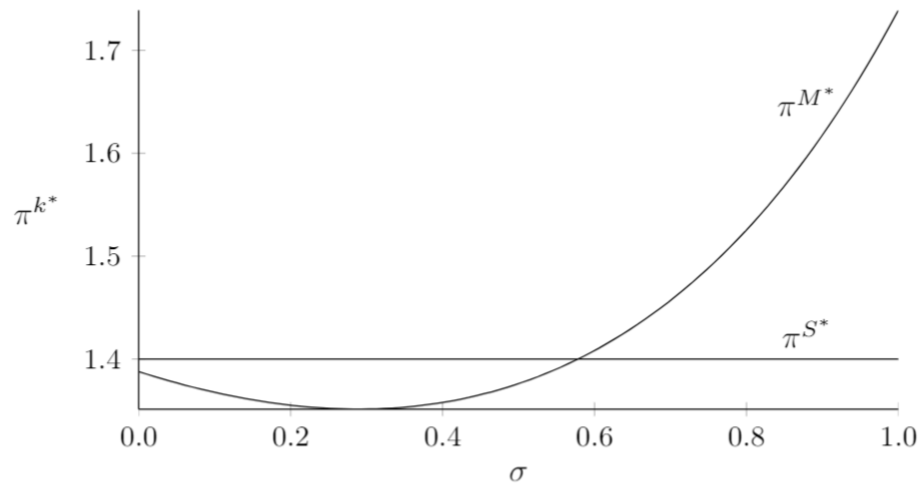

Figure 3: Equilibrium profits.

From the analysis of subscription prices we know that consumers who subscribe to only one platform are better off in a multi-homing regime when $\sigma > \frac{2}{3}$, since $p^{S^*} \geq p^{M^*}$.

Moreover, we find that at least some consumers prefer multi-homing over single-homing if $\sigma > \frac{2}{9}$.

The following proposition sums up the comparison of equilibrium outcomes:

Proposition 3 *Assume that condition 2 holds and that* $\sigma > \frac{2}{3}$.

Compared to pure single-homing, multi-homing provides

(i) lower subscription prices and higher consumer utility

(ii) higher ad revenues

(iii) higher platform profits

Proof. See Appendix. ■

By nature, single-homing profits do not depend on the value of shared consumers (σ). Multi-homing profits, on the other hand, are either increasing in σ or have a U-shaped relationship with σ. An increase in σ means that shared consumers are more valuable to advertisers. Since this allows the platforms to charge a higher ad price, one might expect that it would lead to greater platform profits. For most parameter values, profits are indeed unambiguously increasing in σ. An increase in the value of shared consumers also makes the platforms eager to attract more of them. But suppose that consumers have very strong platform preferences (high t). Attracting a larger audience may then require a price drop that is more costly than the additional revenue from gained consumers. This could be the case if the value of shared consumers, even after an increase, remains fairly low. Consequently, the overall impact on profits could be negative. However, as σ takes on higher values, profits will eventually start to increase. Figure 3 illustrates this U-shaped relationship between σ and multi-homing profits.

6.2 The existence of equilibria

Next, we investigate whether single-homing and multi-homing constitute potential equilibria. If shared consumers are sufficiently valuable, it pays off to charge lower subscription fees and forgo some subscription revenues in order to extract more ad-side revenues. Moreover, if the platforms set multi-homing prices, some consumers will actually subscribe to both platforms.

If, on the other hand, the advertiser valuation of shared consumers is low (small sigma), multi-homing might not constitute an equilibrium. In a situation with weak platform preferences (low t), a reduction in the subscription price would be efficient in attracting many consumers, making it tempting to undercut the rival's subscription price and only serve more valuable exclusive consumers. Both platforms would in that case deviate from multi-homing. However, as long as $\sigma > \sigma^* = 0.03$, we find that it is never beneficial for a platform to deviate from multi-homing. Recall that $\sigma \in (0, 1)$, which means that there is only a small interval where deviation from multi-homing might be feasible.

Then, consider the single-homing regime. Unless shared consumers have very little value for advertisers, the platforms have strong incentives to deviate from setting the single-homing price. More precisely, we find that it is profitable for a platform to deviate from single-homing for all $\sigma > 0.1$.

The most obvious reason is that deviation enables the platforms to sell more subscriptions and ad impressions. But even if shared consumers are not that valuable (i.e. $\sigma < 0.1$), single-homing does not constitute an equilibrium.

The single-homing prices would still be so low that some consumers would like to deviate and subscribe to both platforms.

We can state:

Proposition 4 *Assume that condition 2 holds. Then, there exists* *(i) a unique equilibrium with multi-homing for* $\sigma > \sigma^*$

(ii) no equilibrium with single-homing for all $\sigma > 0$

Remark 1 *Multi-homing could also constitute an equilibrium for* $\sigma < \sigma^*$*, but only if consumers have sufficiently strong platform preferences.*

Proof. See Appendix. ■

The second result of Proposition 4 is particularly interesting. Previous literature has typically made the stark assumption of single-homing, which we find never takes part in a targeting equilibrium, and hence might not be an appropriate assumption to make.

7 Concluding remarks

This paper has two major contributions: First, we demonstrate the importance of consumer multi-homing. Multi-homing allows the platforms to attract more subscribers, which is increasingly valuable in the ad market when platforms use targeting technologies with increasing returns to scale in the audience size. Moreover, targeting does not trigger an aggressive price response from the rival platform, as would be the case in a single-homing regime. Altogether, we find that targeting can only be profitable if we relax the typically made single-homing assumption.

The second key contribution is an even more important one: we find that pure single-homing never occurs in equilibrium. This means that existing literature assuming single-homing might be misleading, and emphasizes that assessing the nature of consumer purchasing behavior (ie. single-homing or multi-homing) is vital to fully understand the impact of targeting.

Our set-up is partly motivated by the rise of first-party data. Until recently, consumer data could easily be purchased from third parties. However, increased demand for privacy has led to new regulations, such as the General Data Privacy Regulation (GDPR). Since GDPR came into force in 2018, compliance has been high on the business agenda, limiting the utilization of externally collected consumer information. Web browsers increasingly block third-party cookies, and platforms are moving away from third-party data and towards permission-based, internally collected first-party data.[15]

[15] See e.g. Goswami, S. (2020, November 9) and Walter, G. (2021, January 13).

The industry is already adapting to the new privacy-oriented landscape. Major platforms like The New York Times and The Washington Post have recently developed in-house solutions in order to control data and targeting.[16] The VP of commercial technology and development at The Washington Post, Jarrod Dicker, says (Washington Post Press release, July 16, 2019): *"User privacy is paramount to us, so we are deeply invested in building sophisticated tools powered by first-party data"*. The machine learning-based tools enable the newspaper to benefit from data on how the users engage with the platform, and reduce its reliance on cookie-driven information. The head of ad product for RED, Jeff Turner, elaborates (Washington Post Press release, July 16, 2019): *"Data points like a user's current page view and session on The Post's site are much more relevant to that user's current consumption intention than the information a cookie-driven strategy can offer"*. Advertisers cannot find this insight elsewhere, which gives the platforms a competitive advantage. The focus on privacy has increased the strategic importance of first-party data, which is also the key to successful targeting in our model.

Our model makes the assumption of ad-neutral consumers. Targeting could, however, either increase or reduce ad nuisance. On the one hand, privacy concerns might lead to less consumer satisfaction (Johnson, 2013; Kox *et al.*, 2017). On the other hand, more relevant ads could please them (Gong *et al.*, 2019). The overall effect is therefore ambiguous. We leave this analysis for future research. Our model specification says that the platforms' targeting ability increases at a constant rate as more consumers subscribe. While some sources claim that the more data, the better targeting results, others suggest that the benefit of more data will be diminishing at some point. We have not formally analyzed this issue, but we do not believe that it would change the key properties. Targeting would presumably still provide similar, but somewhat smaller effects.

We assume that advertisers' willingness to pay to reach a shared consumer is weakly lower than their willingness to pay to reach an exclusive consumer, i.e. $\sigma \in (0, 1)$. In an empirical study of US magazines, Shi (2016) finds that shared consumers are about half as valuable as exclusive consumers, i.e. $\sigma = \frac{1}{2}$. This estimate indicates that it is reasonable to assume that platforms must charge less for consumers that can be reached elsewhere.

One may also ask whether stricter privacy regulations provide greater incentives to cooperate in order to share data. On the one hand, joining forces to increase the total data

[16]See Fischer, S. (2020, May 19).

pool could be seen as an alternative to purchasing third-party data. On the other hand, stricter regulations might make cooperation less feasible. Furthermore, the perhaps greatest advantage of first-party data is that it provides the platforms with exclusive insight. This advantage clearly goes against sharing. Future studies could explore this issue further.

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Appendix

A.1 Conditions for multi-homing

Without targeting, second-order and stability conditions are always satisfied. From the equilibrium price, which is given by $p^M = \frac{v-\sigma\alpha}{2}$, we see that $v > \sigma\alpha$ is required for the price to be positive. The equilibrium demand functions are given by

$$x^e = \frac{1}{2}\frac{2t - v - \alpha\sigma}{t} \text{ ; } x^{sh} = \frac{v + \alpha\sigma - t}{t} \text{ and } D^M = \frac{1}{2}\frac{v + \alpha\sigma}{t}.$$

The restriction of the analysis to partial multi-homing implies that we need $x^e > 0$ and $x^{sh} > 0$. This places some additional constraints on the parameter values: $\frac{1}{2}(v+\sigma\alpha) < t < v + \sigma\alpha$.

With targeting, stability requires $t > \frac{1}{2}\alpha(\sigma + 1)$ and the second-order condition is satisfied for $t > \sigma\alpha$. The equilibrium price is given by equation (15), and non-negative prices require that $t > \frac{\alpha v(3\sigma-1)}{v-\alpha}$. The equilibrium demand functions are

$$x^{e^*} = \frac{2t - v - 3\alpha\sigma}{2t + \alpha(1-3\sigma)} \text{ ; } x^{sh^*} = \frac{\alpha + 2v + 3\alpha\sigma - 2t}{2t + \alpha(1-3\sigma)} \text{ and } D^{M^*} = \frac{\alpha + v}{2t + \alpha(1-3\sigma)},$$

for which partial multi-homing is ensured by $\frac{1}{2}(v + 3\alpha\sigma) < t < v + \frac{1}{2}\alpha(1 + 3\sigma)$.

Summarizing, this leaves us with the two binding constraints, depending on the value of σ:

Condition A.1 *(Multi-homing)* $\max\{\frac{1}{2}(v + 3\sigma\alpha), \frac{\alpha v(3\sigma-1)}{v-\alpha}\} < t < v + \sigma\alpha$ *for* $\sigma > \frac{1}{3}$.

Condition A.2 *(Multi-homing)* $\max\{\frac{1}{2}(v + 3\sigma\alpha), \frac{1}{2}\alpha(1 + \sigma)\} < t < v + \sigma\alpha$ *for* $\sigma \leq \frac{1}{3}$.

Finally, Condition A.1 constrains $v > \alpha\left(\sigma + \sqrt{\sigma(\sigma + 1)}\right)$.

A.2 Omitted proofs

Proof of Lemma 2

Under single-homing, this follows directly as $p^{S^*} - p^S = t - 2\alpha - (t - \alpha) = -\alpha < 0$. Under multi-homing, we have $p^{M^*} - p^M = \frac{1}{2}\frac{\alpha}{2t+\alpha(1-3\sigma)}(v - 2t(1 + \sigma) - 3v\sigma + \alpha\sigma - 3\alpha\sigma^2)$, which is negative if conditions A.1 and A.2 hold. ■

Proof of Lemma 3

Consider the single-homing regime. Suppose platform i targets ads, while platform j does not. The best-response functions are then

$$p_i(p_j) = \frac{t\,(t-2\alpha) + p_j\,(t-\alpha)}{2t-\alpha} \text{ and } p_j(p_i) = \frac{t + p_i - \alpha}{2}.$$

The equilibrium prices are given by (superscript 'd' for deviation)

$$p_i^d = \frac{3t^2 - 6\alpha t + \alpha^2}{3t-\alpha} \text{ and } p_j = \frac{3t^2 - 5\alpha t + \alpha^2}{3t-\alpha},$$

yielding profits

$$\pi_i^d = \frac{9}{4}t^2\frac{2t-\alpha}{(3t-\alpha)^2} \text{ and } \pi_j = \frac{1}{2}t\frac{(3t-2\alpha)^2}{(3t-\alpha)^2}.$$

From equation (17), we have the symmetric equilibrium profits when both platforms target and when none of the platforms targets.

The decision of whether or not to deviate can be formulated as a game matrix in Table A.1.

		Platform i	
		Target	Not target
Platform j	Target	$\frac{2t-\alpha}{4}, \frac{2t-\alpha}{4}$	$\frac{9}{4}t^2\frac{2t-\alpha}{(3t-\alpha)^2}, \frac{1}{2}t\frac{(3t-2\alpha)^2}{(3t-\alpha)^2}$
	Not target	$\frac{1}{2}t\frac{(3t-2\alpha)^2}{(3t-\alpha)^2}, \frac{9}{4}t^2\frac{2t-\alpha}{(3t-\alpha)^2}$	$\frac{1}{2}t, \frac{1}{2}t$

Table A.1: Prisoner's dilemma.

If platform j targets, it is optimal for platform i to target iff $\frac{2t-\alpha}{4} - \frac{1}{2}t\frac{(3t-2\alpha)^2}{(3t-\alpha)^2} = \frac{1}{4}\alpha\frac{3t^2-\alpha^2}{(\alpha-3t)^2} > 0$, which is always the case.

If platform j does not target, it is nevertheless optimal for platform i to target iff $\frac{9}{4}t^2\frac{2t-\alpha}{(3t-\alpha)^2} - \frac{1}{2}t = \frac{1}{4}t\alpha\frac{3t-2\alpha}{(3t-\alpha)^2} > 0$, which is also always the case.

Hence, platform i's dominant strategy is to target, regardless of the rival's decision, and despite targeting yielding lower profits than not targeting. This means that the platforms end up in a prisoner's dilemma when consumers single-home. ■

Proof of Proposition 2

To prove Proposition 2, we start by decomposing profits into ad revenues and subscription revenues.

Subscription revenues First, we show that subscription revenues are always lower with targeting:

$$p^{M^*}D^{M^*} - p^M D^M = -\frac{1}{4}\alpha^2\left(2t - v - \sigma\alpha - 2t\sigma + 3v\sigma + 3\alpha\sigma^2\right)\frac{2t-v+\alpha\sigma+2t\sigma+3v\sigma-3\alpha\sigma^2}{t(\alpha+2t-3\alpha\sigma)^2} < 0.$$

Ad revenues Ad revenues with targeting minus ad revenues without targeting are given by

$$A^{M^*}-A^M = \frac{1}{2}\alpha\frac{4t^2(1-\sigma)(v+2\alpha\sigma)+2tv^2(2\sigma-1)+2tv\alpha\sigma(9\sigma-5)+2t\alpha^2\left(5\sigma-13\sigma^2+12\sigma^3-1\right)-\alpha^2(2\sigma-1)(3\sigma-1)^2(v+\alpha\sigma)}{t(2t+\alpha-3\alpha\sigma)^2}.$$

We find it useful to consider $\sigma > \frac{1}{3}$ and $\sigma < \frac{1}{3}$ separately.

For $\sigma > \frac{1}{3}$, we have that $v_{\min}$ ensures higher profits with targeting:

$$A^{M^*} - A^M|_{v=\alpha} = \frac{\alpha}{2t}\frac{\left(4t^2\alpha(2\sigma+1)(1-\sigma)+4t\alpha^2\left(\sigma-2\sigma^2+6\sigma^3-1\right)-\alpha^3(\sigma+1)(2\sigma-1)(3\sigma-1)^2\right)}{(2t+\alpha-3\alpha\sigma)^2} > 0.$$

Moreover, the difference between profits with and without targeting becomes greater as v increases:

$$\frac{d(A^{M^*}-A^M)}{dv} =$$
$$\frac{1}{2t}\frac{\alpha}{(2t+\alpha-3\alpha\sigma)^2}\left(4t^2\left(1-\sigma\right)+2t\alpha\sigma\left(9\sigma-5\right)+4tv\left(2\sigma-1\right)-\alpha^2\left(2\sigma-1\right)\left(3\sigma-1\right)^2\right) > 0.$$

Hence, $A^{M^*} > A^M$ for all $\sigma > \frac{1}{3}$.

We then consider $\sigma \leq \frac{1}{3}$. If t is low, a reduction in the subscription price will turn many exclusive consumers into shared consumers. But if the shared consumers are not worth much in the ad market, the benefit for the platform is limited. This implies that even though targeting increases the ad price, it does not necessarily increase ad revenues when $\sigma < \frac{1}{3}$. We illustrate with an example:

Suppose that $\sigma = 0$, which yields

$$A^{M^*} - A^M = \alpha\frac{\left(2t-v\right)\left(2tv-\alpha^2\right)}{2t\left(2t+\alpha\right)^2}$$

The expression is positive if $2tv > \alpha^2$. If t and v are not sufficiently high relative to α, this is not the case. Consider, for instance, $v = 0.6$, $t = 0.55$ and $\alpha = 1$. The numerator $\left(2t-v\right)\left(2tv-\alpha^2\right)$ then equals -0.17, which implies that $A^{M^*} - A^M < 0$.

Platform profits We then analyze the platform profits. By inserting equations (15) and (14) into (8), we find the equilibrium profits with and without targeting, respectively, when consumers multi-home:

$$\pi^{M^*} = \frac{(t-\alpha\sigma)(\alpha+v)^2}{(\alpha(1-3\sigma)+2t)^2} + \frac{\alpha(2t-v)}{(\alpha(1-3\sigma)+2t)} - \alpha\sigma\left(1+\frac{2\alpha-v}{\alpha(1-3\sigma)+2t}\right) \tag{A.1}$$

and

$$\pi^M = \frac{1}{4t}(v^2+2v\alpha(2\sigma-1)+3\alpha^2\sigma^2-2\alpha^2\sigma-4t\alpha(1-\sigma)). \tag{A.2}$$

Whether higher ad revenues compensate for lower subscription revenues is dependent on σ. We start by considering $\sigma > \frac{1}{3}$. The minimum $v-$value is given by $v_{\min} = \alpha + \varepsilon$. By definition, $\sigma_{\min} = \frac{1}{3} + \varepsilon$. Evaluating multi-homing profits of equation (A.1) and (A.2) at $v_{\min}$ and $\sigma_{\min}$, we have that targeting in both cases provides greater profits ($\pi^{M^*} > \pi^M$):

$$\pi^{M^*} - \pi^M|_{v\min} \approx \frac{1}{4}\alpha^2\frac{(2t-\alpha)^2+9\alpha\sigma^3(4t-3\alpha\sigma)+6\sigma^2\left(-2\alpha t+3\alpha^2-2t^2\right)-4\sigma\left(\alpha t+2\alpha^2-2t^2\right)}{t(2t+\alpha-3\alpha\sigma)^2} > 0$$

and

$$\pi^{M^*} - \pi^M|_{\sigma\min} = \frac{1}{12}\frac{\alpha}{t^2}\left(4tv-(v+\alpha)^2\right) > 0.$$

Moreover, for $v_{\min}$ we have that $(\pi^{M^*} - \pi^M)$ is increasing in σ :

$$\frac{d(\pi^{M^*}-\pi^M)}{d\sigma}|_{v\min} \approx \frac{1}{2}\frac{\alpha^2(\alpha^3(3\sigma+1)(3\sigma-1)^3-8t^3(3\sigma-1)-2\alpha^2t\left(-9\sigma-27\sigma^2+81\sigma^3+11\right)+12\alpha t^2\left(-2\sigma+9\sigma^2+1\right)}{t(\alpha+2t-3\alpha\sigma)^3} > 0.$$

Similarly, $(\pi^{M^*} - \pi^M)$ is increasing in v for $\sigma_{\min}$:

$$\frac{d(\pi^{M^*}-\pi^M)}{dv}|_{\sigma\min} = \frac{1}{6t^2}\alpha\left(2t-v-\alpha\right) > 0.$$

Finally, higher $v-$values enhance the increase in $(\pi^{M^*} - \pi^M)$ in response to a change in σ :

$$\frac{d\left(\frac{d(\pi^{M^*}-\pi^M)}{d\sigma}\right)}{dv} = \frac{1}{2}\alpha\frac{2\alpha^3(3\sigma-1)^3+12\alpha t^2(5\sigma-1)-4\alpha^2t\left(5-18\sigma+27\sigma^2\right)+4tv(4t-\alpha-3\alpha\sigma)-8t^3}{t(\alpha+2t-3\alpha\sigma)^3} > 0.$$

The numerator is increasing in v, and since it is positive for $v_{\min}$ it is also positive for larger $v-$values. In sum, targeting is profitable if $\sigma > \frac{1}{3}$.

Consider then the case where $\sigma < \frac{1}{3}$. In this case, targeting does not necessarily increase ad revenues. Since targeting also reduces subscription revenues, it might lead to lower profits. ■

Proof of Proposition 3

The proof consists of three parts:

(i) a) **Subscription prices:** The subscription prices are given in equation (15). The difference in the prices, $p^{S^*} - p^{M^*} = \frac{15\alpha^2\sigma - 5\alpha t - 5\alpha^2 + 2t^2 - 3\alpha t\sigma}{\alpha + 2t - 3\alpha\sigma}$, is greater than zero for $\sigma > \frac{2}{3}$. For lower values of σ, this requires the additional constraint

$$t > \frac{1}{4}\left(\sqrt{\alpha^2(9\sigma^2 - 90\sigma + 65)} + 3\alpha\sigma + 5\alpha\right).$$

b) **Consumer utility:** To show that consumer utility is higher with multi-homing, we need to compare the utility from subscribing to one platform with that of subscribing to two platforms. The utility from subscribing to one platform only is $u_i^{p=p^{S*}}(x = 1/2) = 5\alpha - \frac{3}{2}t$, whereas the utility from subscribing to both platforms is given by $u_{i+j}^{p=p^{M^*}}(x = 1/2) = t\frac{7\alpha - 2t + 3\alpha\sigma}{\alpha + 2t - 3\alpha\sigma}$. Multi-homing is preferred if

$$u_{i+j}^{p=p^{M^*}} - u_i^{p=p^{S*}} = \frac{1}{2}\frac{30\alpha^2\sigma - 3\alpha t - 10\alpha^2 + 2t^2 - 3\alpha t\sigma}{\alpha + 2t + 3\alpha\sigma} > 0,$$

which holds for $\sigma > \frac{2}{9}$. From the analysis of subscription prices, we know that consumers who subscribe to only one platform are also better off when $\sigma > \frac{2}{3}$. Hence, a sufficient condition for all consumers to be better off with multi-homing is that $\sigma > \frac{2}{3}$.

(ii) **Ad prices:** The ad prices are given by

$$\alpha^{M^*} = \alpha\frac{(2t + 5\alpha - 3\alpha\sigma)}{(2t + \alpha - 3\alpha\sigma)} \text{ and } \alpha^{S^*} = \frac{3}{2}\alpha.$$

The difference in ad prices $\alpha^{S^*} - \alpha^{M^*} = -\frac{1}{2}\alpha\frac{(7\alpha - 2t + 3\alpha\sigma)}{2t + \alpha - 3\alpha\sigma} < 0$, which states that the ad price is always higher with multi-homing.

(iii) **Profits:** The platform profits are given by equations (A.1) and (A.2). The difference is given by

$$\pi^{S^*} - \pi^{M^*} = \frac{\left(36\alpha^3\sigma^3 - 3\alpha^2\sigma^2(7\alpha + 10t) + 2\alpha\sigma\left(16\alpha t + 17\alpha^2 - 4t^2\right) + 4t^2(2t - 3\alpha) - \alpha^2(50t - 11\alpha)\right)}{4(\alpha + 2t - 3\alpha\sigma)^2}.$$

We find that single-homing profits cannot be greater than multi-homing profits if $\sigma \geq 0.65$ when Condition 2 holds.

■

Proof of Proposition 4

A stable equilibrium is one in which no player will deviate from a given strategy. We first examine the incentives to deviate from a multi-homing price setting, and then the incentives to deviate from a pure single-homing outcome. Finally, we find that the consumers facing single-homing prices will always deviate and subscribe to an additional platform, such that single-homing can never be part of an equilibrium.

(i) Deviation from multi-homing

Suppose that platform i believes that the rival prices according to the multi-homing regime: $p_j = \frac{v(t+\alpha)-\alpha(t+3v\sigma)}{2t+\alpha(1-3\sigma)}$. Could it be optimal for platform i to charge a higher price and only sell to consumers that do not subscribe to platform j?

We insert $p_j = \frac{v(t+\alpha)-\alpha(t+3v\sigma)}{2t+\alpha(1-3\sigma)}$ into the location of the indifferent consumer:

$$\widetilde{x} = \frac{1}{2} + \frac{p_j - p_i}{2t},$$

which yields

$$D_i = \frac{1}{2}\frac{2t^2 + 3\alpha t + 3\alpha^2 - 3\alpha\sigma\left(3\alpha + t\right) - p_i\left(\alpha + 2t - 3\alpha\sigma\right)}{t\left(\alpha + 2t - 3\alpha\sigma\right)}$$

and subscription price (superscript 'd' for deviation):

$$p_i^d = \frac{\left(2t - 3\alpha\right)\left(\alpha t + \alpha^2 + t^2\right) - 3\alpha\sigma\left(t^2 + \alpha t - 3\alpha^2\right)}{\left(2t - \alpha\right)\left(\alpha + 2t\right) - 3\alpha\sigma\left(2t - \alpha\right)}.$$

Compared to the equilibrium price with multi-homing, the deviation price is always higher if $\sigma > \frac{2}{3}$. The deviation profit is given by

$$\pi_i^d = \frac{1}{4}\frac{\left(12\alpha^2\sigma - 5\alpha t - 4\alpha^2 - 2t^2 + 3\alpha t\sigma\right)^2}{\left(2t - \alpha\right)\left(-\alpha - 2t + 3\alpha\sigma\right)^2}.$$

Comparing deviation profits with the multi-homing equilibrium profit, we find that

$$\pi_i^d - \pi^{M^*} = \frac{1}{4}\frac{4t^4+4\alpha t^3(5\sigma-3)-3\alpha^2t^2\left(10\sigma+29\sigma^2+13\right)+8\alpha^3t(3\sigma+7)\left(2-3\sigma+3\sigma^2\right)-4\alpha^4(\sigma-1)\left(1-30\sigma+9\sigma^2\right)}{(2t-\alpha)(-\alpha-2t+3\alpha\sigma)^2}.$$

The above shows that deviation is never profitable if $\sigma > 0.03$.

However, for multi-homing to be an equilibrium, it must be true that consumers will actually purchase both products when $p = p^{M^*}$.

From

$$u_i^{p=p^{M*}}(x=\frac{1}{2})=\frac{1}{2}t\frac{7\alpha-2t+3\alpha\sigma}{\alpha+2t-3\alpha\sigma}$$

and

$$u_{i+j}^{p=p^{M*}}(x=\frac{1}{2})=t\frac{7\alpha-2t+3\alpha\sigma}{\alpha+2t-3\alpha\sigma},$$

we see that $u_{i+j}^{p=p^M}-u_i^{p=p^M}=\frac{1}{2}t\frac{7\alpha-2t+3\alpha\sigma}{\alpha+2t-3\alpha\sigma}>0$ whenever Condition 2 holds, which confirms that some consumers want to multi-home. Hence, (some) multi-homing consumers have no incentives to deviate (subscribe to only one platform) when facing multi-homing prices, and there is a unique equilibrium with multi-homing. ■

(ii) Deviation from single-homing

If both platforms price according to single-homing, prices and profits are given by $p^{S^*}=t-2\alpha$ and $\pi^{S^*}=\frac{1}{4}(2t-\alpha)$. Suppose that platform i believes that platform j sets the single-homing price, p^{S^*}. If platform i deviates and sets the prices that maximize profits if also selling to some consumers who buy the rival's product, we get:

$$p_i^d=\frac{\alpha t(\sigma+1)-\alpha^2(11\sigma-5)}{2(t-\alpha\sigma)}.$$

Deviation profit is given by:

$$\pi_i^d=\frac{1}{4}\alpha\frac{25\alpha^3(\sigma-1)^2+8t^3(1-\sigma)+\alpha t^2(2\sigma+9\sigma^2+5)-10\alpha^2t(1-\sigma)(5-3\sigma)}{t^2(t-\alpha\sigma)}$$

and

$$\pi_i^d-\pi^{S^*}=\frac{1}{4}\frac{-2t^4+25\alpha^4(\sigma-1)^2-10\alpha^3t(1-\sigma)(5-3\sigma)+\alpha^2t^2(\sigma+9\sigma^2+5)+3\alpha t^3(3-2\sigma)}{t^2(t-\alpha\sigma)}.$$

Examining the above equations shows that deviation is profitable when $\sigma>0.1$. For $t-$values in the higher end of Condition 2, it might also be the case for $\sigma<0.1$.

Suppose next that for some $\sigma<0.1$, it is optimal for the platforms to set the single-homing price. This can only be an equilibrium if the consumers do not subscribe to both platforms at this price. We insert p^{S^*} into (1) and (3) for $x=\frac{1}{2}$ and find

$$u_i^{p=p^{S^*}}(x=\frac{1}{2})=\frac{1}{2}(10\alpha-3t)$$

and

$$u_{i+j}^{p=p^{S^*}}(x=\frac{1}{2})=10\alpha-3t.$$

We see that $u_{i+j}^{p=p^{S^*}} > u_i^{p=p^{S^*}}$, which implies that there exist consumers who want to subscribe to both platforms when $p=p^{S^*}$. By the same token, deviation is only possible if some consumers actually subscribe to both platforms at the deviation price. We insert p^{S^*} and p_i^d into (1) and (3), respectively, and find

$$u_i^{p=p^{S^*}}(x=\frac{1}{2})=\frac{1}{2}(10\alpha-3t)$$

and

$$u_{i+j}^{p=p_i^d}(x=\frac{1}{2})=\frac{1}{2}\frac{3\alpha t(\sigma+5)-5\alpha^2(\sigma+1)-4t^2}{t-\alpha\sigma}.$$

At $x=\frac{1}{2}$, $u_{i+j}^{p=p_i^d} > u_i^{p=p^{S^*}}$, and there exist consumers who want to multi-home. Some consumers have incentives to deviate and subscribe to more platforms when facing single-homing prices. Therefore, single-homing can never take part in an equilibrium. ■

A.3 Robustness

In the equilibrium analysis, we assume that $v=3\alpha$. This provides us with a more tractable set of constraints. The drawback is that it might bring the robustness of the findings into question. To shed some light on this issue, we take a closer look at how the results depend on v.

First, note that π^{S^*} does not depend on v, while $\partial\pi^{M^*}/\partial v>0$ if $v>\mu\equiv\frac{\alpha^2\left(1-2\sigma+3\sigma^2\right)-2\alpha t\sigma}{2(t-\alpha\sigma)}$. Since $\partial\mu/\partial t<0$, the requirement is strictest for $t_{\min}$. From the conditions, we know that the lowest possible t is given by $\frac{5}{2}\alpha$. This gives $\mu|_{t=2\alpha}=\alpha\frac{7\sigma-3\sigma^2-1}{2\sigma-5}$, which is at its highest when $\sigma=0$ and yields $\mu=\frac{1}{5}\alpha$. Hence, multi-homing becomes relatively more profitable compared to single-homing for all $v>\frac{1}{5}\alpha$.

We then consider how v affects the platforms' incentives to deviate from committing to single-homing and multi-homing. Suppose that we do not fix v. If the rival commits to single-homing, the deviation profit is given by

$$\pi_i^{d,S} = \frac{1}{4}\frac{t^2v(v+2\alpha\sigma)+\alpha^2(\sigma-1)^2(2\alpha+v)^2+8\alpha t^3(1-\sigma)-2\alpha tv(\sigma-1)(-4\alpha-v+3\alpha\sigma)+\alpha^2t^2\left(-4\sigma+9\sigma^2-4\right)-4\alpha^3t(3\sigma-2)(\sigma-1)}{t^2(t-\alpha\sigma)}.$$

Consequently, $\frac{d(\pi_i^d-\pi^{S^*}))}{dv} > 0$ if $v > \lambda \equiv \frac{\alpha^2t(\sigma-1)(3\sigma-4)-2\alpha^3(\sigma-1)^2-\alpha t^2\sigma}{(t-\alpha+\alpha\sigma)^2}$. This is ensured by condition $(t < v + \sigma\alpha)$ for all $\sigma > 0.01$.

Proof:

$v > t - \sigma\alpha > \lambda$ if $t - \sigma\alpha - \lambda > 0$. We have that:

$$t - \sigma\alpha - \lambda = \frac{t^3 + \alpha^3 (2-\sigma)(\sigma-1)^2 + t\alpha^2(4\sigma-3)(1-\sigma) - 2t^2\alpha(1-\sigma)}{(t-\alpha+\alpha\sigma)^2}$$

The expression is greater than 0 for all $\sigma > 0.01$. ■

Unless multi-homing consumers are almost worthless in the ad market, it is certainly more tempting to deviate from single-homing if v increases from 3α.

Similarly, we consider the case with the rival committing to multi-homing. Deviation profit is then

$$\pi_i^{d,M} = \frac{1}{4}\frac{(2t\alpha + v\alpha + \alpha^2 - 3\alpha^2\sigma + tv + 2t^2 - 3t\alpha\sigma - 3v\alpha\sigma)^2}{(2t-\alpha)(2t+\alpha-3\alpha\sigma)^2}$$

From the profit expression, we find that $\frac{d(\pi_i^d-\pi^{M^*})}{dv} < 0$ if

$$v > \mu \equiv \frac{2t^3 - \alpha^3(3\sigma+1)(1-\sigma) - t^2\alpha(17\sigma-4) + t\alpha^2(7-16\sigma+21\sigma^2)}{(t-\alpha+\alpha\sigma)(7t+\alpha-9\alpha\sigma)}.$$

This is ensured by condition $(t < v + \sigma\alpha)$ for all $\sigma > 0.02\dot{6}$.

Proof:

$v > t - \sigma\alpha > \mu$ if $t - \sigma\alpha - \mu > 0$. We have that

$$t - \sigma\alpha - \mu = \frac{5t^3 + \alpha^3(1-\sigma)(4\sigma - 9\sigma^2 + 1) - 4t\alpha^2(2 - 8\sigma + 7\sigma^2) + 2t^2\alpha(4\sigma-5)}{(t-\alpha+\alpha\sigma)(7t+\alpha-9\alpha\sigma)}$$

is positive for $\sigma > 0.026$. ■

Deviation is less tempting if v increases from 3α, at least if $\sigma \geq 0.02\dot{6}$.

Suppose that $\sigma = 0$, which yields $\mu_{\max} = \frac{1}{(t-\alpha)(7t+\alpha)}\left(2t^3 + 4t^2\alpha + 7t\alpha^2 - \alpha^3\right)$ since $\frac{d\mu}{d\sigma} < 0$. For $t \equiv t' \leq 6.57\alpha$, we find that $v \geq 3\alpha \geq \mu$.

$\frac{d\mu_{\max}}{dt'} > 0$. We have that $v > t'$ (condition $(t < v + \sigma\alpha)$). This implies that if $t' > \mu_{\max}$, then $v > \mu_{\max}$. Since

$$t' - \mu_{\max} = \frac{\alpha^3 - 8t\alpha^2 - 10t^2\alpha + 5t^3}{(t-\alpha)(7t+\alpha)} > 0$$

Deviation is less tempting for $v > 3\alpha$ also if $\sigma = 0$.

In this section, we have shown that our results are quite robust. Given that multi-homing consumers are not negligible in the ad market, our results hold for all $v > 3\alpha$. A higher v does not make single-homing more attractive relative to multi-homing and it does not reduce the incentives to deviate from single-homing. Moreover, a higher v makes it less imperative to deviate from multi-homing. Evaluating $v-$values below 3α is less interesting since the lower boundary is given by $v = 2.75\alpha$.

Consumers

For a general v, we check whether consumers will subscribe to both platforms when $p = p^{M^*}$. Inserting (15) into (1) and (3), we find:

$$u_i^{p=p^{M^*}}(x = \frac{1}{2}) = \frac{1}{2}t\frac{2v + \alpha + 3\alpha\sigma - 2t}{2t + \alpha - 3\alpha\sigma}$$

and

$$u_{i+j}^{p=p^{M^*}}(x = \frac{1}{2}) = t\frac{2v + \alpha + 3\alpha\sigma - 2t}{2t + \alpha - 3\alpha\sigma}.$$

It follows that if $u_{i+j}^{p=p^{M^*}} > u_i^{p=p^{M^*}}$ for $v = 3\alpha$, the same is true for any other v-value as well. ■

In this paper, we address how targeting and consumer multi-homing impact platform competition and market equilibria in two-sided markets. We analyze platforms that are financed by both advertising and subscription fees, and let them adopt a targeting technology with increasing performance in audience size: a larger audience generates more consumer data, which improves the platforms' targeting ability and allows them to extract more ad revenues. Targeting therefore increases the importance of attracting consumers. Previous literature has shown that this could result in fierce price competition if consumers subscribe to only one platform (i.e. single-home). Surprisingly, we find that pure single-homing possibly does not constitute a Nash equilibrium. Instead, platforms might rationally set prices that induce consumers to subscribe to more than one platform (i.e. multi-home). With multi-homing, a platform's audience size is not restricted by the number of subscribers on rival platforms. Hence, multi-homing softens the competition over consumers. We show that this might imply that equilibrium profit is higher with than without targeting, in sharp contrast to what previous literature predicts.

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