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**A non-parametric model of the timecharter-equivalent spot  
freight rate in the Very Large Crude oil Carrier market**

**by**

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# **A non-parametric model of the timecharter-equivalent spot freight rate in the Very Large Crude oil Carrier market**

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## **Non-technical Summary**

This paper presents a new way of modelling the timecharter equivalent spot freight rate in the VLCC market. Using monthly data from January 1989 to December 1998, the empirical results indicate that high freight rates are expected to drift downwards while the expected change at low to medium freight rate levels is zero. Moreover, the rate of change (volatility) is increasing progressively in the freight rate level. The market price of freight rate risk is close to zero for most freight rate levels indicating that shipowners are not compensated for the risk associated with trading in the spot market.

# 1. Theoretical background

Consider a continuous-time diffusion process, satisfying a time-homogeneous stochastic differential equation (SDE):

$$dX_t = \mu(X_t)dt + \sigma(X_t)dZ_t \quad (1)$$

where  $Z$  is a standard Brownian motion, and the drift and diffusion terms  $\mu$  and  $\sigma$  are, respectively, the instantaneous mean and variance of the process. Following the notation of Stanton (1997), the conditional expectation of an arbitrary function  $f$  can be written, under suitable restrictions on  $\mu$ ,  $\sigma$ , in the form of a Taylor series expansion:

$$E_t[f(X_{t+\Delta}, t + \Delta)] = f(X_t, t) + Lf(X_t, t)\Delta + \frac{1}{2}L^2f(X_t, t)\Delta^2 + \dots + \frac{1}{n!}L^n f(X_t, t)\Delta^n + O(\Delta^{n+1}) \quad (2)$$

where  $L$  is the infinitesimal generator of the process  $\{X_t\}$ . Ignoring all higher-order terms gives us a first-order approximation for  $Lf$ :

$$Lf(X_t, t) = \frac{1}{\Delta} E_t[f(X_{t+\Delta}, t + \Delta) - f(X_t, t)] + O(\Delta) \quad (3)$$

To derive approximations of the drift,  $\mu$ , consider the function  $f(x,t) \equiv x$ . From the definition of  $L$  we have that the drift  $\mu(x) = Lf(x,t)$ . Substituting into equation (3) leads to the following first-order approximation for  $\mu$ :

$$\mu(X_t) = \frac{1}{\Delta} E_t[X_{t+\Delta} - X_t] + O(\Delta) \quad (4)$$

Similarly, to construct approximations to the diffusion,  $\sigma$ , consider the function

$f(x,t) \equiv (x - X_t)^2$ . From the definition of  $L$ , we have:

$$Lf(x, t) = 2(x - X_t)\mu(x) + \sigma^2(x) \quad \text{and so} \quad (5)$$

$$Lf(X_t, t) = \sigma^2(X_t) \quad (6)$$

Substituting into equation (3) yields a first-order approximation for  $\sigma^2$ :

$$\sigma^2(X_t) = \frac{1}{\Delta} E_t \left[ (X_{t+\Delta} - X_t)^2 \right] + O(\Delta) \quad (7)$$

In general, the higher the order of the approximation, the faster it will converge to the true drift and diffusion of the process given in equation (1), as we observe the variable  $X_t$  at finer and finer time intervals. Unfortunately, the software used in this exercise (Eviews) does not allow non-parametric estimation with more than two variables.

An approximation to the market price of freight rate risk can be constructed in an analogous manner. Following Stanton (1997), the first order approximation is:

$$\lambda(X_t) = \frac{\sigma(X_t)}{\Delta(\sigma^{(1)}(X_t) - \sigma^{(2)}(X_t))} E_t (R_{t,t+\Delta}^{(1)} - R_{t,t+\Delta}^{(2)}) + O(\Delta)$$

where  $R_{t,t+\Delta}^{(i)}$  is the holding period return on asset  $i$  between times  $t$  and  $t + \Delta$  and  $\sigma^{(i)}$  is the instantaneous volatility of asset  $i$ .

## 2. Empirical results

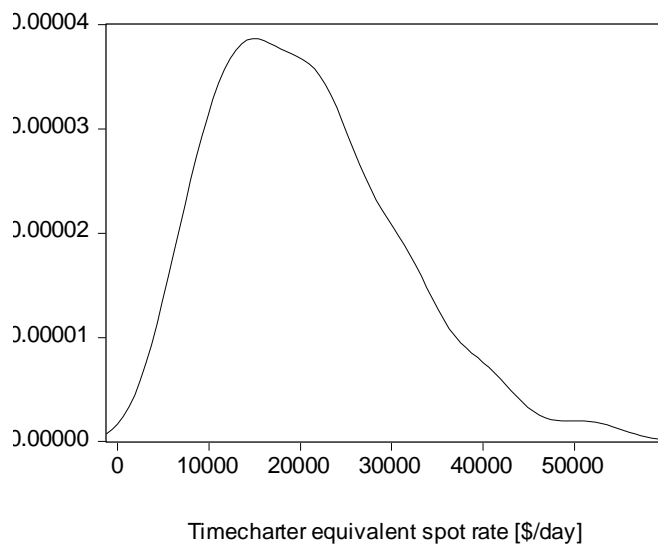
A kernel estimation procedure is used to estimate the conditional expectations in (4) and (7). The data used to form the density estimators consists of discrete observations of the spot freight rate  $\{X_1, \dots, X_n\}$  sampled at interval  $\Delta$ . The non-parametric kernel estimator of the marginal density is given by:

$$\hat{f}(u) = \frac{1}{n} \sum_{i=1}^n \frac{1}{h_n} K\left(\frac{u - x_i}{h_n}\right) \quad (8)$$

where  $K(\cdot)$  is a kernel function that integrates to one and  $h_n$  the bandwidth.

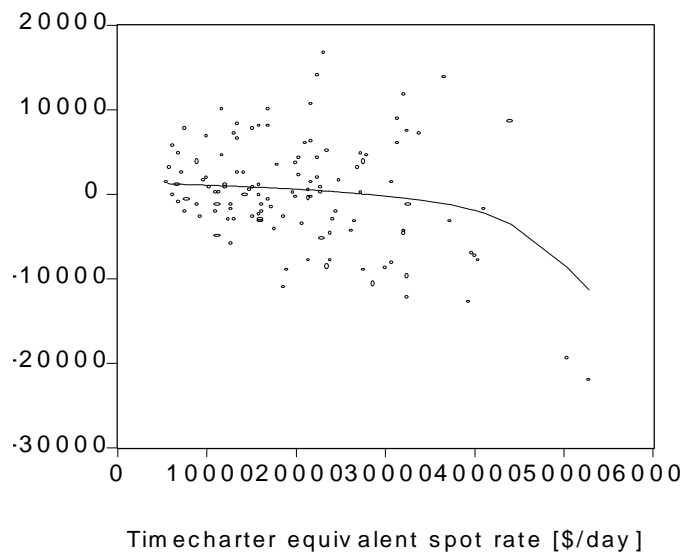
One can think of (8) as being a "smooth histogram" where the density at any point is estimated as the average of densities centered at the actual data points. Some regularity conditions and restrictions on the choice of the kernel and bandwidth apply. However, results in the kernel estimation literature show that any reasonable kernel gives almost optimal results. In this case, the Gaussian kernel has been used. Using Silverman (1986) rule of thumb, which is incorporated in the software, the optimal bandwidth for our data is  $h = 3424.2$ .

**Figure 1: Estimated marginal density of spot freight rate**



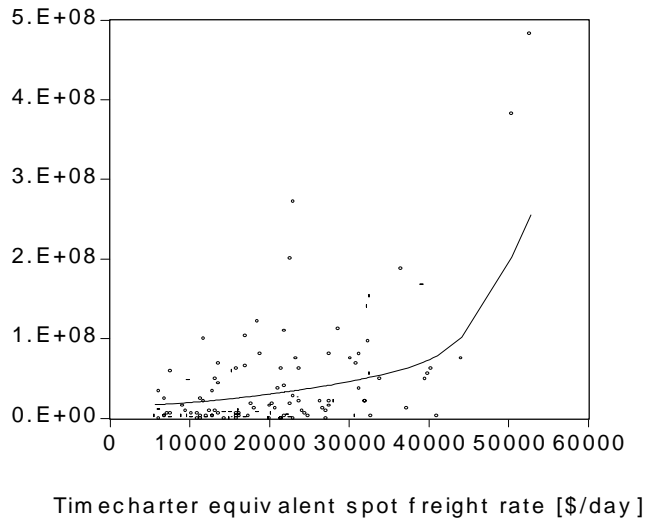
Given the estimated density, one can calculate any desired moments from the distribution. Note that the first-order approximations to the conditional expectations (4) and (7) is equivalent to determining a non-linear regression line between the monthly changes/squared changes and the freight rate the previous month.

**Figure 2: Estimated drift of freight rate process**



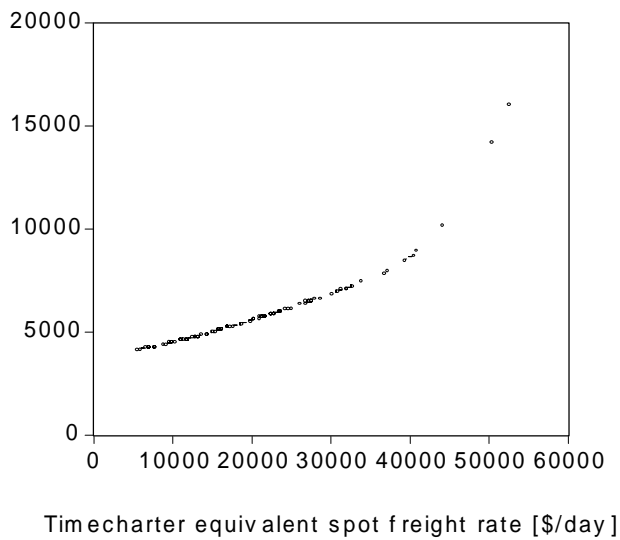
Looking first at figure 2, we see that the estimated drift does not look linear. For low and medium values of the spot freight rate, there is only very slight mean reversion. As the freight rate increases beyond about \$35,000 per day, the estimated drift drops sharply. This is in line with the notion of mean reversion. The decline in the drift that we estimate at high freight rates has the effect of preventing freight rates from exploding towards infinity, despite the increase in volatility. However, there are too few observations to get a statistically confident estimate. This emphasizes the greater data requirements of non-parametric techniques compared with their parametric counterparts. Unfortunately, the kernel regression option in Eviews does not incorporate the calculation of confidence bands. Numerical methods such as the Kunsch (1989) block bootstrap algorithm could have been used for this purpose, but it seems clear that the low number of highly scattered observations would lead to wide bands. Presumably it would not be possible to reject that the drift  $\mu = 0$  for most freight rate levels at the 95% level of confidence.

**Figure 3: Estimated  $\sigma^2(X)$  of freight rate process**



As for the estimated  $\sigma^2(X)$ , it increases with the freight rate, implying increasing volatility in the freight rate level. However, the diffusion process is given by the square root of the estimates, depicted in the figure below.

**Figure 4: Estimated diffusion function**

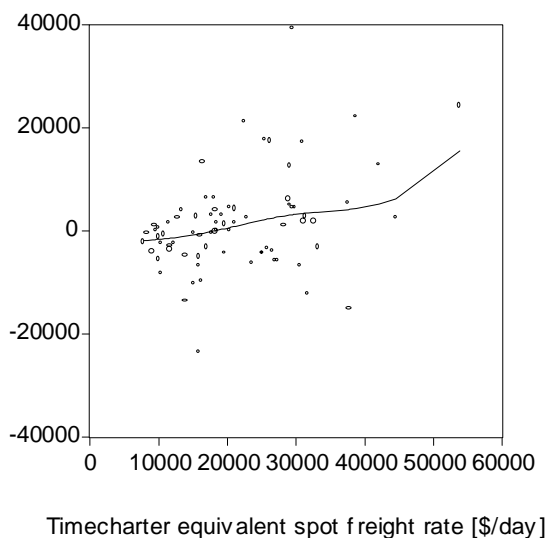




The estimated instantaneous rate of change (volatility) clearly increases in the freight rate level. For low and medium freight rates, the diffusion function  $\sigma(X)$  is close to linear, while it is increasing progressively for very high freight rates. Again, however, the low number of observations makes inference dubious. Moreover, it is natural to assume that  $\sigma(0) = 0$ , a condition which prevents freight rates from becoming negative. This is not imposed in the estimation above.

The only available market prices for freight rate dependent assets are the vessels themselves. However, these assets pay dividends, in the sense that any daily profit from operation is paid to the owner. Thus, to get a consistent time series for a "non-dividend" paying asset, these profits & losses need to be added to the asset value in any given period. The two assets are a five-year old and a ten-year old VLCC. The resulting non-parametric estimate for the market price of freight rate risk is illustrated below.

**Figure 5: Estimated market price of freight rate risk**



The market price of risk is close to zero for low and medium freight rates and increasing in the freight rate level, corresponding to an increasing premium for bearing freight rate risk. For high freight rates, although there are few observations, the estimates suggest a large positive market price of freight rate risk. This is consistent with the observed freight rate premium in the spot market (on average). The drift in a risk-neutral world is given by the difference  $(\mu(X) - \lambda(X))$ , thus the market price of risk has the effect of reinforcing the mean reverting property of the risk-neutral drift term.

### 3. Implications for vessel valuation

By using third- and fourth-order polynomial approximations for the drift, diffusion, and market price of risk (ref. appendix), it is straightforward to calculate the vessel value (the expected present value of future earnings) using Monte Carlo simulation of the risk-adjusted freight rate process. In the table below, the results are compared to vessel values calculated using  $\lambda = 0$  as in previous research (e.g. Tvedt 1997).

**Table 1 : Effect of price of risk on vessel valuation**

Freight rate	Vessel value [million]	Vessel value ( $\lambda = 0$ )	Difference
\$10,000/day	5.257	5.041	4.1%
\$20,000/day	5.717	5.517	3.4%
\$30,000/day	8.235	7.522	8.7%
\$40,000/day	14.373	11.307	21.3%

Calculated using risk-free rate of  $r = 6\%$  p.a., scrap value  $S = \$5$  million, lay-up level  $m = \$2,000/\text{day}$  and a maximum remaining trading life of  $T = 10$  years.

The introduction of a non-zero market price of risk has a large impact on the vessel valuation, and more so for high freight rates.

## **4. Conclusions**

By not specifying a particular parametric form, non-parametric techniques avoid the possibility of misspecification, but at the expense of greater estimation error than their parametric counterparts. As Jiang (1998) points out, the approximations used in this paper can be extremely non-robust in that the estimates can be very sensitive to the sampling path. Moreover, the performance of the "naïve" first-order approximations deteriorates as the sampling frequency and the number of observations decrease, and the approximation errors introduced may be significant when monthly observations are used. However, as a first cut, the results are interesting. The hypotheses regarding mean reversion and increasing volatility in the freight rate level have support in the data, and a functional form of the market price of risk has never been documented in this market previously. Moreover, the introduction of a non-zero market price of risk has a large impact on the vessel valuation.

## Appendix A: Parameterizations of $\mu$ , $\sigma$ and $\lambda$

Dependent Variable: SIGMA

Method: Least Squares

Sample(adjusted): 1 119

Included observations: 119 after adjusting endpoints

SIGMA=C(1)+C(2)\*TCE+C(3)\*TCE^2+C(4)\*TCE^3

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	2572.952	118.1853	21.77049	0.0000
C(2)	0.277964	0.016633	16.71162	0.0000
C(3)	-1.02E-05	6.85E-07	-14.87203	0.0000
C(4)	1.81E-10	8.37E-12	21.62400	0.0000
R-squared	0.991593	Mean dependent var	5803.109	
Adjusted R-squared	0.991374	S.D. dependent var	1704.033	
S.E. of regression	158.2658	Akaike info criterion	12.99946	
Sum squared resid	2880526.	Schwarz criterion	13.09288	
Log likelihood	-769.4681	Durbin-Watson stat	0.256956	

Dependent Variable: MU

Method: Least Squares

Sample(adjusted): 1 119

Included observations: 119 after adjusting endpoints

MU=C(1)+C(2)\*TCE+C(3)\*TCE^2+C(4)\*TCE^3+C(5)\*TCE^4

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	524.6533	81.28997	6.454097	0.0000
C(2)	0.205725	0.016279	12.63764	0.0000
C(3)	-2.06E-05	1.08E-06	-19.13366	0.0000
C(4)	6.92E-10	2.83E-11	24.44634	0.0000
C(5)	-8.63E-15	2.54E-16	-33.98978	0.0000
R-squared	0.998733	Mean dependent var	239.1783	
Adjusted R-squared	0.998689	S.D. dependent var	1577.832	
S.E. of regression	57.12908	Akaike info criterion	10.96961	
Sum squared resid	372065.4	Schwarz criterion	11.08638	
Log likelihood	-647.6919	Durbin-Watson stat	0.434870	

Dependent Variable: LAMBDA

Method: Least Squares

Sample(adjusted): 1 75

Included observations: 75 after adjusting endpoints

SLAMBDA=C(1)+C(2)\*STCE+C(3)\*STCE^2+C(4)\*STCE^3

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-6426.709	629.2856	-10.21271	0.0000
C(2)	0.604703	0.081710	7.400579	0.0000
C(3)	-1.73E-05	3.12E-06	-5.532314	0.0000
C(4)	2.45E-10	3.58E-11	6.850036	0.0000
R-squared	0.967162	Mean dependent var	1060.938	
Adjusted R-squared	0.965774	S.D. dependent var	2681.105	
S.E. of regression	496.0094	Akaike info criterion	15.30293	
Sum squared resid	17467801	Schwarz criterion	15.42652	
Log likelihood	-569.8597	Durbin-Watson stat	0.560160	

## Appendix B: Data

TC equivalent, built  
mid-1970's

Values refer to average of period  
[USD/day] from Fearnleys

Jan-89	18665	Jan-93	20929	Jan-97	20301
Feb-89	7675	Feb-93	17504	Feb-97	22413
Mar-89	5634	Mar-93	15990	Mar-97	24341
Apr-89	6981	Apr-93	15826	Apr-97	21251
May-89	11900	May-93	13549	May-97	27272
Jun-89	16451	Jun-93	15986	Jun-97	27594
Jul-89	14425	Jul-93	23869	Jul-97	31395
Aug-89	14403	Aug-93	16013	Aug-97	37347
Sep-89	17020	Sep-93	17031	Sep-97	33986
Oct-89	27126	Oct-93	16272	Oct-97	41059
Nov-89	30181	Nov-93	15116	Nov-97	39304
Dec-89	21499	Dec-93	12512	Dec-97	26388
Jan-90	21923	Jan-94	9560	Jan-98	21833
Feb-90	23136	Feb-94	6823	Feb-98	28198
Mar-90	39651	Mar-94	7846	Mar-98	32711
Apr-90	32649	Apr-94	7104	Apr-98	31533
May-90	22881	May-94	6179	May-98	40490
Jun-90	23681	Jun-94	6035	Jun-98	32700
Jul-90	28721	Jul-94	9179	Jul-98	40100
Aug-90	18086	Aug-94	12986	Aug-98	32700
Sep-90	21498	Sep-94	11210	Sep-98	20400
Oct-90	13561	Oct-94	11317	Oct-98	24700
Nov-90	21854	Nov-94	10189	Nov-98	22600
Dec-90	32313	Dec-94	12213	Dec-98	26700
Jan-91	44128	Jan-95	13315		
Feb-91	52819	Feb-95	10440		
Mar-91	30867	Mar-95	11174		
Apr-91	22599	Apr-95	8998		
May-91	36742	May-95	7781		
Jun-91	50386	Jun-95	15412		
Jul-91	30823	Jul-95	22984		
Aug-91	32157	Aug-95	23038		
Sep-91	27466	Sep-95	17851		
Oct-91	32167	Oct-95	13681		
Nov-91	27718	Nov-95	20149		
Dec-91	18727	Dec-95	19882		
Jan-92	16138	Jan-96	20123		
Feb-92	12999	Feb-96	23795		
Mar-92	7215	Mar-96	19092		
Apr-92	9781	Apr-96	10121		
May-92	11304	May-96	17007		
Jun-92	6329	Jun-96	25100		
Jul-92	12035	Jul-96	26842		
Aug-92	12806	Aug-96	23647		
Sep-92	11511	Sep-96	15027		
Oct-92	11788	Oct-96	15385		
Nov-92	21707	Nov-96	16142		
Dec-92	21456	Dec-96	13277		

## References

Tvedt, Jostein (1997): «Valuation of VLCCs under income uncertainty». *Maritime Policy and Management*, Vol. 24, No. 2, pp. 159-174.

Stanton, Richard (1997): «A Nonparametric Model of Term Structure Dynamics and the Market Price of Interest Rate Risk», *Journal of finance*, 52, pp 1973 - 2002.

Jiang, George (1998): "Nonparametric Modeling of U.S. Interest Rate Term Structure Dynamics and Implications on the Prices of Derivative Securities", *Journal of Financial and Quantitative Analysis*, Vol 33, No. 4, December 1998.

Kunsch, H. R. (1989): "The Jackknife and the bootstrap for general stationary observations", *Annals of Statistics* 17, 1217 - 1241.

Silverman, B. W. (1986): "Density estimation for Statistics and Data Analysis", Chapman and Hall, London