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A Steel Industry Model

A simulator for analyzing effects on demand for ocean shipping
from regulating the carbon emissions of steel production

by

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Abstract

An international regulation of carbon dioxide emissions could affect the shipping industry *directly* in terms of increased fuel cost, which when passed on to customers as higher freight rates presumably would reduce demand for transportation. In addition, there would be *indirect* effects. A large user of shipping services, like the steel industry depending on long transportation legs for its inputs of iron ore and coal, has its own emission problems from burning of fossil fuel. Almost 10% of global emissions of carbon dioxide are attributable to this industry. It turns out that the production technology that has the largest emissions per unit of steel, namely the basic oxygen furnace, also is the most transportation intensive. Under a carbon regulation scenario one could expect a reallocation towards the electric arc technology, which rely on electricity and scrap metal with much lower transportation needs. A structural model of the markets involved is built in order to learn more about the magnitudes of transportation changes that could result from regulation of carbon emissions from the steel industry.

This paper deals with the modeling of such a simulator, while Mathiesen and Mæstad (2001) and Mæstad et.al. (2000) report the results from simulations.

1. Introduction

The objective of this paper is to describe a simulator for assessing some effects for the shipping industry of an international carbon emission regulation. A carbon regulation could obviously affect the shipping industry *directly* in terms of increased fuel cost or otherwise. Mæstad et. al. (2000) find that '[t]he share of transport costs in cif-prices is so small in most cases that [...] the price increase for final consumers will be negligible.'

In this paper we consider *indirect* effects. A large user of shipping services, like the steel industry depending on long transportation legs for its inputs of iron ore and coal, has its own emission problems from burning of fossil fuel.[1] An international carbon regulation will affect its operations, and thereby its demand for shipping services. Our choice of the steel industry is not arbitrary. Transportation of iron ore and metallurgical coal amounts to 50 % of total dry bulk, and hence the demanded volumes and choice of transportation legs for these two factors of steel production determine freight rates in dry bulk shipping. The interesting observations from our point of view are that different steel technologies rely on very different factor input combinations, and the production technology that has the largest carbon emissions per unit of steel, namely the basic oxygen furnace, also most intensively employs shipping transportation. Under a carbon regulation scenario one could expect a reallocation towards the electric arc technology, which rely on electricity and scrap metal with negligible demand for shipping services. Substitution between inputs to steel production could thus imply a fairly sizeable reduction in demand for shipping transportation.

Our modeling strategy is based on the following ideas. In order to learn the direction and magnitude of changes in demand for shipping services we go behind the demand functions and study a customer industry, namely steel production and its possible future. We employ the concept of a market equilibrium. This is an idealized situation obtained after all repercussions from a shock like a carbon regulation, have worked their way through the economy. We do not consider the process whereby markets regain equilibrium; our analysis is that of comparative statics. This is a model for the short to medium run, i.e., a time frame within which capacities at the various stages of the industry are largely fixed. Even though we model several related markets, the model is partial equilibrium in nature, where prices in the remaining markets are exogenously given. As usual, the delineation of the relevant markets, that is, what should be inside the model and what

[1] The production of iron and steel accounts for about 7% of man-made carbon emissions. Adding the emissions from mining of iron ore and metallurgical coal and related transportation raises this percentage to about 10.

should remain outside, is a delicate modeling issue. Our model is a collection of conditions for the various sectors in the industry to be in equilibrium. This yields a complementarity model, which is not novel[2], but also is not the mainstream modeling framework. Finally, our opinion is that an equilibrium model is not mainly a predictor of prices and activity levels, but rather a provider of insight into direction and strength of the net consequences of the various mechanisms in the markets.

Mathiesen and Mæstad (2001) and Mæstad et al. (2000) report results from our analyses. Let us here just observe that our main hypothesis is borne out. A tax of $25 per ton CO_2 reduces aggregate steel production by 2.8% and the total shipping demand from the steel industry by 5.3%. It turns out that in addition to our hypothesized mechanism of *reallocation* of production between the basic oxygen furnace and the electric arc, the so-called mini-mills, a fairly large *substitution* away from iron ore toward scrap metal takes place within the basic oxygen furnace.

The layout of the paper is as follows. In order to develop an understanding of the economic content of our modeling format, the next chapter presents a simple model that consists of three sets of equations describing supply, demand and equilibrium in a spatial market. We observe which issues this model can provide insight into. In Chapter 3, we describe the economic contents of extensions to this core model. Chapter 4 presents the full model that consists of eleven sets of equations. It includes features like supply of and demand for various factors of production and shipping requirements for the transportation of these factors. It describes production of different types of steel by use of different technologies, the transportation of produced steel, and the demand for these types of steel. Finally, a carbon regulation introduced through a carbon tax or a price for a carbon quota, adds to the cost of factor usage and hence steel production, and the emission of carbon dioxide from the use of factors is computed. Throughout we are particularly concerned with the sources of substitutability in the industry and how these can be modeled. Chapter 5 concludes.

[2] See Mathiesen (1985), Mathiesen and Lont (1984), Mathiesen, et.al. (1986), Rutherford (1995) and Light (1999).

2. The core model

This is a model of the market for one product being produced in several regions and shipped to meet demand in possibly another set of regions. The model may be applied to describe the market for a wide range of commodities: A resource like crude oil, natural gas or coal, renewables like timber or other forest or agricultural products, or a round fish or a lightly processed fish product. Further examples are the outputs of process industries, like paper, pulp or newsprint, a primary metal, some basic material, a chemical or a petrochemical, or output from some manufacturing industry or even a service like shipping.

For the sake of simplicity of presentation, let us assume that the product is homogeneous, that is, the product of one region is held by customers to be a perfect substitute of the product of other regions. Eventually, in our model of the steel industry, steel is distinguished both by technology and by region of origin.

We shall assume that all agents behave competitively so that this is a model of a competitive industry. Even though there certainly are a few large steel producers who may exert some market power at least locally, we think it is realistic to assume that no producer has any market power in the world market.[3]

2.1 The modeling format

The model consists of three types of conditions and variables:

- demand in a region is met by shipments from this and other regions,
- quantities shipped from a region are supported by production in that region, and
- for any shipment, the price obtained in the market exactly covers (marginal) costs.

Let indices i and j denote supply and demand regions respectively, i=1,…,M and j=1,…,N. Further let

x_{ij} denote the quantity shipped from region i to region j,

p_j denote the price paid in market region j,

[3] Mathiesen (1984) developed the MAREQ modeling format for analyses of competitive or non-competitive behavior. Mathiesen and Lont (1984) and Mathiesen and Wergeland (1986) applied this format to the world steel

c_i denote the supply price (marginal cost of production) of region *i*.

These are the variables of the model. Let

$d_j(p_j)$ denote demand in region *j*,

$s_i(c_i)$ denote supply from region *i*, and

t_{ij} denote the cost of shipping one unit from region *i* to region *j*.

The model of a spatial market equilibrium is: Find variables x_{ij}, p_j and c_i such that

(1) $\Sigma_i x_{ij} \geq d_j(p_j)$, $p_j \geq 0$, and $p_j (\Sigma_i x_{ij} - d_j(p_j)) = 0$, $j=1,\ldots,N$,

(2) $s_i(c_i) \geq \Sigma_j x_{ij}$, $c_i \geq 0$, and $c_i (s_i(c_i) - \Sigma_j x_{ij}) = 0$, $i=1,\ldots,M$,

(3) $c_i + t_{ij} \geq p_j$, $x_{ij} \geq 0$, and $x_{ij}(c_i + t_{ij} - p_j) = 0$, $i=1,\ldots,M$ and $j=1,\ldots,N$.

The format of model (1)-(3) is called a complementarity problem[4]. See Mathiesen (1985). Each line of the model consists of three conditions. The interpretation of (1) is as follows. First, the sum of volumes offered to region *j* has to be at least as large as demand at the prevailing price in region *j*. Secondly, this price has to be non-negative. The third condition that the product of the price of region *j* and excess supply to region *j* has to be zero may look unfamiliar, but has the following natural economic interpretation. If the price is positive, supply has to equal demand, while, if supply exceeds demand, the price has to be zero.

The interpretation of (2) is similar. Production in region *i* has to be at least as large as the sum of supplies from that region. The supply price (marginal cost) has to be non-negative. Finally, if marginal cost is positive, excess production is zero, while if there is excess production, marginal cost must be zero.

Condition (3) is of a slightly different style; it concerns the profitability of flow (*i*,*j*). First, the cost of supplying region *j* from region *i* has to be at least as large as the price paid in region *j*. Obviously, in a competitive equilibrium price cannot exceed cost; if so, the flow should profitably be expanded. Secondly, the shipment has to be non-negative.[5] Finally, if the shipment

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market. The modeling format has been applied to a range of commodities: natural gas, crude oil, electricity, grain, wheat, salmon, pulp, cement, steel, aluminum, ferro-alloys, petroleum products, rock and shipping of chemicals.

[4] Let $z = (z_1,\ldots,z_n)$ and $F:R^n \rightarrow R^n$. The complementarity problem is: Find z such that $F(z)\geq 0$, $z\geq 0$ and $z^T F(z) = 0$. Mathiesen (1985) compares this format to that of Samuelson (1952) and Takayama and Judge (1971).

[5] One may formulate the model so that a negative flow can be considered a shipment in the opposite direction.

is positive, the price paid in region *j* exactly covers the marginal costs of supplies from region *i*, while if these costs exceed the price, there is no shipment. Of course, there is no room for excess profits in a competitive equilibrium. Observe that these are *economic* costs containing an imputed rent on supply capacity. In an *accounting* context such a solution may imply a handsome profit. (A broader interpretation is given below, in Section 2.2) This equilibrium is short to medium term, describing operating decisions within given capacities. The model user will have to modify capacities and thereby the supply-function in order to simulate a longer time horizon.

From these conditions we infer that if marginal cost of production is positive for all positive levels of production, there will be no excess production (2). Also, there will be no excess supply and yet prices will be positive. Cf (1). To see the latter conclusion consider two types of demand: I) demand d(p) tends to zero when price tends to infinity, e.g. demand of constant price elasticity, II) there exists a price p' > 0 such that d(p) = 0 for all p > p', e.g. a linear demand function.
For demand-type I the solution will be: $p_j^* > 0$ such that $\Sigma_i x_{ij} = d_j(p_j^*) > 0$, *j*=1,...,N, while for demand-type II there is also the possibility: $p_j^* = p_j' > 0$ such that $\Sigma_i x_{ij} = d_j(p_j') = 0$, for some j.

Model (1)-(3) extends the famous *transportation problem* of linear programming[6], where d_j and s_i are constants, and not functions of endogenous variables as here, and where the issue is how one company should supply its N warehouses from M factories at the lowest overall transportation cost. When used to describe trade between regions and presumably involving several independent producers, model (1)-(3) suffers from two shortcomings:

i) It only computes *net flows*[7] (and not gross flows), and,

ii) although prices (p_j) and marginal costs (c_i) and hence demanded and (total) supplied quantities are unique, the optimal shipments (x_{ij}) are generically *non-unique*. Consequently, the pattern of optimal shipments may change in a seemingly 'spurious' way as a response to a slight change in some parameter.

For our purpose of studying the transportation work implied by carbon regulation of the steel industry, both shortcomings may seem critical. The transportation of steel, as the flows (x_{ij}) are

[6] The LP is: minimize $\Sigma_i \Sigma_j t_{ij} x_{ij}$ subject to $\Sigma_i x_{ij} \geq d_j$, $\Sigma_j x_{ij} \leq s_i$, $x_{ij} \geq 0$. The corresponding Kuhn-Tucker conditions of this LP is (1)-(3) where functions s(.) and d(.) are replaced by constants s and d.

about, accounts for about 20% of the total sea borne transportation work related to the steel industry. The transportation of iron ore and metallurgical coal account for the remaining 80%. Because input volumes depend on production, the related transportation work is not subject to the above mentioned deficiencies. Thus, model (1)-(3) would mainly err by systematically under-estimating the transportation work required by steel as a product and by possibly providing a measure of such transportation work that would be unstable to parameter shocks.

2.2 Production and supply

The output of region *i* is described by the supply function $s_i(c_i)$, which can be interpreted as the inverse of an industry cost curve ($c_i(q_i)$) of this region, that is, marginal cost as a function of produced quantity q_i ($\equiv \Sigma_j x_{ij}$). Typically this quantity is the sum of individual supplies by several companies. We are not particularly interested in any single company and employ the following functional form for the industry cost curve: (For convenience, we drop the regional subscript *i*.)

(4) $c(q) = c_0 + c_1 q^e = c_0 + (q/b)^e$,

where c_0, c_1 ($\equiv b^{-e}$) and e are parameters, and where the latter form is most convenient from a numerical point of view. Two versions of the curve are illustrated in Figure 1. Curve A has $c_0 = 0$, while curve B has $c_0 > 0$. The supply curve corresponding to (4) is

$$s(c) = b(c-c_0)^{1/e}.$$

Figure 1. The industry cost curve

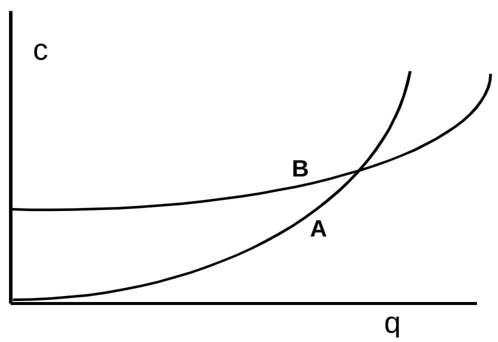
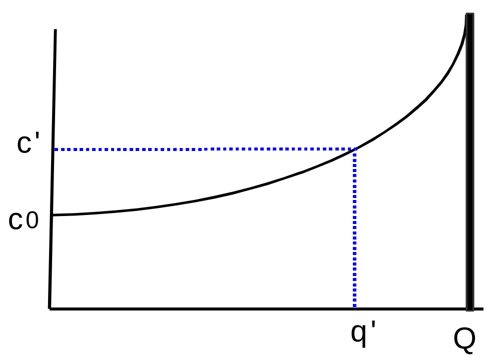


Figure 2. Calibration of the cost-curve



[7] This is the well-known condition of 'no cross-hauling' in the competitive trade equilibrium.

How to arrive at appropriate values of parameter c_0, c_1, and e, depends on what kind of data is available on this cost-volume relationship. Observations from individual plants[8], in particular plants positioned along the segment of the curve in the vicinity of the equilibrium price, would certainly be most welcome. Typically, however, we shall have to use cruder measures, as for example the average (unit) cost over the ensemble of plants within a region and some idea of the elasticity of supply at this volume.

Assume that the available data is a reference point, (q', c'), and some information on c_0 and a capacity Q. A curve that passes through (q', c') and in addition has the properties that $c(0) = c_0$ and c(q) becomes large when production (q) approaches Q, can easily be calibrated. See Figure 2. Of course, c(q) in (4) does not have an asymptote at Q; rather it is defined for any q. The relevant observation is that by choice of parameter e, c(Q) is made large compared to the value-range of the equilibrium price in the model. Hence, any imposed inaccuracy of c(q) for q close to Q would never significantly affect the computed equilibria.

Economic Interpretation

Let us return to the interpretation of condition (3) and the issue of economic versus accounting profit. Assume for this discussion that the curve in Figure 2 represents one company. Consider an equilibrium where price equals c' and the producer supplies a volume q'. Revenue to producer is c'q' (the rectangle). According to the model, his operating costs are represented by the area under the cost curve, while the area between the line p = c' and the cost curve is termed a quasi profit or rent. This is a contribution to the fixed factors that are involved in allowing production to take place, but that are not represented by the curve c(q). These factors and their related costs are irrelevant to the decision of choosing the particular volume q'.

The computed rent is implied by demand for the services of capacity Q and thus looks forward to the market, while the accounting representation of the corresponding costs of providing capacity looks backward to its acquisition. Whatever its historic cost may have been and whatever tax authorities or accounting principles allow as depreciation, the imputed rent would almost certainly not match such accounting costs.

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[8] Cf. the Salter diagram. See Salter (1960).

For the producer to stay in business, he has to cover his fixed costs over the long run. The rent in any short or medium run equilibrium, however, may imply an accounting profit or a loss. Consider for example an equilibrium where p = 2c', and where the producer would operate close to capacity. If p = c' represents a situation where books balance, his revenue at p = 2c' would imply a handsome accounting profit, while economic profit still is zero, because the rent consumes the entire contribution.

Different production technologies

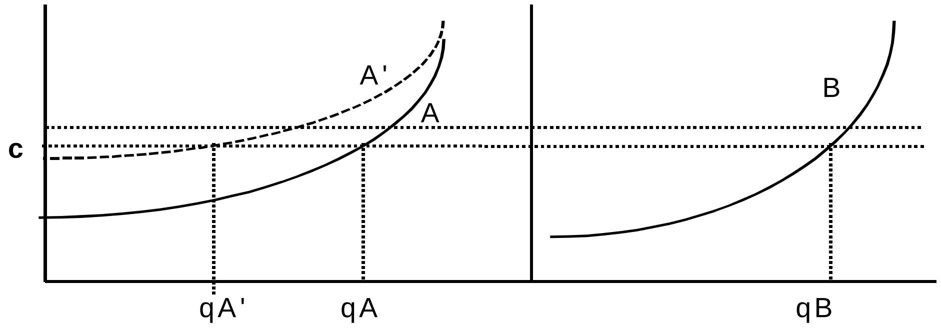
For many reasons the cost of producing may differ considerably between producers. This is conveyed by the industry cost curve; low-cost producers are positioned at the left and high-cost producers at the right. If one is not interested in any particular producer, or other consequences of their level of production than aggregate output, this curve suffices. It summarizes the essentials of supplying a given level of the product, viz. the supply cost, or alternatively it provides the supply at a given price.

In some instances, however, certain aspects of the differences between individual producers are of interest to the analysis. It may be the case, for example, that they are quite differently affected by some shock, whereby the ranking along the curve could be switched. If so, there may be no easy way to describe how a given shock shifts the cost curve. We would do better to distinguish different types of producers, assign individual producers (plants) within a region to separate groups and construct an industry cost curve for each group. Then each group's cost curve could be shifted in an easily described manner by a given shock.

The working of such a model is illustrated in Figure 3. Assume that the producers in a region employ two technologies. At a marginal cost (or supply price) of **c**, the production from the technologies represented by curves A and B are q_A respectively q_B, and total supply from this region is (q_A+q_B). When, for example, a carbon tax raises the cost of producing with technology A, as depicted by the shifted curve A', production with this technology is reduced to $q_{A'}$. At a reduced supply the feedback from the market, as computed by the new equilibrium, implies a somewhat higher price than **c**, whereby production with technology B increases and production with technology A becomes higher than $q_{A'}$**,** while total supply is lower than (q_A+q_B).

Although the aggregate supply may not change much, the reallocation of production between technologies have consequences for the aggregate use of factors of production. If, in our example, technologies A and B represent iron ore based and scrap based production respectively, the new equilibrium would imply reduced demand for iron ore and metallurgical coal. This translates into reduced demand for transportation of these commodities. The increased demand for scrap on the other hand would imply only a minor increase in shipping transportation because scrap is largely a regionally supplied commodity with relatively small sea borne transportation.[9]

Figure 3. Industry cost curves of two technologies



2.3 Demand.

Demand for a homogeneous product can be described by several demand curves of the following functional form

$$y = d(p) = \alpha + \beta p^{\gamma}.$$

y and p denotes quantity respectively price, and α, β and γ are parameters. In Figure 4 three different curves are portrayed. Curve A has $\alpha = 0$, $\beta > 0$ and $\gamma < 0$. This is demand of constant price elasticity equal to ε, so that $\gamma = \varepsilon$. Curves B and C both have $\alpha, \gamma > 0$ and $\beta < 0$, where C has $\gamma > 1$ and B has $\gamma < 1$. The linear demand curve is the special case where $\gamma = 1$.

As drawn in Figure 4, the three curves provide much the same information within the indicated box, while their asymptotic behavior is tremendously different. So what do we know about demand at very high or very low prices? Does this product have a clear substitute whereby a price increase above a certain level would reduce demand considerably as customers switch to the

[9] This version does not account for within-region transportation costs. Such costs can easily be added, however.

substitute? If so, curve C would be relevant. There is, however, no general answer to which curve provides the best description; in each and every case that would depend on the particular features of the product and the customers. (In the present version of the model we employ curve A.)

Figure 4. Demand curves

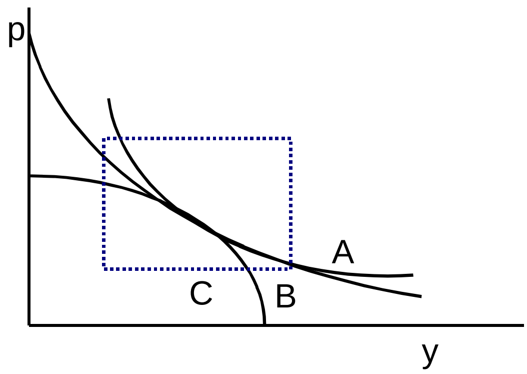
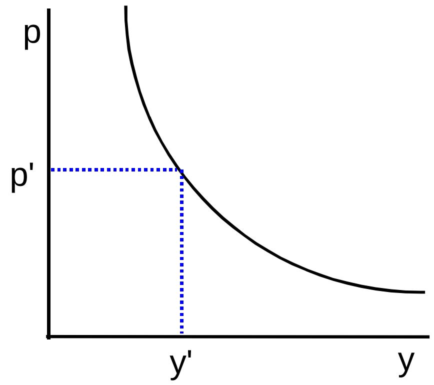


Figure 5. Calibration of demand curve A



Again, calibration of parameters depends upon which data is available. If a sufficient data set is available, one may chose to estimate parameters for these curves by econometric methods. Curve A and the linear demand seem to be the most popular ones for this purpose. If such data is not available, we shall have to employ what there is. Suppose we want to use the curve with constant price elasticity, of which we have an estimate ε'. As stated above $\gamma = \varepsilon'$, whereby β can be calibrated from an observation (y', p') and the estimate ε'. (See Figure 5.)

$$\beta = y'p'^{-\varepsilon'}.$$

2.4 Transportation of steel and inputs to steel production.

The total transportation work for steel products (in terms of ton-miles) will be

(5) $TW_S = \Sigma_i \, \Sigma_j \, m_{ij} \, x_{ij}$,

where m_{ij} denotes the distance between (ports of) regions *i* and *j*. The transportation work for inputs to steel production, e.g. iron ore, would be computed as

(6) $TW_O = \Sigma_i \, m_{Oi} \, \Sigma_s \, O_{is} = \Sigma_i \, m_{Oi} \, (\Sigma_s \, a_{Ois} q_i)$, where $q_i \equiv \Sigma_j \, x_{ij}$.

a_{Ois} is the input of iron ore per ton steel in technology *s* in region *i*, O_{is} is the aggregate input of ore in production with technology *s* in region *i*, and m_{Oi} denotes the average sailing distance per ton iron ore used in region *i*. Transportation work of metallurgical coal is similarly computed.
In model (1)-(3), all measures of transportation work can be computed *ex post* because unit costs (t_{ij}) and marginal cost curves are exogenous to the solution. That is, while these cost parameters determine the equilibrium, the implied aggregate factor usage and transportation volumes do not feed back to factor prices or shipping rates. This is the partial equilibrium framework. While activities and prices within the model (the sectors and markets under analysis) are endogenously determined, the prices of markets outside these sectors are exogenously stipulated.[10]

A major carbon regulation, however, is likely to cause large relative shifts in individual cost curves. This will cause a reallocation of steel production between technologies and influence demand for corresponding inputs. Shifts in factor demand are likely to affect the prices of these inputs as well as the demand for shipping and thus the corresponding freight rates. That is, supply costs of inputs would change as a function of changes in the steel production. The only way to account for such feedback is to pull factor markets and the transportation of factors into the model and thus account for all these markets explicitly. That is the aim of the full model.

2.5 Calibration of a benchmark.

Let boldface characters ($\mathbf{x}_{ij}$, $\mathbf{p}_{ij}$ and $\mathbf{c}_i$) denote benchmark values for the corresponding variables. When parameters of supply and demand are calibrated such that conditions (1)-(3) are satisfied, we know that the equilibrium values will be

$$x_{ij} = \mathbf{x}_{ij},\ \ p_{ij} = \mathbf{p}_{ij},\ \ \text{and}\ \ c_i = \mathbf{c}_i,$$

that is, equal to benchmark values. For the purpose of debugging the coding of the model and the computation of the various parameters, the value of this feature should not be underestimated. Until computed equilibrium values equal benchmark values, some error is present. In models of this size, locating such errors may be time consuming.

[10] Observe that accounting for different technologies and their corresponding cost-curves only allowed us to translate shocks into cost in a more precise way. It did not affect factor prices underlying the cost curves.

3. Extensions

Our objective is to analyze how carbon regulation of the steel industry translates into shifts in transportation requirements for inputs to this industry. We argued that while model (1)-(3) might provide some insights, it would most likely be inappropriate as it did not account for feedback from the steel industry on its factor prices and hence the volumes of factors supplied and their transportation pattern. In this chapter we will lay out the industrial structure of the full model, whereas its mathematical formulation and economic interpretation are left for the next chapter.

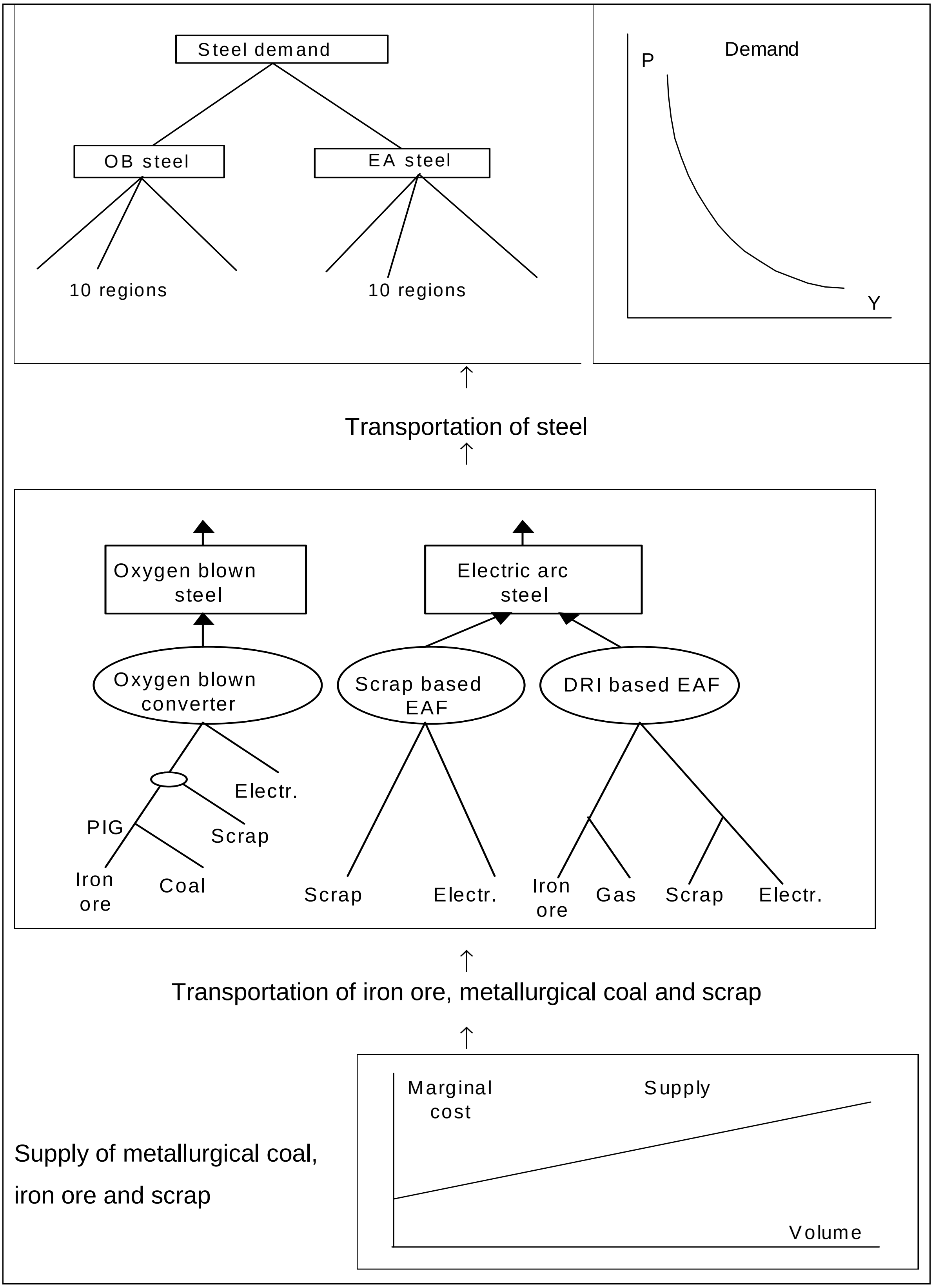
3.1 The industrial structure of the steel model

Figure 6 provides a schematic view of the steel industry. At its bottom the global supply of three factors of production are represented: Iron ore, coal and scrap. Each factor is supplied at a price that is determined by the equilibration of its supply and demand from all steel production, and such that the implied transportation and the corresponding freight rate are determined simultaneously. We also consider explicitly two other production factors: Electricity and natural gas. These factors we assume to be supplied locally, and we assume that the steel industry is too small a customer in these (local) markets to affect their prices. This is equivalent to assuming horizontal supply curves, or exogenously stipulated prices. These (local) prices may differ between regions and be subject to shifts from carbon regulation, however.

In the middle section of Figure 6, factors enter production in three technology specific combinations together with other factors, e.g. labor, that are only accounted for through their cost. The technologies are the basic oxygen furnace and two kinds of electric arc mini-mills, a scrap-based and one based upon directly reduced iron ore. The oxygen blown steel (or OB steel for short) is, at least for some applications, considered a better product than the electric arc (EA) steel. Hence we treat these variants of steel as imperfect substitutes. We assume, however, that the two electric arc technologies produce the same (homogeneous) quality of steel. Their factor usage and hence their costs of production differ and will determine to which extent each is used.

In principle there may be room for substitution between the factors. After all, we do observe different factor combinations between regions. In reality, however, the substitution possibilities seem to be negligible. Apart from substitution between so-called pig iron and scrap in the

Figure 6. A schematic view of the steel industry in the model



production of OB steel, our model assumes fixed combinations. While each technology is represented by the same structure in all regions, the actual factor volumes per ton steel differ between regions. Some differences may be explained by relative factor prices that differ between regions, others may result from managerial competence and experience. We take all such observed differences as given.

Produced steel of the two types is supplied to the world market and transported between regions. We have adopted the so-called Armington assumption that each type of steel is differentiated by region of origin and therefore is not considered homogeneous.[11] The assumption both stabilizes flows and allows gross flows, which are two major drawbacks of the model with a homogeneous product. On the other hand, the assumption implies that the trade pattern for steel is rather rigidly tied to the benchmark structure and will only to a lesser degree be determined by competitive pressures. Within our timeframe of the short to medium run this may not be too critical a restriction. Also, the formulation complicates the model. Final demand is thus represented by a two-level function breaking steel into two types, each of which in turn is broken down into steel originating in different regions.

In addition to these stages of transforming factors into steel, the model accounts for the transportation work and the cost of transporting the various commodities. Iron ore and metallurgical coal are large commodities in dry bulk shipping[12] whereby changes in shipped volumes affect freight rates. Thus the freight rate of dry bulk is endogenously determined. To some degree, other ships than those involved in transporting iron ore and coal, transport steel products. For simplicity, however, and because there is some overlap, we have added steel transportation to that of iron ore and coal as if these ships were close substitutes. We have made another simplification that may be more critical for some purposes of the model. While the transportation of steel is accounted for explicitly, as in (5), there is no similar network of transportation routes of iron ore and met-coal. The transportation work is computed from the equivalent of (6), implying that imports to region i from different suppliers of iron ore (or met-coal) are scaled proportionally when usage in steel production within region i changes. [13]

[11] See Armington (1960).
[12] Scrap metal, which is another major input to steel production, is mainly supplied within each region and requires modest shipping transportation. Hence, we disregard the transportation requirements for scrap.
[13] There are indications that average distances for such commodities may change from one year to another. Light (1999) bases his model of the world steam coal market on the premise that the US is a swing supplier, implying that

In order to simulate the effects of carbon regulation on the steel industry and its factor demand, carbon dioxide emissions and the regulatory regime have to be specified. Although the steel industry and its related operations cause as much as 10% of man made emissions, we shall assume that the regulatory regime (the price of an emission quota or the carbon tax) is not affected by a change in emissions from the steel industry.

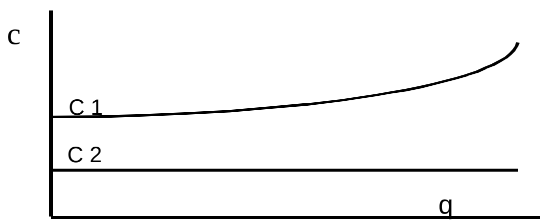
3.2 Accounting for factors of production

(4) relates unit cost of production to the quantity of production. Prices and consumed volumes of factors are disguised behind parameters c_0 and c_1, that are assumed to be fixed. In our analysis, however, we would like prices of industry specific factors like iron ore, coal and scrap, and the dry bulk freight rate to be endogenously determined, while prices of other factors like labor and electricity are assumed to be unaffected by changes in steel production activities. Let us therefore divide factors of production into two groups: one group where factor prices are assumed not to be affected by steel production activity levels, and one group of *F* factors whose prices (v_f, f=1,…,F) are affected. This distinction is made explicit by rewriting (4) as the sum of two parts

(4') $c(v_1,\dots, v_F; q) = (c_0 + c_1 q^{\varepsilon}) + \chi_2(v_1,\dots, v_F)$,

where c_0 and c_1 are parameters that describe the unit cost of the first group of factors, and χ_2 is a cost function of factor prices of the second group. The first part corresponds to the graph of Figure 2, that is, a production with decreasing returns to scale. For the second part we assume constant returns to scale production. (Further details on such a function are given below.) This breakdown is portrayed in Figure 7, where curves C_1 and C_2 represent the two parts. Admittedly, it is arbitrary when this formulation implicitly assumes that all plants (of a certain technology within a region) operate with the same factor input shares for the second group of factors. Thus,

Figure 7. Two groups of operating costs



marginal and average transporting distances differ. Strandenes and Wergeland (1983) find that average distance in both the iron ore and the coal market vary with the freight rate.

observed differences in production cost between plants are entirely assigned to the first term and hence to factors like labor, maintenance, administration, etc.

3.3 Demand for a differentiated product.

We are concerned with products that are inputs to production. This applies at two levels in the model: i) the combination of pig iron and scrap into steel, and ii) the aggregation of steel of different types and from different regions of origin in further processing into final products. (See the preceding paragraph.)

We employ a function with constant elasticity of substitution (CES) to represent both stages. Following Rutherford[14], we employ the calibrated share form. Here we present in general terms this function and its adaptation to the two uses. Let *i*, *i*=1,…, n, index factors. The calibrated share form of the CES-function is specified by:

- benchmark factor *demands* ($\mathbf{x}_i$) and corresponding *prices* ($\mathbf{p}_i$)[15],
- benchmark *cost* (**C**) and *output* (**y**),
- benchmark *value shares* (θ_i), and
- *elasticity* of substitution (σ).

The calibrated share form of the CES production function is written:

$$y = \mathbf{y}\ [\ \textstyle\sum_i \theta_i\ (x_i\ /\ \mathbf{x}_i)^{\rho}\ ]^{1/\rho},$$

where $\rho = (\sigma\text{-}1)/\sigma$. The corresponding cost function is written:

(7) $$C(p_1,\ldots, p_n) = \mathbf{C}\ [\ \textstyle\sum_i \theta_i\ (p_i\ /\ \mathbf{p}_i)^{1-\sigma}\ ]^{1/(1-\sigma)},$$

where **C** and θ_i are defined as

$$\mathbf{C} \equiv \textstyle\sum_i \mathbf{p}_i\ \mathbf{x}_i, \qquad \text{and}$$

$$\theta_i \equiv \mathbf{p}_i\ \mathbf{x}_i\ /\mathbf{C}, \qquad i=1,\ldots,n.$$

[14] http://www.gams.com/solvers/solvers.htm#MPSGE. CES-functions: Some hints and useful formulae.

[15] Boldface characters are used to represent benchmark (observed) values in order to distinguish these parameters from the corresponding variables of the model.

Conditional factor demand may be obtained from (7) by Shepard's lemma:

$$(8)\qquad x_i(p_1,\ldots,p_N) = \partial C/\partial p_i = \mathbf{x}_i\,[(C(p_1,\ldots,p_N)/\mathbf{C})(\mathbf{p}_i/p_i)]^{\sigma}$$
$$= \mathbf{x}_i\,[\textstyle\sum_i \theta_i\,(p_i/\mathbf{p}_i)^{1-\sigma}\,]^{\sigma/(1-\sigma)}\,(\mathbf{p}_i/p_i)^{\sigma},\quad i=1,\ldots,n.$$

(8) is employed to describe the demand for pig iron and scrap. See Figure 6. *Unconditional* demand will be

$$(9)\qquad d_i(p_1,\ldots,p_N;\,y) = (y/\mathbf{y})\,x_i,\quad i=1,\ldots,n.$$

In a general equilibrium model, the activity level of production (y) would be determined from a unit profit condition. (Cf. (3)). In our partial equilibrium model, however, demand for steel can be thought of as a final demand and we will assume this final demand to be a function of its price. Interpret C(p) as the price(-index) for this particular aggregate, and postulate the following demand function (i.e., curve A of Figure 4)

$$y = \beta C^{\varepsilon},$$

where ε denotes the price elasticity and β is a constant. β may be calibrated such that $\beta=\mathbf{y}/\mathbf{C}^{\varepsilon}$, where **y** and **C** are benchmark volume and price respectively. Thus final demand becomes

$$(10)\qquad y = \mathbf{y}(C/\mathbf{C})^{\varepsilon}.$$

When (8) and (10) are substituted into (9), **y** cancels out and one yields demand for steel (factor *i*)

$$D_i(p_1,\ldots,p_N) = \mathbf{x}_i\,(C(p_1,\ldots,p_N)/\mathbf{C})^{\sigma+\varepsilon}\,(\mathbf{p}_i/p_i)^{\sigma},\qquad i=1,\ldots,n.$$

In our model there are N regions, and demand in region *j* for the product of region *i* is[16]

$$(11)\qquad D_{ij}(p_{1j},\ldots,p_{Nj}) = \mathbf{x}_{ij}\,(C(p_{1j},\ldots,p_{Nj})/\mathbf{C})^{\sigma+\varepsilon}\,(\mathbf{p}_{ij}/p_{ij})^{\sigma}$$
$$= \mathbf{x}_{ij}\,[\textstyle\sum_i \theta_i\,(p_{ij}\,/\,\mathbf{p}_{ij})^{1-\sigma}\,]^{(\sigma+\varepsilon)/(1-\sigma)}\,(\mathbf{p}_{ij}/p_{ij})^{\sigma},\quad i=1,\ldots,M,\ j=1,\ldots,N.$$

Returning for a while to the core model, equilibrium condition (1) would now be replaced by

$$(1')\qquad x_{ij} \geq D_{ij}(p_{1j},\ldots,p_{Nj}),\quad p_{ij}\geq 0,\quad \text{and}\quad p_{ij}\,(x_{ij} - D_{ij}(p_{1j},\ldots,p_{Nj})) = 0,\ i=1,\ldots,M,\ j=1,\ldots,N.$$

[16] For notational simplicity we suppress the indices on parameters ε and σ.

Observe that (1') involves MN equations compared to N in (1). The N variants of a differentiated product cannot be meaningfully summed as in (1), but have to be aggregated according to their substitutability as described by D_{ij} in (11). Further, observe that (for a finite σ) the non-uniqueness of flows (x_{ij}) is no longer a generic problem. The obvious drawback of *assuming* differentiation by region of origin is that we impose a trade pattern on the model rather than have the model infer a trade pattern from the description of the market[17]. The lower the elasticity of substitution between variants, the more rigid the trade pattern will be to parameter shocks.

3.4 The regulation of carbon emissions

We assume that a carbon regulation will be implemented as either of two systems, which from our modeling point of view are identical: A tax on the use of carbon inputs as coal and natural gas, or alternatively as purchases of carbon emissions quotas. The model also imposes carbon tax shifts on the local production of electricity. Such carbon taxes (purchase prices of emissions quotas) and cost shifters are included in the subsequent formulation. See τ_{fti} and c_{0ti} in conjunction with the cost function of steel production, i.e., (14) and (4") below.

We assume there is no feedback from the operations of the steel industry and its use of fossil material on the regulatory regime, i.e., the carbon tax. Thus, the total carbon emissions are computed from the equilibrium production and factor usage after the equilibrium is obtained.

[17] The choice of modeling approach is a long standing issue. See e.g. Haaland m.fl. (1987).

4. The full model

Let us consider a model that contains:

- Supply of and demand for production factors: Scrap, iron ore, coal, natural gas, and electricity.
- Transportation of production factors: Scrap, iron ore, and coal.
- Two types of steel: Oxygen based and scrap based steel.
- Three technologies for producing steel: Integrated steelworks for oxygensteel, and two kinds of mini-mills: Based on scrap and directly reduced iron ore.
- Transportation of steel between regions.
- Final demand for steel differentiated by type and by region of origin.

Let us employ the following indices:

f denotes factor inputs, $f = 1,..., F$,
t denotes steel producing technology, $t = 1,..., T$,
s denotes steel type, $s = 1,..., S$,
i denotes steel producing region, $i = 1,..., I$, and
j denotes steel consuming region, $j = 1,..., J$.[18]

The model consists of eleven types of relations, variables and complementary slackness conditions as compared to the previous model's three types. Table 1 (cf. Figure 6) provides an overview of these variables describing the transformation of factors of production into steel and the corresponding build up of the cost of producing steel.

Starting at the top of Table 1, the prices (v_f) of production factors are determined in a world market for each factor. The factors coal and iron ore are transported to the consuming regions, and these shipments together with steel shipments and the supply of shipping capacity, determine the freight rate (r). Next, region specific carbon taxes and local costs are added to the *cif* factor price in order to arrive at regional and technology specific factor prices (w_{fti}). These factor prices are arguments of cost functions (4') that describe how the corresponding factor inputs (a_{fti}) can be combined into steel within that specific technology at a unit cost (c_{ti}) and of volume (Q_{ti}). Within each region, steel is produced in a cost minimizing way such that the cost of steel of type *s* (sc_{si})

[18] In the present version of the model $F = 5$, $T = 3$, $S = 2$, and $I = J = 10$.

is the lowest cost over the available technologies. This is the supply price of steel type *s* out of region *i*. Next, transportation costs, tariffs, etc. are added to obtain the price in region *j* of steel of type *s* originating in region *i* (p_{sij}). Corresponding shipments x_{sij} are aggregated, first by region of origin (Figure 6) into aggregates Y_{sj} with corresponding prices P_{sj}, and finally by type into aggregate steel consumption of region *j* (Y_j) with price (P_j). P_j and Y_j, however, are not explicit variables in the model. The remaining seven types of prices and four types of quantities in Table 1 comprise the variables of the model. For each variable the model contains one inequality and one complementary slackness condition connecting the inequality and the corresponding variable.

Table 1. Model structure and price formation

Price and added costs	**Price** variable	**Transformation**	**Quantity** variable	**Type of quantity**
World market factor price	v_f			Factor supply and demand
Transportation cost	r			Shipping supply and demand
Carbon tax and local costs on factor input	↓			
Factor input price	w_{fti}		a_{fti}	Input coefficient of factor *f*
	↓	Cost function of technology *t* (4’)		
Marginal cost of technology *t*	c_{ti}		Q_{ti}	Production of steel With technology *t*
	↓	Choice among technologies		
Marginal cost of steel type *s* (supply price)	sc_{si}			
Transportation cost & tariffs on steel	↓			
Price in region *j* of steel type *s* originating in region *i*	p_{sij}		x_{sij}	Shipment of steel type *s* from *i* to *j*
	↓	Aggregation over regions of origin		
Price of steel type *s* in region *j*	P_{sj}		Y_{sj}	Consumption of type *s* in region *j*
	↓	Aggregation over steel types		
Price of steel in region *j*	(P_j)		(Y_j)	Consumption of steel in region *j*

Inputs to steel production are central elements of the model. (See the nesting of factors of production in Figure 6.) Some of these inputs are required in fixed combinations, as the technology allows no substitution. The inputs of pig iron and scrap in oxygen blown steel, however, are treated as imperfect substitutes. The corresponding unit cost function is

(4") $$c_{ti}(w_{1ti},\ldots, w_{Fti}; Q_{ti}) = (c_{0ti}+(Q_{ti}/c_{1ti})^{e}) + \boldsymbol{C}_{2ti}\,[\sum_f \kappa_{fti}\,(w_{fti} / \mathbf{w}_{fti})^{1-\gamma}]^{1/(1-\gamma)},$$

where γ denotes the elasticity of substitution between factors of production, κ_{gti} denotes the value share and $\boldsymbol{C}_{2ti}$ is the benchmark unit cost of the second part. (Cf. Figure 7. See also (4') of section 3.2 and (5).) The cost minimizing input of factor f at the prevailing user-costs (w_{fti}) is derived as

$$\varphi_{fti} \equiv \partial c_{ti}(w_{1ti},\ldots, w_{Fti}; Q_{ti})/\partial w_{fti} = \mathbf{a}_{fti}\,[\sum_g \kappa_{gti}\,(w_{gti}/\mathbf{w}_{gti})^{1-\gamma}\,]^{\gamma/(1-\gamma)}\,(\mathbf{w}_{fti}/w_{fti})^{\gamma}.$$

See the conditional factor demand (8). Coefficients (φ_{fti}) are derived from the second part of the cost function (4").

Supply of and demand for (traded) factors of production:

(12) $s_f(v_f) \geq \sum_i \sum_t a_{fti}\, Q_{ti}$, $v_f \geq 0$, and $v_f\,[s_f(v_f) - \sum_t \sum_i a_{fti}\, Q_{ti}] = 0$, $f = 1,\ldots, F$.

Supply of factor f is given by $s_f(v_f) = A_f + B_f v_f$, where $B_f > 0$, i.e., a higher supply comes at a higher price. A_f and B_f are constants. Demand for factor f is the sum over all technologies and regions, where factor usage is the product of conditional input (a_{fti}) and activity level (Q_{ti}). When v_f is positive, factor supply equals demand. Excess supply will never occur.

Supply of and demand for dry bulk shipping:

(13) $s(r) \geq \sum_f \sum_i \sum_t d_{fi}\,(a_{fti}\, Q_{ti}) + S$, $r \geq 0$, and $r[s(r) - (\sum_t \sum_i d_{fi}\, a_{fti}\, Q_{ti} + S)] = 0$,

Shipping supply is specified as $s(r) = A+Br$, with $B > 0$, and where A and B are constants. Again, we assume that a higher supply demands at a higher rate. Regional factor demand (first term on the right) is weighted by the average distance (d_{fi}) per ton of factor f used in steel production in region i to obtain total demand for shipping services measured in ton-miles[19]. The second term $S = \sum_s \sum_i \sum_j d_{ij}\, x_{sij}$ accounts for the total transportation work for steel. There is a large overlap between the fleets of ships carrying coal respectively iron ore to warrant their summation into one fleet. The addition of shipping requirements for steel is more difficult to defend. When r is positive, shipping supply equals demand. Excess supply will never occur.

[19] For factors that are supplied locally, like scrap, electricity and natural gas, the corresponding d_{fi} is zero.

User cost of factors of production:

(14) $w_{fti} \geq v_f + d_{fi}\, r + \tau_{fti}$, $a_{fti} \geq 0$, and $a_{fti}\,[w_{fti} - (v_f + d_{fi}\, r + \tau_{fti})] = 0$.

The user-cost of factor *f*, denoted w_{fti}, is a sum of the world market price, the transportation cost and a regional or technology specific cost (τ_{fti}) that could include a carbon tax.

Input coefficients for variable factors:

(15) $a_{fti} \geq \varphi_{fti}$, $w_{fti} \geq 0$, and $[a_{fti} - \varphi_{fti}] = 0$.

(14) and (15) are merely definitions of two intermediate variables: the user cost (w_{fti}) respectively the input coefficient (a_{fti}). Their complementary slackness conditions are superfluous – the user cost is always equal to the sum of its components, and the input coefficient always equals its cost minimizing level. In fact, (14) could be eliminated and the sum on the right could replace w_{fti} wherever it appears. The same goes for (15); a_{fti} could be replaced by the function of factor prices. We think, however, that both the model description and the resulting GAMS code look more transparent by including these variables.

Marginal cost of producing with technology *t*:

(16) $c_{ti} \geq c_{ti}(w_{1ti},\dots, w_{Fti}; Q_{ti})$, $c_{ti} \geq 0$, and $c_{ti}\,[c_{ti} - c_{ti}(w_{1ti},\dots, w_{Fti}; Q_{ti})] = 0$,

$t = 1,\dots, T$, $i = 1,\dots, I$.

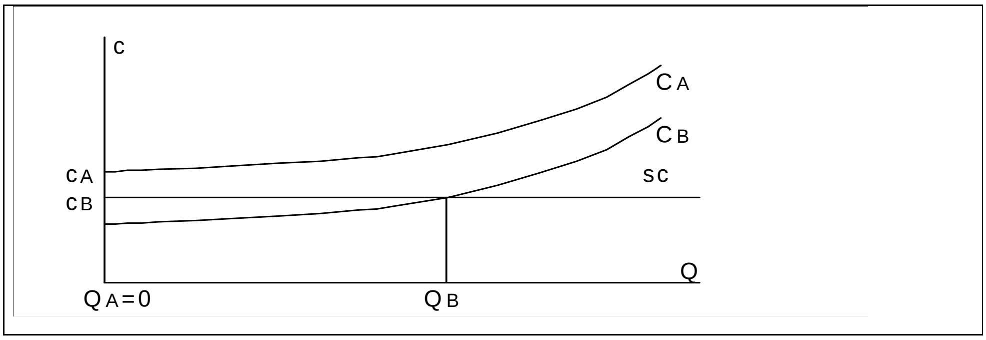
The complementary slackness condition $c_{ti}\,[c_{ti} - c_{ti}(..)] = 0$ may seem odd, but should be considered in conjunction with (17) and (18). (See also Table 2 below.) $c_{ti} > 0$, implies $c_{ti} = c_{ti}(..)$. Furthermore, as long as $c_{ti}(..) > 0$ for $Q_{ti} \geq 0$, $c_{ti} > c_{ti}(..)$ never applies.

Marginal cost of producing steel type *s*:

(17) $c_{ti}\ \$ST(t,s) \geq sc_{si}$, $Q_{ti} \geq 0$, and $Q_{ti}\,[c_{ti}\ \$ST(t,s) - sc_{si}] = 0$, $t = 1,\dots,T$, $i = 1,\dots, I$.

ST(t,s) is an array with values 1 or 0 according to whether technology *t* produces steel type *s* or not. \$ST(t,s) is a GAMS-logical (true or false) providing the connection between the marginal cost of the *T* technologies and the marginal cost of *S* steel types. The complementary slackness condition states that if technology *t* is employed (i.e., $Q_{ti} > 0$), then the marginal cost of production equals the marginal cost of its steel. If, on the other hand the marginal cost of employing technology *t* to produce steel type *s* (i.e., $c_{ti}\ \$ST(t,s)$), exceeds the marginal cost of providing steel type s (because another technology provides this steel at a lower cost), technology *t* is not used, i.e., $Q_{ti} = 0$. Figure 8 illustrates these points. There are two technologies: A and B. Technology B is operated, i.e., $Q_B > 0$ and $c_B = C_B(Q_B) = sc$. Technology A, however, has a too high cost. $c_A = C_A(0) > sc$, and hence $Q_A = 0$.

Figure 8. Two technologies for producing the same type of steel



Output and supply of steel type *s*:

(18) $\sum_{ST(t,s)} Q_{ti} \geq \sum_j x_{sij}$, $sc_{si} \geq 0$, and $sc_{si} [\sum_{ST(t,s)} Q_{ti} - \sum_j x_{sij}] = 0$, $s = 1,..., S$, $i = 1,..., I$.

The summation on the left is over technologies *t* that produce steel type *s*, and the summation on the right is over shipments of steel type *s* to consuming regions. The complementary slackness condition is obvious: If marginal cost of steel type *s* is positive, the sum of shipments equals the sum of production. If, on the other hand there is excess production, the corresponding marginal cost of steel type *s* must be zero, a case that is ruled out by (17).

Now we come to the other use of the calibrated share function (see Section 3.3), namely the two-level aggregation of the different kinds of steel.

Supply and conditional demand for steel type *s* from region *i*:

(19) $x_{sij} \geq D_{sij}(p_{s1j}, \ldots, p_{sNj}; Y_{sj})$, $p_{sij} \geq 0$, and $p_{sij}(x_{sij} - D_{sij}(p_{s1j}, \ldots, p_{sNj}; Y_{sj})) = 0$,

$s = 1,..., S$, $i = 1,..., I$, $j = 1, \ldots, J$.

x_{sij} is the shipment of steel and p_{sij} is its delivered price. Y_{sj} is the Armington-aggregate of steel type *s* in region j consisting of the individual D_{sij}, being demand for steel type *s* from region *i*. (See (9) and (8).) The complementary slackness condition reads: If price (p_{sij}) is positive, supply (x_{sij}) equals demand (D_{sij}), and oppositely, if there is excess supply, price has to be zero. (Of course, price is never zero, hence supply equals demand. See discussion of (1) in Section 2.1.)

Unit profit of supply:

(20) $sc_{si}(1+T_{ij}) \geq p_{sij}$, $x_{sij} \geq 0$, and $x_{sij}\,(c_{si}(1+T_{ij}) - p_{sij}) = 0$,

$s = 1,..., S, \quad i = 1,..., I, \quad j = 1,\ldots,J.$

T_{ij} aggregates all costs of transporting steel from region *i* to region *j*, including tariffs. Condition (20) states that if shipment (x_{sij}) is positive, the price in region *j* exactly covers the cost of supplying this region from region *i*. If supply cost exceeds price, however, there is no shipment. This condition is the equivalent of condition (3) in the core model.

Demand for Armington aggregate *s*:

(21) $Y_{sj} \geq D_{sj}(P_{1j}, P_{2j})$, $P_{sj} \geq 0$, and $P_{sj}\,(Y_{sj} - D_{sj}(P_{1j}, P_{2j})) = 0$

$s = 1,..., S, \quad j = 1,\ldots,J.$

D_{sj} is (final) demand for steel type *s* in region *j* and P_{sj} is its price. See (11). The complementary slackness condition is analogous to (19). The aggregate (Y_{sj}) interpreted as supply has to be at least as large as demand. If price (P_{sj}) is positive, supply equals demand and oppositely, if there is excess supply, price has to be zero. Of course, price is never zero, hence supply equals demand.

Unit cost of Armington aggregate *s*:

(22) $C_{sj}(p_{s1j},\ldots, p_{sNj}) \geq P_{sj}$, $Y_{sj} \geq 0$, and $Y_{sj}\,(C_{sj}(p_{s1j},\ldots, p_{sNj}) - P_{sj}) = 0$

$s = 1,..., S, \quad j = 1,\ldots,J.$

C_{sj} denotes the unit cost of the Armington aggregate Y_{sj}. In equilibrium, this cost has to be at least as large as the price. If the aggregate is positive, the cost exactly equals the price of the aggregate.

The eleven sets of conditions ((12)-(22)) have to be solved simultaneously. They are, however, related to each other in a circular manner that is quite illustrative for the interpretation of the results from this type of model. Starting (arbitrarily) with the demand for factors of production ($a_{fti}\,Q_{ti}$), we have the sequence of causal relations laid out in Table 2. (Cf. Table 1.)

Table 2. Causal relationships in the model

	Variables
(12) determines factor prices (v_f) from factor shares and produced quantities of steel.	a_{fti} Q_{ti} $\rightarrow$ v_f
(13) determines freight rate (r) from factor shares, produced quantities of steel and distances.	a_{fti} Q_{ti} & x_{sij} $\rightarrow$ r
(14) determines user cost (w_{fti}) from worldmarket prices and transportation costs.	v_f & r $\rightarrow$ w_{fti}
(16) determines marginal cost (c_{ti}) of technology *t* from factor prices and production volumes.	Q_{ti} & w_{fti} $\rightarrow$ c_{ti}
(17) determines marginal cost (sc_{si}) of steel type *s*.	c_{ti} $\rightarrow$ sc_{si}
(20) determines supply price (p_{sij}) of steel type *s*.	sc_{si} $\rightarrow$ p_{sij}
(22) determines price (P_{sj}) of Armington-aggregate.	p_{sij} $\rightarrow$ P_{sj}
(21) determines the level (Y_{sj}) of Armington-aggregate.	P_{sj} $\rightarrow$ Y_{sj}
(19) determines demand (x_{sij}) for steel type *t* from region *i*.	Y_{sj} & p_{sij} $\rightarrow$ x_{sij}
(18) determines production (Q_{ti}) from total supply.	x_{sij} $\rightarrow$ Q_{ti}
(15) determines input coefficients (a_{fti}) from factor prices.	w_{fti} $\rightarrow$ a_{fti}

5. Conclusions

We have described a model for conducting simulations of the responses in the steel industry to carbon regulations. It provides a fairly detailed representation of the various steps from supply of (sector specific) factors of production to the consumption or further processing of steel to final products. The time horizon for the equilibrium of such a structural model is the short to medium term. Within this time frame, capacities behind supply of factors and shipping services as well as production of steel and the rigidity of supply-demand relations implied by the Armington assumption, stay fixed. The evaluation of the goodness of such a model rests with its results and they are reported elsewhere (Mathiesen and Mæstad (2001) and Mæstad et.al. (2000)). We do not think of such a model as a predictor of prices or volumes, but rather a provider of insight into the net consequences of all the mechanisms that are pictured.

The notion of substitutability is a central issue, may be *the* central one in a model like ours. Substitution is represented both through model structure, e.g. alternative production technologies and supplying regions facilitating reallocation between sectors, and through elasticity parameters allowing reallocation within sectors. Roughly speaking, the kinds of substitutability that might be important to our analysis rest with the following features:

1. A producer’s choice among suppliers of iron ore and metallurgical coal,
2. his possibilities of substitution between inputs to iron and steel production,
3. a consumer’s choice among suppliers of steel (both by region and steel type), and
4. the price elasticity of demand for steel.

Points 1 and 3 are directly related to transportation. Usage of iron ore (and met-coal) in region *i* is based upon fixed import-shares, translating a lower steel production into proportional reductions for all suppliers. Hence, only the reallocation of world’s steel production among regions will affect the relative contributions of factor suppliers. While the model most likely underestimates the potential for substitution in factor markets and the resulting changes in aggregate transportation, shipments of steel are determined by the Armington-structure, whereby the flexibility of reallocation depends on the magnitude of this elasticity of substitution.

We think there is some substitutability between factors of production in the steel technologies in addition to the one that is included between scrap and pig iron for oxygen blown steel. We also think the model could benefit from including a third level in the aggregation of steel, whereby supplying regions are divided into groups, e.g. according to some measure of (how industrialized the economies of the various regions are, and hence) the quality of the steel produced. This would allow higher substitutability between steel from regions within each group than from regions across groups. Of course, none of these extensions would facilitate the estimation (or calibration) of the various elasticities, and certainly not the magnitude of one elasticity versus another.

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