<u>Working Paper No. 71/03</u>

The Accounting Rate of Return: Some analytical results

by

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SNF Project No. 7680
Kapitaltilgang som faktor for suksess i maritim verksemd

The project is financed in part by the Norwegian Shipowners Association

INSTITUTE FOR RESEARCH IN ECONOMICS AND BUSINESS ADMINISTRATION
BERGEN, December 2003
ISSN 1503-2140

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October 2003

Preliminary: Comments welcome

Abstract

This paper analyzes the accounting rate of return assuming expected cash flow is linearly decreasing over time. An interval estimate for the rate of return is obtained using two different depreciation plans. The paper derives conditions under which the expected interval estimate will contain the expected return on investment (internal rate of return). The conditions hold for a single investment project and for an arbitrary collection of projects (a firm). Accounting rates of return are analyzed using a model with explicit cash flow uncertainty. If return measures are interpreted as estimates of expected return on investment, they will be subject to random errors as well as biases caused by accounting policies. The ideas of the paper are illustrated using case study data.

1. Introduction

The rate of return on capital invested in a commercial enterprise is potentially relevant information under different circumstances. Equity owners may find rates of return useful measures of management performance. Financial analysts may find it to be relevant inputs in their valuation models. Economists have been known to use the rate of return as a gauge of market competitiveness. They are also frequently used in social science research involving business firms.

The accounting rate of return is a periodic measure of the return on investment (the internal rate of return). Accounting rates of return are calculated using transactions based accrual accounting. Neither fair value accounting, nor cash flow accounting is directly relevant at least not in general.[1] It is well known that there exist accruals that will make the accounting rate of return equal to the return on investment in every period under quite general conditions. On the other hand it is equally well known that the accounting rate of return is very sensitive with respect to the accrual policy. If, for example, linear depreciation is chosen for a T-period project with constant cash flows, the accounting rate of return will increase T-fold over the lifetime of the project.

Theory provides valuable insights into the measurement process. Using steady state theory Stauffer (1972) and others have derived relationships between growth rates, accounting policies and biases in the accounting rate of return. Although few firms may be in steady state, strictly speaking, the theory may still provide useful insights of a qualitative nature. A mature firm using a conservative accrual policy will, for example, almost certainly report accounting rates of return that overestimates the true return. Steady state theory also teaches caution; increasing “sample size” does not necessarily make the bias go away or even reduce it. In steady state a firm may report biased rates of return forever.

The potential biases involved have led some observers to give up on the accounting rates of return as reliable return measures (see Fisher and McGowan (1983)). Others see the problems as one of making the available data speak by devising more effective estimation methods Edwards et. al.(1987) and Peasnell(1982)). This paper adopts the latter view. Notwithstanding its considerable accomplishments, existing theory is limited by the steady state assumption and does not in any case provide much precise information about the magnitude of the biases. This paper attempts to extend the theory of accounting rates of return in three different directions. First of all the effects of imposing restrictions on the cash flow vector rather than

[1] Hill(1979) argues that cash flow based measures may be preferable in a second best world. Salamon(1985) shows that cash flow data may be used to calculate returns without accrual accounting.

the composition of the investment portfolio will be explored. Specifically it will be assumed that the cash flow vector generated by investments is declining linearly over time. While this assumption may not necessarily be a better approximation to reality than the steady state assumption, it extends existing theory in a different direction. Secondly, this paper explores the possibility of calculating two rates of return using different accrual policies. The objective is to estimate upper and lower bounds for the return on investment, which will provide indications of the magnitude of the biases involved.

The third extension of the theory is conceptual in nature. Existing theory does not explicitly acknowledge uncertainty. Here uncertainty is introduced by first of all reinterpreting the return concepts. Received theory may be said to address the relationship between the expected accounting rate of return and the expected return on investment. Secondly, random cash flows, taking the form of unexpected windfalls, are introduced, and the effects of accrual policies on return measures under such condition, are tentatively explored.

The paper is organized in the following way: In section 2 the model is presented, and the existing theory is briefly reviewed. In section 3 the relationship between the expected accounting return and the expected return on investment is explored assuming linearly decreasing cash flows. Conditions under which a dual set of return measures may be used to construct bounds on the expected return on investment are derived. Section 4 introduces the actual accounting rates of return and their relationship to expected returns conditional on the choice of accounting policies. Section 5 briefly summarizes a case study implementing some of the ideas introduced in this paper. The case study also serves as an illustration. Section 6 provides concluding remarks.

2. The Model

The following notation will be used: An amount K is invested at time 0 (end of period 0). An investment generates a vector of expected end of period cash flows $\mathbf{c} = (c_1, c_2, \ldots, c_T)$ per dollar invested. T will be referred to as the life of the project. The discount factor that makes the present value of the cash flow vector equal to 1, will be referred to as the (expected) internal rate of return and denoted r[2]. Earnings measurement involves an accrual vector $\mathbf{a} = (a_0, a_1,. a_T)$. a_0 is equal to –1 if the investment cost is capitalized in full. $a_1, \ldots a_T$ is referred to as the depreciation plan (per dollar invested). The accrual vector always satisfies the following condition:

[2] r is not equal to the expectation of the ex post (realized) rate of return.

$$\sum_{j=0}^{T} a_j = 0,$$

The start of period book value B_t per dollar invested and the expected accounting rate of return ρ(t) at time t, are defined as follows:

$$B_t = -\sum_{j=0}^{t-1} a_j, \text{ for t} = 1, \ldots, \text{T}$$

$$\rho(t) = \frac{c_t - a_t}{B_t}, \text{ for t} = 1, \ldots., \text{T}$$

There is of course a one-to-one correspondence between the accrual vector and the sequence of book values, making it possible to derive the accrual vector from book values. It is well known that there exist a unique accrual vector with the property that a_0 = -1 and ρ(t) = r for all t. This vector, usually called IRR-depreciation, may be defined by the sequence of book values as follows:

(1) $$B_t^*(\mathbf{c}) = \sum_{j=0}^{T-t} \frac{c_{t+j}}{(1+r)^{j+1}}$$

Two specific depreciation plans (both independent of the cash flow vector) will be used. The linear method (LIN) and the annuity method with some interest rate i (ANN(i)) are identified by superscripts L and A respectively.

(2) $$B_t^L = \frac{T-t+1}{T}$$

(3) $$B_t^A = \sum_{j=1}^{T-t} \frac{c(i)}{(1+i)^{j+1}}, \text{ where c(i)} = \frac{i(1+i)^T}{(1+i)^T - 1}$$

The annuity method is thus really a family of depreciation plans indexed by i. It is easy to show that the linear plan is actually a member of the annuity family with i = 0. This paper will focus on the case of i equal to r (in addition to i = 0). When the index i is dropped, it is understood that i = r. The associated rates of return are denoted ρ_t^A and ρ_t^L (or ρ_t^i if it is necessary to indicate which value of i is used).

It is possible to define an incomplete ordering of depreciation plans, with book values B and B' respectively, called "conservatism" as follows,

B is more conservative than B' if $B_t \bullet B_t'$ for all t

It is easy to show that within the annuity family conservatism is increasing in the index i. Linear depreciation is the most conservative member (if negative values of i are excluded).

A firm is a collection of investment projects of different vintages. Investments are assumed to have identical cash flow vectors. The investment profile is described by the vector (K_{-T}, $K_{-(T-1)}$, … , K_0). K_0 is the amount invested at the end of the current period, and K_{-T} is the amount invested in the oldest vintage (being retired at the end of the current period). Assuming a_0 = -1, the expected accounting rate of return for a firm is a weighted sum of the returns on currently active projects:

(4) $$\rho = \sum_{j=1}^{T} \frac{K_{-j}(c_j - a_j)}{\sum_{j=1}^{T} K_{-j} B_j} = \sum_{j=1}^{T} \frac{K_{-j} B_j}{\sum_{j=1}^{T} K_{-j} B_j} \rho(j)$$

To study firm rates of return, steady state with growth rate g is usually assumed. Then the investment profile is characterized by the following relations: $K_{-t} = K_{-(t+1)}(1+g)$ for t = 1,…T-1). A celebrated theorem due to Stauffer, 1972, derives the expected rates of return in steady state. The following discrete time expression of firm returns holds for arbitrary accrual vectors (as long as a_0 = -1):

(5) $$\rho = g\,\frac{A(g) - (C(g) - 1)}{A(g)}$$

where, $C(g) = \sum_{t=1}^{T} \frac{c_t}{(1+g)^t}$, and $A(g) = \sum_{t=1}^{T} \frac{a_t}{(1+g)^t}$

To use this relationship to calculate returns it is necessary to know **c**, which is not observable[3]. The expression may also be used to derive general relationships between r, ρ, g and the depreciation plan (via A(g)). Since C(r) = 1, g = r implies ρ = g = r. Differentiating with respect to A keeping g fixed, it is readily apparent that ρ will increase in conservatism if g < r, and vice versa. Since IRR-depreciation always produces ρ = r, ρ is close to r if g is close to r or **a** is close to IRR-depreciation.

Restricting attention to steady state is not really satisfactory. Usually it is not clear whether a firm is close to steady state. Nor is it easy to judge whether depreciation is close to IRR-depreciation. The objective of this paper is to provide estimates of internal rates of return for firms without invoking steady-state assumptions. Instead restrictions will be put on cash flow profiles. First of all it will be assumed that flow profiles are linear (in time):

(6) $c_t = c_0 + b\,t,\ \ t = 1, 2, \ldots\ldots, T.$

It will also be assumed that cash flows are decreasing (weakly), but not decreasing too fast. Keeping r constant, c_0 and b must satisfy the following conditions:

(6i) $$\sum_{t=1}^{T}(c_0 + bt)\frac{1}{(1+r)^t} = 1$$

and

(6ii) $-(1/T)\, r \le b \le 0$

This set of cash flow profiles will be denotes L (r). Letting r vary over the set of r's that have positive probability (the support of r), generates the complete set of possible cash flow vectors.

For cash flow profiles in the set L, IRR-depreciation is more conservative than ANN(r) and less conservative than LIN. ANN(r) equals IRR-depreciation if b = 0. LIN equals IRR-depreciation if b = -r/T. It follows immediately from steady state theory that expected return would be bounded by ρ^L and ρ^A regardless of the growth rate and the cash flow vector in L. Calculating both ρ^L and ρ^A first of all makes it unnecessary to know the sign of (g – r) since it equals the sign of (ρ^L - ρ^A). More significantly the bounds will provide information about the size of the potential bias in steady-state not only its sign. The idea of calculating two

[3] This paper will follow the traditions of this literature and assume that realized firm cash flows are only observable in the aggregate.

accounting rates of return to produce an interval estimate on the IRR is the main focus of this paper. Steady state being a serious limitation, the next section is devoted to relaxing this assumption.

3. More about the expected rate of return

Throughout this section expected cash flow profiles are assumed to belong to the set L.

The first theorem characterizes ρ^L and ρ^A for a single project and states that r is bounded by ρ^L and ρ^A at any time t. The theorem thus extends the idea of an interval estimate from steady state to a single project.

Theorem 1

Assume that $\mathbf{c} \in L$.

Then there exists a real number t_0, such that:

For $t < t_0$; $\rho^L \leq r \leq \rho^A$, and

For $t > t_0$; $\rho^L \geq r \geq \rho^A$,

Moreover $\rho^L(t)$ is increasing, and $\rho^A(t)$ is decreasing in t. $\rho^L(t) - \rho^A(t)$ is strictly increasing in t.

The proof of theorem 1 is in the appendix.

Theorem 1 may be illustrated numerically. Assume investment (K) equals 1, that the life of the project (T) is 25, and that the internal rate of return (r) is .12. Then constant cash flows (b=0) implies an annual cash flow of .1275. Maximally decreasing cash flows (b = .0048) will start at .16 in period 1 and decline to .0448 in the last period. Figure 1 shows the ARR as a function of age using different accrual vectors. ARR identically equal to .12 obtains when constant cash flow is combined with annuity depreciation or maximally decreasing cash flows is paired with the linear method. The increasing line shows how the ARR develops over time with linear depreciation and constants cash flows. Finally, the decreasing line exhibits the accounting return when annuity depreciation is used even though cash flows are decreasing at the maximum rate within the set L (the dotted line may be ignored for the time being).

(Figure 1 here)

Figure 1 shows that the effect of the accrual vector is quite strong when investments get old. This is particularly true for long-lived investments as in this example. The lines in the figure shows the maximal ‘measurement errors’ that may be made when the cash flow vector is constrained to be in the set L. However, for any particular cash flow vector in L, the interval estimate for the IRR obtained when using both depreciation plans will be much narrower – approximately half the distance between the two lines. The reason is of course that ρ^L and ρ^A cannot both miss by the maximum amount simultaneously.

As noted above, a firm is a collection of investment projects of different vintages. The accounting rate of return for a firm is a weighted average of the returns on the different projects using the relative book values as weights (see (4)). As the accounting rate of return is a nonlinear function of time, and the book value depends on the depreciation plan, it is by no means obvious that theorem 1 extends to a firm. However in section 2 it was shown to hold for a firm in steady state. Theorem 2 demonstrates that it ρ_t^A and ρ_t^L provides bound on the IRR for any collection of investment projects i. e. for any firm (provided the cash flow vector belongs to L).

Theorem 2

Assume that $\mathbf{c} \in L$.

Then,

$$(\rho^L - r)\ (\rho^A - r) \bullet 0, \text{ for all investment profiles } (K_{-T}, K_{-(T-1)}, \dots, K_{-1}).$$

The proof of theorem 2 is in the appendix.

The boundedness result of course implies that one ARR overestimates and the other underestimates the true IRR. It is of interest to know under what conditions ρ_t^A overestimates and ρ_t^L underestimates the IRR and vice versa. Roughly speaking this follows immediately from the one-project analysis of theorem 1. LIN leads to underestimation of IRR when the firm is young i. e. when new projects predominate, and to overestimation for older firms. For ANN the opposite result holds.

More precise statements are possible. Using LIN and assuming as before a linear cash flow profile, the accounting return is a function of the weighted average age of investment projects. To put it a little differently, the average age is sufficient for the investment profile vector. When the average age is below t_0 (defined in theorem 1), the ARR underestimates the IRR. When the average age is above t_0, overestimation occurs.

With ANN the relationship between measurement error and age is more complicated. It is still true that the sign of the measurement error is a function of average age only. Again t_0 is the pivotal age. However, ceteris paribus the size of the measurement error is (weakly) increasing in the age spread when the annuity method is used[4]. These results are collected in the following theorem.

Theorem 3

Assume that **c** $\in$ L.

The firm is described by an investment profile K = (K_{-T}, $K_{-(T-1)}$, … , K_{-1}). Let t_0 be defined as in theorem 1, and let $\bar{t}$ be the weighted average age of the firm:

(7) $$\bar{t} = \sum_{j=1}^{T} \frac{K_{-j}\, j}{\sum_{j=1}^{T} K_{-j}}$$

Then:

For $\bar{t} < t_0$; $\rho^L \bullet r \bullet \rho^A$, and $\rho^L < \rho^A$
For $\bar{t} = t_0$; $\rho^L = r = \rho^A$, and
For $\bar{t} > t_0$; $\rho^L \bullet r \bullet \rho^A$, and $\rho^L > \rho^A$

Moreover, with respect to ρ^L $\bar{t}$ is sufficient for the investment profile K. On the other hand $\left|\rho^A - r\right|$ is increasing in the spread of K.

The proof of theorem 3 is in the appendix.

Theorem 3 is clearly related to steady-state theory discussed above. Steady-state with growth rate equal to r, implies an average asset age of t_0. A high growth firm is a young firm whereas

[4] By spread is meant the usual second order dominance ranking of distributions – here age distributions.

an old firm has experienced low growth. However, steady-state is a very strong condition to impose on an investment profile. Theorem 3 requires no assumptions on investment profile, and is therefore a much more useful result. On the other hand theorem 3 requires a linear cash flow profile, which may not obtain even as an approximation in many cases.

Figure 1 can also be used to illustrate theorems 2 and 3. Theorem 3 says that the increasing line also depict ARR for firms. It is sufficient to interpret age as average age. For the annuity method matters are more complicated; age spread matters as well. The dotted line in figure 1 represents ARR as a function of average age for a two-investment firm when annuity depreciation is used (along with maximally decreasing cash flows). The firm has been constructed to maximize age spread; a firm of age 5 has 1 and 9 year old investments whereas average age of 16 means ages 25 and 7. Clearly the effect of the age spread is larger for older firms. The reason is that non-linearities are stronger here. Note in particular the change between ages 12 and 13 in figure 1. All firms of age 13 and older include a 25 year old investment.

This section concludes by a discussion of some of the assumptions made. The cash flow vector **c**, which is assumed to be linear and decreasing, represents nominal cash flows. Inflation may present a potential problem by introducing non-linearities in the cash profile and by making the case for also allowing cash flows which increase over time. If, for example, real cash flows are constant and inflation is positive and constant, nominal cash flows will be increasing and convex. However, the theory may be adapted to handle inflation. If it is assumed that real (rather than nominal) cash flow vectors belong to the set L, theorems 1 through 3 will still be valid if inflation adjustments are introduced to either cash flows or accruals.

The particular annuity method considered in theorems 1 through 3 is ANN(r). This method requires knowledge of r. The assumption may be justified if management chooses the accrual vector. Management may know r even if they do not know the slope of the cash flow vector. On the other hand, if an external analyst picks the depreciation plan, knowledge of r cannot very well be assumed.

In case r is not known, the analyst must start the estimation process by making an informed guess of r to be used in calculating the accrual vector. If there is little variation in the annual cash flows (in particular if cash flows are non-random) the assumed value of the IRR may be tested against the calculated values of the ARR. Recalling the numerical example, assume that cash flow is constant, and that the analyst guesses that the internal rate of return is .16 rather than .12. In that case ρ_1^{16} is .1235 whereas is ρ_1^{L} is .0875. The analyst may then conclude that r is between .1235 and .0875, which should lead her to start over with a downwardly revised estimate of r.

If cash flows are random, which is the more realistic case, it is more difficult to evaluate initial assumptions. A low ARR may be due to negative deviations from the mean rather than overestimation of the mean. In that case the analyst might try a different approach. If the objective of the analysis is to construct a band, which covers the true expected IRR, i should be chosen such as to broaden the band rather than shrink it. In that case the analyst should choose a high rather than a low value of i. Figure 2 illustrates the consequences of using ANN(16) rather than ANN(12) in the numerical example with b = -.0048 (the lower bound).

(Figure 2 here)

Figure 2 illustrates the following general facts: $\rho^{16} - \rho^{12}$ is fairly small in absolute value. Except for the last four periods, the difference is smaller than one percentage point, and for the most part considerably smaller (it is less than .4 percentage points until period 20). Thus ρ^i is not very sensitive with respect to overestimation of i. $\rho^{16} - \rho^{12}$ is positive for low values of t, and negative for high values of t. However, the three lines in figure 2 do not cross in the same point.[5] Although this may be hard to see from figure 2, ρ^{16} and ρ^{12} cross between periods 13 and 14 whereas ρ^L and ρ^{12} cross at t_0 (which lies between periods 7 and 8). Thus using ρ^{16} rather than ρ^{12} widens the band in periods 1 – 7 and periods 14 – 25, but makes the band narrower in periods 8 through 13. Fortunately, the band is reduced by very little (less than .2 percentage points) in this interval.

4. Actual rates of return

Thus far the theory has only dealt with expected values. The main results specify conditions under which expected accounting returns brackets the expected internal rate of return. Actual, observed rates may of course differ from their expected values. Introducing uncertainty explicitly raises a host of new issues, which are outside the scope of this work. The most fundamental question is perhaps which internal rate of return one would like to measure.[6]

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[5] I have not been able to show that ρ^i and ρ^j only cross once in general.

[6] Alternatives are: The ex ante expected rate of return, the revised expected rate of return of the project taking realized cash flows into account and the expected return on the remainder of the project.

Here uncertainty is introduced in the simplest manner. Actual cash flow is assumed to be equal to expected cash flow plus random error terms are identically distributed and independent of everything else[7]:

(8) $$\tilde{c}_t = c_t + \varepsilon$$

It follows that the actual accounting rate of return may be expressed as follows (k = ANN, LIN and γ denote the accrual bias):

(9) $$\tilde{\rho}_t^k = \rho_t^k(\bar{t},b) + \frac{\varepsilon}{B_t^k(\bar{t},b)} = r + \gamma^k(\bar{t},b) + \frac{\varepsilon}{B_t^k(\bar{t},b)}$$

The accounting rate of return for a firm be may derived by aggregation. Since the analysis is not going to probe very deeply, a looser approach will be adopted. The time subscript in the two expressions will be dropped, and ρ and ε may refer to firm as well as project returns.

The accounting rate of return has two different interpretations. First of all it is a measure of the (expected) internal rate of return. Secondly, it is a measure of the return on capital in period t (including windfalls). It is not at all clear that the second interpretation is meaningful, and it will not be pursued further here.[8] As a measure of the internal rate of return, the accounting rate has two sources of error. The first one - γ - has been studied above. It is created by the accrual vector, and for a given $\bar{t}$ its sign is known. However, from a Bayesian perspective it may be considered a random error as long as b is an unknown parameter. The second source of error is the cash windfalls ε.

It is obvious that the effect of the cash flow error depends on accruals. For a given ε the last term in (9) will decrease in book value (in absolute terms). Since book value is negatively related to age and conservatism, the first part of the next theorem follows immediately. Thus the effects of random windfalls on the ARR depend on the depreciation plan. The second part of the theorem states the relationship between actual accounting returns when ε varies. The final result provides a way to estimate r on the basis of the accounting rates of return; the estimator will be discussed below:

[7] Hence observed cash flows have no relvance for future cash flows; e is a pure windfall in the parlance of income measurement.
[8] It may be argued that the effect of windfalls on profitability cannot be expressed by their relationship to current book value. Such effects must be seen in the context of the project as a whole.

Theorem 4

(i) $\left|\tilde{\rho} - \rho\right|$ is increasing in age and conservatism.

(ii) $\tilde{\rho}^L = \dfrac{a^A - a^L}{B^L} + \dfrac{B^A}{B^L}\tilde{\rho}^A$

(iii) Assume that $\mathbf{c} \in$ L, $\gamma^A = \gamma^L + \Gamma$, and $E\gamma^A = -E\gamma^L = \Gamma/2$, then the following estimate of r is unbiased and independent of ε:

$$\hat{r} = \alpha_0 + \alpha_1\tilde{\rho}^L + (1-\alpha_1)\tilde{\rho}^A$$

where,

$$\alpha_0 = -\left(\frac{B^A + B^L}{B^A - B^L}\right)\frac{\Gamma}{2}, \quad \text{and } \alpha_1 = -\left(\frac{B^L}{B^A - B^L}\right)$$

Proof:

Using (9) for i = A and solving for ε, yields $\varepsilon = B^A(\tilde{\rho}^A - \rho^A)$. Inserting this expression into (9) for i = L and collecting terms yields (ii).

To prove (iii) first note that α_1 has been chosen to make the error term vanish. Taking expectation, again using (9) as well as the assumptions in the theorem, shows that the estimator is unbiased.

The accounting rate of return will reflect periodic windfalls (positive or negative). Because conservatism implies lower book values, the variance in the accounting rate of return will increase. Conservative accounting will imply more extreme measures, reporting higher returns in good times and lower in bad times. This insight may be of some interest for financial analysis. Similarly book value is decreasing over time, and the impact of windfalls increases.[9]

The accounting rate of return may also be interpreted as a measure of the (ex ante) expected internal rate of return. From this perspective ε is a source of measurement error. This error is amplified by conservatism, and will be superimposed on the error caused by the choice of

[9] It may be argued that the variance of ε should also decrease over time - in particular if expected cash flow declines. However, the varance of returns would still be decreasing with age even if it were assumed that the error variance were proportional to expected cash flow.

accrual vector. Sometimes these sources of error work in the same direction. In other cases they may cancel out.

Figure 1 illustrates the relationship between $\tilde{\rho}^A$ and $\tilde{\rho}^L$ for three different firms (or projects). When $\varepsilon = 0$, ρ^A and ρ^L over- and underestimates expected return respectively for a young firm. See point a in figure 1. In bad times ($\varepsilon < 0$) $\tilde{\rho}_t^L$ will underestimate r a fortiori and $\tilde{\rho}^A$ may underestimate r as well. In good times on the other hand ($\varepsilon > 0$) $\tilde{\rho}^A$ and $\tilde{\rho}^L$ may both overestimate r, and linear depreciation may very well produce the higher accounting rates of return. $\tilde{\rho}^A$ - $\tilde{\rho}^L$ (measured as the vertical difference between the 45° line and the young firm line) decreases in ε.

For an old firm (or project) the effects go in opposite directions. The difference between accounting rates of returns increases in ε. For firms whose average age is close to t_0, the accrual errors vanish, and the ε-error determines the difference between accounting rates of return and between the expected internal rate of return and the former.

Under certainty r may be measured perfectly if $\bar{t} = t_0$ regardless of the accrual vector. When cash flows are random, this does not work since ε is unobservable. With two ARRs, however, it is possible to identify ε by triangulation, and measure r without error. Theorem 4(iii) generalizes this insight. The restriction on the cash flow vector implies that the accrual errors γ has opposite signs. The assumptions of the theorem impose a certain symmetry on their subjective probability distributions; the sum of their absolute values are equal to a known constant Γ, and their means are equal. Under these assumptions it is possible to derive an unbiased estimate of r that is independent of ε.

There are of course many unbiased estimators of r, but $\hat{r}$ may be attractive if ε is the major concern; for example if the variance of ε is large, or there are few available observations. The distribution of γ depends primarily on the distribution of (beliefs about) the cash flow parameter b. The theory implies that the symmetry assumptions may represent a reasonable approximation if the distribution of b is symmetrical. It should be noted that the variance of $\hat{r}$ might be considerable if B^L and B^A are close.

It follows from theorem 4 that the slopes of the lines in figure 3 are always greater than 1. It is also interesting and perhaps unintuitive that slopes as well as constant terms are independent of the expected cash flow profile **c**. However, it is important to note that ρ^A and ρ^L (the expected accounting rates of return) will depend on **c** as well as on asset age. ρ^A and ρ^L in figure 3 corresponds to a young firm and a particular (downward sloping) expected cash flow profile.

(Figure 3 here)

5. Implementation

This section reports the results of an implementation of the ideas outlined in sections 3 and 4. As the calculations involved in remeasuring accounting rates of return are quite extensive, a large sample study is beyond the scope of this work. Instead a longitudinal case study has been chosen. The case study will also serve as an illustration.

The accounting rates of return for the Norwegian shipping company Odfjell have been calculated for the period 1986-99, using linear as well as annuity depreciation. Odfjell operate 50 tankers for the transportation of chemicals. A shipping company has been chosen because the theoretical model seems to fit its investment activities reasonably well. Odfjell acquires similar ships on a regular basis. The ships are operated for period of 25-30 years until they are scrapped. Real (and perhaps even nominal) cash flows may be expected to decrease over time due to falling freight rates (caused by technical progress) and increasing maintenance costs.

However, a moment's reflection makes clear that there are many features of an actual shipping company that are not captured by the theoretical construct. Odfjell invests in financial assets and real estate (terminals) as well as ships. Of the 50 tankers, about 20 are leased on various types of contracts rather than owned. Moreover, used vessels are bought and sold, and transactions may take place any time during the year (not only at the end of the year). Hence the methods are implemented at less than perfect conditions (and the results are not tautological).

Odfjell currently depreciates its ships linearly over a period of 25 years. However, the assumed life of the vessels has been increased twice during the period (in 1987 and 1995). For that reason linear depreciation has been recalculated using T=25 uniformly. Returns have then been derived using the annuity method with an interest rate of 12%. The rate was chosen after looking at the "linear" returns and deciding that 12% was in the ballpark (and probably on the high side). The rate of return reported here is the rate on operational capital. Financial assets, financial income and "interest free" debt have been deducted where appropriate. These adjustments are potentially subject to considerable measurement error.[10]

Table 1 Accounting rates of return, average age and book values, Odfjell 1985 - 1999

	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	Mean
ρ	8.5	6.1	26.2	18.3	18.4	11.5	3.7	3.1	8.1	14.4	13.6	22.0	9.0	6.4	12.09
ρ^L	10.4	4.9	19.7	15.2	18.8	12.7	5.5	4.8	9.0	11.3	10.9	18.0	7.4	5.2	10.98
ρ^A	11.4	5.7	17.6	14.8	19.8	14.6	7.6	6.5	9.7	11.5	11.3	16.1	8.0	6.6	11.51
ρ^A-ρ^L	1.0	0.8	-2.2	-0.4	1.0	1.9	2.1	1.7	0.7	0.3	0.4	-1.8	0.6	1.3	0.53
r(est)	12.65	8.79	3.49	-3.46	12.07	16.58	15.11	16.34	14.74	13.83	16.21	6.24	10.36	11.44	11.03
AGE	6.6	7.5	6.0	4.1	4.6	5.2	6.2	7.6	8.0	7.8	8.4	7.6	7.6	7.2	
B^L	959	903	828	1507	1975	2913	3109	3298	3396	3678	3913	4181	5340	6344	
B^A	1113	1084	1026	1723	2217	3222	3517	3804	3991	4355	4676	5047	6268	7379	
B^A/B^L	1.16	1.20	1.24	1.14	1.12	1.11	1.13	1.15	1.17	1.18	1.20	1.21	1.17	1.16	1.17

ρ - Accounting rate of return as reported
ρ^L - Accounting rate of return with uniform linear depreciation (25 years), beginning book value is B^L.
ρ^A - Accounting rate of return with annuity depreciation (25 years, interest rate 12%), beginning book value is B^A.
AGE - average (relative) age of fleet.
r(est) - estimate of internal rate of return using theorem 4(iii).

Table 1 reports three return series based on actual, 25-year linear depreciation, and 25 year/12% annuity depreciation respectively. They are highly correlated, although changing depreciation plan twice has clearly affected the reported returns. The absolute difference between ρ^A and ρ^L exceeds 2 percentage points only twice. Not surprisingly, in view of theorem 4, larger differences tend to occur when times are particularly good or bad. The mean difference is just .53%.[11] The modest differences should be interpreted in light of the average age of the tanker fleet. The age does not differ very much from t_0, which is 7.8 in this particular case.[12]

To describe the relationship between ρ^A, ρ^L and AGE, the following regression has been estimated:

$$\rho^A - \rho^L = .1527 - .243(\rho^A - 11.5) - .497(AGE - 7.5)$$

[10] The case study is reported in more detail in Gjesdal (2001). Only public information has been used. Book values and depreciation has been recalculated using a spreadsheet. The main problems involve calulating beginning values for the fleet in 1986, and calculating annual depreciation when vessels have been bought and sold during the year. For the accounting returns beginning values of capital have been used throughout.

[11] This is the arihmetic mean which is more meaningful than the geometric mean in this context.

[12] Average age is here measured as relative age. Relative age means that a used ship will have age 0 when acquired and depreciation starts. Odfjell acquired a number of older ships in the late eighties which implied that the absolute age increased and the relative age decreased.

The negative coefficient on ρ^A confirms theorem 4(ii). (Using the mean of B^A/B^L, the "theoretical" value is -.17). The negative coefficient on AGE confirms theorems 2 and 3.[13] The relationship between (ρ^A - ρ^L) and AGE (assuming ρ^A is 11.5) is shown in figure 4 along with the actual values. Note that the estimated line crosses the AGE axis at 7.8, which equals t_0.

(Figure 4 here)

Finally using theorem 4(iii) to estimate r does not work very well. Assuming the true value of r equals 11, r(est) beats the average of ρ^A and ρ^L in only 8 of 14 cases.

6. Summary and concluding remarks

In this paper return measurement has been addressed in a model reflecting explicit uncertainty. The expected return on long-term investment has been assumed constant, but subject to random cash flow shocks that are independent over time. This is obviously a first pass at introducing uncertainty. A natural second step would be to formulate a stochastic process that better reflects the empirical evidence of mean reversion in returns (Penman (1992)).

Rather than assuming steady state, which is common in the literature, the cash flow profiles generated by investments (nominal or real) are restricted to be linear and decreasing. If cash flows are not declining too sharply, it can be shown that the expected accounting rates of return, calculated using linear and annuity depreciation respectively, effectively bounds the expected internal rate of return in every period of the life of an investment project.

Considerable deeper is the conclusion that this result also holds for an arbitrary collection of investment projects (firms). Furthermore it is shown that with linear depreciation the expected accounting rate of return is only a function of the average age of projects (holding cash flow profile constant). Using the annuity method of depreciation, the accounting rate of return is also a function or the age spread of the portfolio.

The assumption of linear cash flow profiles is at best a useful approximation. The robustness of this approximation should be investigated further. It seems likely that the bounds

[13] The relationship is not predicted to be linear.

constructed by using different depreciation plans should hold for more general cash flow profiles – at least at the project level.

In the illustrative case study it is demonstrated that the bounds provided by the two different measures of return are quite sharp. The maximum absolute difference is 2.2 percentage points over the 14-year period, and the average difference is .53. The average age of the firm's fleet of ships varies between 4.1 and 8.4 (The expected accounting rates of return equal the return on investment when the average age is 7.8).

Modeling cash flow uncertainty explicitly, it immediately becomes evident that the sensitivity of the accounting rates of returns with respect to variations in cash flows depends on book values. Book value depends on age as well as accrual policy. It is also agued that using several measures of return may reveal the effect of windfalls via triangulation. However, the power of this method depends on the variance of the slope of the cash flow profile (and other possible measurement errors), and it does not work very well in the case study.

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APPENDIX

(Proof of theorems)

Proof of theorem 1

First it will be shown that a linear cash flow profile will satisfy (6i) if and only if there exists a pair of constants t_0 and d (which depend on r) such that the parameters c_0 and b satisfy

(A1) $c_0 + b\, t_0 = d$

Solving for c_0 and inserting into (6i) yields:

$$1 = \sum_{t=1}^{T}(-bt_0 + d + bt)\frac{1}{(1+r)^t} = \sum_{t=1}^{T}(-bt_0 + d)\frac{1}{(1+r)^t} + \sum_{t=1}^{T}(bt)\frac{1}{(1+r)^t}$$
$$= (-bt_0 + d)\sum_{t=1}^{T}\frac{1}{(1+r)^t} + b\sum_{t=1}^{T}\frac{t}{(1+r)^t} = b(-t_0 A(r,T) + \hat{A}(r,T)) + dA(r,T)$$

Where A(r,T) and $\hat{A}(r,T)$ are defined by the respective sums.

This equation will hold for all b if $At_0 - \hat{A} = 0$, and d A = 1. These two equations may be used to solve for the parameters t_0 and d. It follows that pairs (c_0, b) satisfying (A1) for these values of the parameters, will also satisfy (6i). These pairs characterize all linear cash flow profiles going through the point (t_0, d).

There cannot be any other linear cash flow profiles satisfying (6i). (If there were, the profile would be parallel to one profile satisfying (A1)). Two parallel profiles cannot both satisfy (6i)). Hence (A1) characterizes all linear cash profiles with a given internal rate of return.

To prove the first part of the theorem, keep b within the bounds specified by (6ii). Then c_t is a function of b: $c_t = c_0(b) + bt$. For $t < t_0$, c_t is decreasing in b. Hence for a given depreciation plan, ρ(t, b) is decreasing in b. It is well known that $\rho^A(t, 0) = r = \rho^L(t, -(1/T))$. Thus it follows directly that $\rho^L(t, b) \leq r \leq \rho^A(t, b)$ for $t < t_0$, and b satisfying (6ii). Similar arguments prove that $\rho^L \geq r \geq \rho^A$ when $t > t_0$. This concludes the proof of part 1.

To prove the second part of the proposition for linear depreciation, assume that

$$\rho^L(2) = \frac{\rho^L(1)B_0 + b}{B_0 - a} < \rho^L(1)$$

(where a is annual depreciation). This is equivalent to

$$-\frac{b}{a} > \rho^L(1)$$

Since a and b are constants, it follows by iteration that $\rho^L(t+1) < \rho^L(t)$ for all t, and hence $\rho^L(t)$ is strictly decreasing. This violates part one of the theorem. Hence $-b/a \leq \rho^L(1)$, and iteration again shows that ρ^L is weakly increasing.

To prove the second part for the annuity method, first note that under this depreciation plan annual depreciation grows at a constant rate: $a_{t+1} = a_t(1+r)$. The accounting rate of return decreases if,

$$\rho^A(t+1) = \frac{\rho^A(t)B_{t-1} + b - ra_t}{B_{t-1} - a_t} \leq \rho^A(t),$$

or equivalently,

(A2) $\quad b \leq (r - \rho^A(t))a_t$

Assume that this condition is violated at time $\hat{t}$, then $\rho^A(\hat{t}) \geq r$ (since $b < 0$). It follows that (A2) is also violated at $\hat{t}+1$ since both factors on the RHS increase in absolute value. Hence $\rho^A(t) \geq r$ for all $t \geq \hat{t}$. This violates part one of the proposition, Hence $\rho^A(t)$ is weakly decreasing.

The above arguments imply that $\rho^A(t)$ is constant if and only if $b = 0$. Similarly $\rho^L(t)$ is constant if and only if $b = r/T$. Both cannot be constant at same time. This concludes the proof.

Proof of theorem 2

If ρ^A_t is equal to r for all t, the theorem is obviously true. Otherwise the theorem is an implication of the following fact

$$\text{(A3)} \qquad \forall t; \ \frac{(\rho_t^L - r)B_t^L}{(\rho_t^A - r)B_t^A} = k \le 0$$

Assuming (A3) holds it easy to show that $(\rho^L - r)$ and $(\rho^A - r)$ have opposite signs. Assume $(\rho^L - r) \geq 0$, then

$$\text{(A4)} \qquad \begin{aligned} \rho^L &= \sum_{j=1}^{T} \frac{K_{-j} B_j^L \rho_j^L}{\sum_{j=1}^{T} K_{-j} B_j^L} \ge r \Leftrightarrow 0 \le \sum_{j=1}^{T} K_{-j} B_j^L (\rho_j^L - r) \\ &= k \sum_{j=1}^{T} K_{-j} B_j^A (\rho_j^A - r) = k(\sum_{j=1}^{T} K_{-j} B_j^A)(\rho^A - r) \end{aligned}$$

If k = 0, the theorem is clearly true. If k < 0, it follows that $(\rho^A - r) \leq 0$.

The case in which $(\rho^L - r)$ is negative, is handled in the same way.

It remains to prove (A3). Interpreting r as the cost of capital (A3) is a ratio of residual incomes. Using R to denote residual income, it is sufficient to show that $R^L_t/R^A_t = R^L_{t+1}/R^A_{t+1}$ for arbitrary t. Annuity depreciation implies that residual income decreases with time as cash flow is decreasing by b every period, and the capital charge (depreciation + interest) is equal to the constant d defined in (A1). With linear depreciation the capital charge decreases by a fixed amount r(1/T) per period. Hence the following equality must be proved:

$$\frac{R_t^L}{R_t^A} = \frac{R_t^L + b + r/T}{R_t^A + b}$$

or,

$$R_t^A \frac{r}{T} = (R_t^L - R_t^A) b$$

The right and left hand sides will be evaluated separately. The right hand side is fairly simple to calculate since the difference in residual incomes equals the difference in capital charges:

$$(R_t^L - R_t^A)b = \frac{b}{T}(dT - 1 - r(T - t + 1))$$

The left hand side is more complex. Using notation defined in (A1) and the proof of theorem 1 above, the following equalities may be derived (the calculations involve evaluating $\hat{A}$ and d which is non-trivial):

$$\begin{aligned}
R_t^A \frac{r}{T} &= \frac{r}{T}(d - bt_0 + bt - d) = \frac{b}{T} r(t - t_0) \\
&= \frac{b}{T}(rt - r\hat{A}d) = \frac{b}{T}\left(rt - r\left[\frac{1}{r}((1+r)\frac{1}{d} - (\frac{1}{1+r})^T T)\right]d\right) \\
&= \frac{b}{T}(rt - (1+r) + (\frac{1}{1+r})^T Td) \\
&= \frac{b}{T}(r(t-1) + Td - 1 - rT) = \frac{b}{T}(dT - 1 - r(T - t + 1))
\end{aligned}$$

This concludes the proof.

Proof of theorem 3

The firm's aggregate cash flow – C – may be expressed as follows:

$$C = \sum_{t=1}^{T} c_0 K_{-t} + \sum_{t=1}^{T} btK_{-t} = (\sum_{t=1}^{T} K_{-t})\left[c_0 + b(\sum_{t=1}^{T} tK_{-t} / \sum_{t=1}^{T} K_{-t})\right] + (\sum_{t=1}^{T} K_{-t})\left[c_0 + b\bar{t}\right]$$

Similarly, total book value – B – and depreciation – D - may be expressed as follows using (2):

$$B = (\sum_{t=1}^{T} K_{-t})\left[1 + \frac{1}{T} + \bar{t}\right]$$

$$D = (\sum_{t=1}^{T} K_{-t})\left[\frac{1}{T}\right]$$

It follows that ρ^L is a function of $\bar{t}$ (as well as the parameters of the cash flow profile: c_0, b and T). In other words $\bar{t}$ is sufficient for the investment profile K = (K_{-T}, $K_{-(T-1)}$, … , K_{-1}).

Since this relationship must hold for a single project as well as for a firm, it follows from theorem 1 that $(\bar{t} - t_0)(\rho^L - r) \geq 0$, and theorem 2 then implies that $(\bar{t} - t_0)(\rho^A - r) \leq 0$.

Next define f as follows:

$$f = \frac{\sum_{j=1}^{T} K_{-j} B_j^L (\rho_j^L - r)}{\sum_{j=1}^{T} K_{-j}}$$

Using (A4) the ARR may then be expressed in the following way:

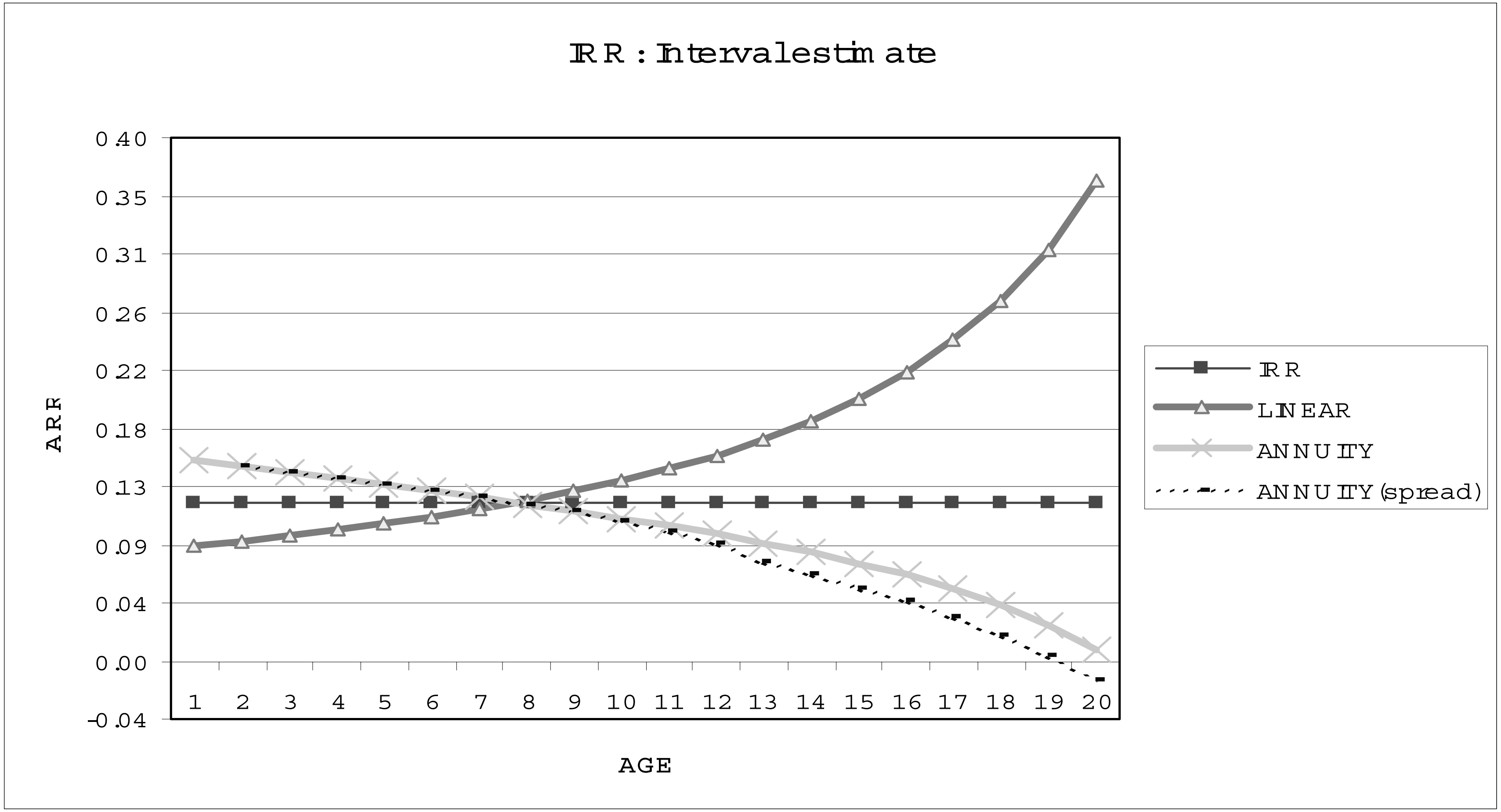
$$\rho^L - r = \frac{\sum_{j=1}^{T} K_{-j} B_j^L (\rho_j^L - r)}{\sum_{j=1}^{T} K_{-j} B_j^L} = \frac{f}{\sum_{j=1}^{T} \frac{K_{-j} B_j^L}{\sum_{j=1}^{T} K_{-j}}}$$

Varying the investment profile K, keeping the weighted average age constant, ρ^L, r and average book value is constant. It follows that f must be constant as well.

Using the same logic and (A3), ρ^A may be expressed as follows:

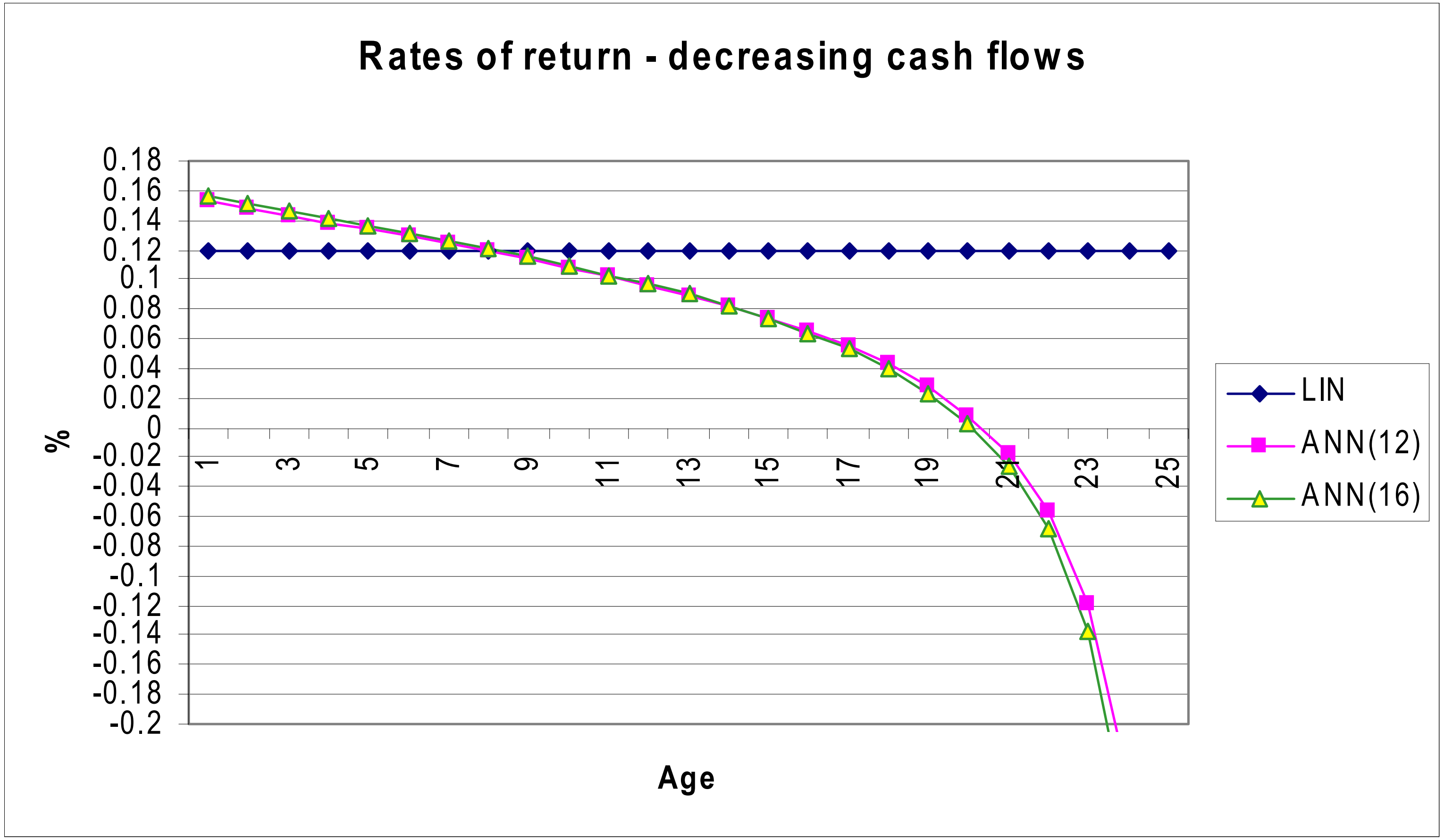
$$\rho^A - r = \frac{kf}{\sum_{j=1}^{T} \frac{K_j}{\sum_{j=1}^{T} K_j} B_j^A}$$

Using annuity depreciation book value is a concave function of age. It follows that the term in the denominator on the RHS of the expression will decrease in the age spread of the investment profile K. Hence keeping the average age (and hence f) constant the absolute value of $\rho^A - r$ will increase in the age spread (defined in the usual way as second order dominance). This concludes the proof of theorem 3.



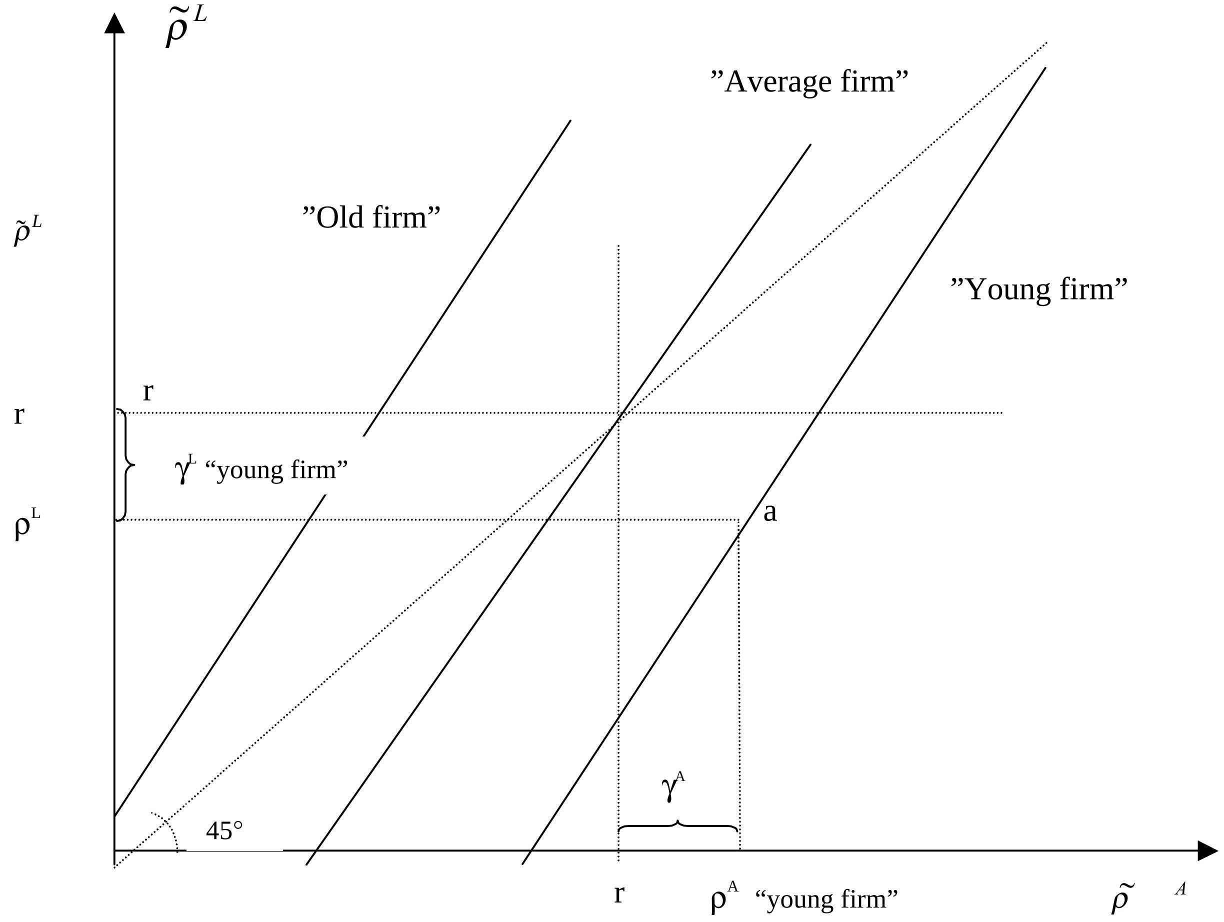
The figure shows the expected accounting rate of return using linear depreciation (LINEAR) as a function of project age when expected cash flow is constant over time. Similarly ANNUITY shows the expected accounting rate of return as a function of age when expected cash flow decreases by rK/T per period. IRR is the internal rate of return r. ANNUITY (spread) shows the expected accounting rate of return for a firm with projects of different vintages (see text). Assumed values of parameters: r = .12, T = 25, and K = 1.

Figure1

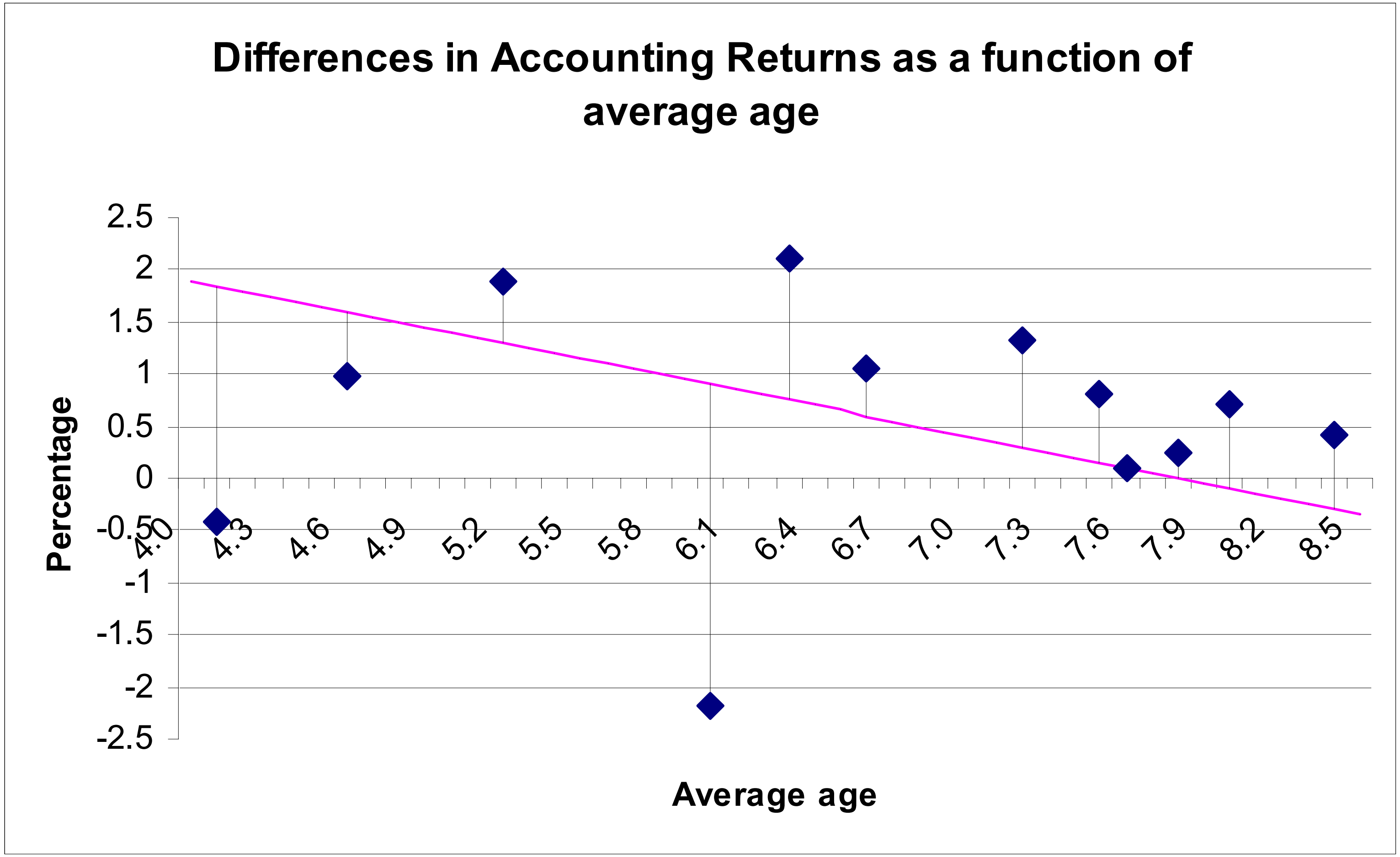


The figure shows the expected accounting rate of return using linear depreciation (LIN) as a function of project age when expected cash flow decreases by rK/T per period.. Similarly ANN(12) and ANN(16) shows the expected accounting rates of return as a function of age when the interest rates are .12 and .16 respectively under the same cash flow assumptions. Assumed values of parameters: r = .12, T = 25, and K = 1.

Figure 2



The figure displays the actual accounting rate of return using linear depreciation – $\tilde{\rho}^L$ – as a function of the actual accounting rate of return using annuity depreciation – $\tilde{\rho}^A$ – for a "young firm", an "old firm" and an "average firm". For a young (fast growing) firm $\tilde{\rho}^A$ has a positive bias γ^A whereas $\tilde{\rho}^L$ has a negative bias γ^L. For an old (mature) firm the biases are reversed (not shown in the figure).



The figure plots the differences between accounting rates of return using annuity and linear depreciation respectively as a function of the average age of the case company's fleet of vessels. The line represents the linear regression

$$\rho^A - \rho^L = .1527 - .243(\rho^A - 11.5) - .497(AGE - 7.5)$$

assuming $\rho^A = 11.5$

Figure 4