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# Estimation of biological and economic parameters of a bioeconomic fisheries model using dynamical data assimilation

**by**

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# Estimation of biological and economic parameters of a bioeconomic fisheries model using dynamical data assimilation

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**Abstract:** A new approach of model parameter estimation is used with simulated measurements to recover both biological and economic input parameters of a natural resource model. The data assimilation technique is the variational adjoint method (VAM) for parameter estimation. It efficiently combines time series of artificial data with a simple bioeconomic fisheries model to optimally estimate the model parameters. Using identical twin experiments, it is shown that the parameters of the model can be retrieved. The procedure provides an efficient way of calculating poorly known model parameters by fitting model results to simulated data. In separate experiments with exact and noisy data, we have demonstrated that the VAM can be an efficient method of analyzing bioeconomic data.

# Introduction

In this paper we use a simple bioeconomic fisheries model to introduce and demonstrate the potential usefulness of a new technique of optimally estimating the parameters of a dynamic fisheries model. The approach is known as data assimilation. In data assimilation mathematical or numerical models are combined with available measurements in order to improve the model itself or to improve the model forecasts. The former application is known as model fitting (Smedstad & O'Brien 1991). A variety of these techniques such as the Kalman filter (Kalman 1960), Optimal Interpolation and the Variational Data Assimilation (Smedstad & O'Brien 1991) are already in common use. These techniques have extensively been used in areas such as groundwater hydrology and petroleum reservoirs (see Carrera & Neuman 1986) and more recently in ecosystem models (Matear 1995). The assimilation technique in this paper is the so-called variational adjoint method (VAM). This method minimizes a preconstructed loss function which is defined by the differences between the data and the model forecasts. The optimal or best fit parameters are obtained by minimizing the cost or loss function subject to the dynamical constraints via the so called adjoint equations which map the predefined loss function into the gradient with respect to the control parameters (Lawson et al. 1995). The gradients are then used iteratively in a descent or Newton type of algorithm in order to search for the minimum of the loss function.

Application of these techniques is spreading very rapidly to many areas such as physical and biological systems (Spitz et al. 1997). Lawson et al. (1995) applied the technique of data assimilation to the well-known and extensively studied predator-prey model in biology. Other techniques of data assimilation are the simulated annealing (Kirkpatrick et al. 1983) and the hybrid Monte Carlo (Harmon & Challenor 1996). These are stochastic in nature and may be very costly to implement.

Much research in bioeconomics of fish stocks has been based on simple aggregated biological models (Clark 1990). Classical growth models such as the logistic and the exponential functions are the commonly employed models in the qualitative analysis of fish stocks. The reality of these models has not yet been rigorously tested against measurements. Data assimilation provides the opportunity to test these models using the available data which are assumed to be more realistic than the models themselves. More realistic bioeconomic models e.g. models of interspecific competition may be multidimensional and complex. They may contain many parameters such as the intrinsic growth rate, the environmental carrying capacity, etc, whose values are extremely difficult to measure. Parameterization of the models becomes mathematically untractable and impractical. As a consequence, biologists and bioeconomists have found it necessary to introduce simpler models. The issue of identification, i.e., the problem of estimating parameters so that the model predictions are more realistic and useful, has been raised by many natural resource economists (Clark 1990). However, adequate attention has not been paid to the problem. Instead, economists have focused on the analytical considerations leading to a neglect of most of the vital questions resource managers/fishers are mostly concerned about, e.g., what is the safe biomass level (Deacon et al. 1998), etc. In this paper, we will use the VAM in an attempt to show how some of these empirical questions may be addressed.

Due to the progress in data collection and processing in recent times from both fisheries biologists and economists and the advances in computer technology, techniques of data assimilation which are both data and computer intensive have good future prospects. The assimilation technique (VAM) introduced in this paper has some advantages compared to the conventional regression analysis. First it provides a more accurate and efficient way of calculating the gradients of the loss function compared to the finite difference approach. Second, it can be used for more realistic and complex dynamical models of bioeconomic systems. Third, it can be used to adjust both the initial conditions and the parameters of the model. Conventional regression methods do not provide the opportunity for estimating initial and boundary conditions. Notice that the VAM is a more general formulation than the traditional regression. It can be used for both linear and nonlinear models and is highly suitable for more realistic models where closed form solutions are unattainable. The method can be used to simultaneously estimate a large number of parameters.

Primarily, the focus of the paper is to demonstrate the applicability of the method in natural resource economics. Hence the use of the simulated data to show the basic steps required to implement the VAM. The model used is also relatively simple but sufficient for this application.

The plan of this paper is as follows. First, the VAM will be presented and the solution algorithm outlined. Second, the bioeconomic fisheries model will also be presented. Third, identical twin experiments will be performed and the results discussed. Finally, we discuss and summarize the work. The mathematical details are relegated to the Appendix.

## The loss function

In adjoint parameter estimation a loss function which measures the difference between the data and the model equivalent of the data is minimized by tuning the control variables (parameters) of the dynamical system. The goal is to find the parameters of the model that lead to model predictions that are as close as possible to the data. A typical loss function takes the form

$$J[X,\alpha]=\frac{1}{2}\int_0^{T_f}(X-\hat{X})^T(X-\hat{X})\,dt \qquad (1)$$

where $\hat{X}$ is the observation vector, $X$ is the model equivalent of the data. The period of assimilation is denoted by $T_f$ and $T$ is the transpose operator. The criterion function is a continuous form of the familiar least squares. This formulation is consistent with the continuous model dynamics. However, in practice, data are often discrete and finite in time thus, the discrete equivalent of the loss function is usually used.

## The adjoint method

Consider the forward model equation

$$\begin{aligned}\frac{\partial X}{\partial t}&=F(X,\alpha)\\ X(0)&=X_0\end{aligned} \qquad (2)$$

where X is a scalar or vector of model variables, $F$ is a linear or nonlinear operator, and $X_0$ is the vector of initial conditions. Minimization of (1) subject to (2) is a constraint optimization.

Formulating the Lagrange function by appending the model dynamics as a strong constraint

$$L[X,\alpha] = J + \frac{1}{2}\int_0^{T_f} M\left(\frac{\partial X}{\partial t} - F(X,\alpha)\right)dt \tag{3}$$

where M is a vector of Lagrange multipliers which are computed in determining the best fit. The original constrained problem is thus reformulated as an unconstrained problem. At the unconstrained minimum the first order conditions are

$$\frac{\partial L}{\partial X} = 0 \tag{4}$$

$$\frac{\partial L}{\partial M} = 0 \tag{5}$$

$$\frac{\partial L}{\partial \alpha} = 0. \tag{6}$$

It is observed that equation (4) results in the adjoint or backward model, equation (5) recovers the model equations while (6) gives the gradients with respect to the control variables. Using calculus of variations or optimal control theory the adjoint equation is

$$\begin{aligned} &\frac{\partial M}{\partial t} + \left[\frac{\partial F}{\partial X}\right]^T M = \left(X - \hat{X}\right) \\ &M(T_f) = 0 \end{aligned} \tag{7}$$

and the gradient relation is

$$\Delta_{\alpha J} = -\int_0^{T_f} \left[ \frac{\partial F}{\partial \alpha} \right]^T M \, dt \tag{8}$$

The term on the RHS of the first equation in (7) is the misfit that acts as forcing term for the adjoint equations. It is worth noting here that we have implicitly assumed that data are continuously available throughout the integration interval. Equations (2) and (7) above constitute the Euler-Lagrange (E-L) system and form a two-point boundary value problem.

Implementation of the adjoint technique on a computer is straightforward. The algorithm is outlined below.

- Choose the first guess for the control parameters.
- Integrate the forward model over the assimilation interval.
- Calculate the misfits and hence the loss function.
- Integrate the adjoint equation backward in time and calculate the gradients.
- Find the minimum of the loss function using an iterative procedure.

The optimization step is performed using standard optimization procedures. In this paper, a limited memory quasi-Newton procedure (Gilbert & Lemarechal 1991) is used. The success of the optimization depends crucially on the accuracy of the computed gradients. Any errors introduced while calculating the gradients can be detrimental and the results misleading. To avoid this incidence from occurring, it is always advisable to verify the correctness of the gradients (see, Smedstad & O'Brien 1991). Verification of the adjoint code is performed by a simple Taylor series expansion of the loss function about a certain vector of the control variables $x$, i.e.,

$$J(x+\gamma u) = J(x) + \gamma u^T \Delta_x J(x) + O(\gamma^2)\,. \qquad (9)$$

Where γ is a small scalar, $u$ is an arbitrary vector. Defining a function ϕ(γ) and rewriting (9) we have

$$\Phi(\gamma) = \frac{J(x+\gamma u) - J(x)}{\gamma u^T \Delta_x J(x)} = 1 + O(\gamma)\,. \qquad (10)$$

Hence, in the limit as γ approaches zero, the value of ϕ(γ) approaches unity. If the scalar γ is small, the values of the function ϕ(γ) will be approximately equal to one.

## The bioeconomic model

This section presents the bioeconomic fisheries model. It is an aggregated or lumped parameter model of a single stock i.e. the biomass model (Clark 1990). The population dynamics of the fishery will be modeled as

$$\frac{dx}{dt} = f(x) - h \qquad (11)$$

where $h(t)$ is the harvest rate from the stock, $x(t)$ is the stock biomass, $f(x)$ is the growth function which for this analysis is the logistic model. That is $f(x) = rx(1-x/K)$ where $r$ and $K$ are positive constants called the intrinsic growth rate and the environmental carrying capacity respectively. The biological model specified above depends on two parameters *(r, K)*. These parameters are little known in the scientific community for most fish stocks around the world. Accurate measurements of their values are difficult if not impossible, thus statistical estimation methods were devised by Schaefer (1964) to find estimates of the parameters. This paper has similar goal of devising mathematical methods of estimating the parameters of the model dynamics (differential equations) using artificial data.

The economics is also simple in this analysis. A hypothetical sole owner is envisaged. The goal of the management is to maximize a given discounted profit or utility

$$max\int_0^{T_f} e^{-\delta t}\pi(h)\,dt \qquad (12)$$

where $h(t)$ is the control variable, $\delta$ is the discount rate, $T_f$ is the time horizon which may be finite or infinite and $\pi(h)$ is a rescaled economic function given by

$$\pi = ah - h^2 \qquad (13)$$

where $a$ is an economic parameter to be estimated. The objective function is assumed to depend explicitly only on the harvest rate from the stock. Note that, $\pi$ can be the net revenue or utility of consumption and it is rescaled to reduce the number of parameters in the model.

Maximizing the above problem (12) subject to the natural constraint is a nonlinear optimal control problem (Clark 1970).

Application of maximum principles to the above problem yields the following system of coupled nonlinear ordinary differential equations [ODEs] (see Clark 1990; Appendix).

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - h \tag{14}$$

$$\frac{dh}{dt} = -0.5(a - 2h)\left(\delta - r\left(1 - \frac{2x}{K}\right)\right) \tag{15}$$

Analytical solutions of the system of nonlinear ODEs are in general unobtainable. However, approximate numerical solutions of these equations are easy to obtain if the necessary initial and boundary conditions are specified. In reality however, the initial conditions that lead to the separatrix solution are not precisely known a priori. Clark (1990; p.98, 184) has offered a more detailed analysis of this model. Given a set of data, a solution could be found by either manually or automatically tuning the poorly known initial and/or boundary conditions as well as the free parameters of the model until a satisfactory solution is obtained. This could easily and efficiently be achieved by the use of the data assimilation technique introduced in this paper.

Identical twin experiments will be used throughout the analysis. That is, data will be generated from the model itself using Monte Carlo simulations. This guarantees that the model and the data are consistent and serves as a good test of the adjoint algorithm. Note that real data are not used here because; the underlying assumption of optimality is inconsistent with most available data. A moderate objective of the paper is to demonstrate the usefulness of this approach in resource modeling, hence the use of this simple model which is fairly well known and much used in bioeconomic fisheries analysis. To carry out the experiments, reasonable parameter values will be chosen for the model and the VAM will be used to recover the parameters.

*Equilibrium analysis*

The equilibrium behavior of the simple fishery model in this paper is studied. Note that in spite of the simplicity of the model it has many interesting real life implications. For an infinite horizon problem, the optimal path for the dynamics is the separatrix leading to the equilibrium point. At the optimal equilibrium, the following conditions hold: $dx/dt = 0$ and $dh/dt = 0$ which implies that $rx^*(1 - x^*/K) - h^* = 0$ and $-0.5(a - 2h^*)(\delta - r(1 - 2x^*/K)) = 0$. This gives rise to these equilibrium points: $(h^* = a/2,\ h^* = rx^*(1 - x^*/K))$ and $(\delta = r(1 - 2x^*/K),\ h^* = rx^*(1 - x^*/K))$. Notice that the first equilibrium is a bliss point (i.e., point at which the benefits are maximum) and most rewarding from the economic viewpoint if achievable. It is the theoretical optimum for the quadratic benefit function defined above i.e. $(d\pi/dh = 0)$.

- Case 1. For the case of the bliss point the optimal equilibrium harvest and stock are $h^* = a/2$ and $x^* = \dfrac{r \pm \sqrt{r^2 - \dfrac{2ar}{K}}}{2r/K}$, where the term under the square root sign is restricted to be nonnegative. Many interesting observations are made here. Notice here that when the term under the square root sign is zero, we recover the maximum sustainable yield (MSY) optimum $x^* = K/2$. However, if the square root term is small compared to $2r$ two different values exist for the equilibrium stock and the larger may be preferred (Clark 1990 showed that this equilibrium is stable). A larger standing equilibrium stock leads to higher sustained yield for the capital asset.

- Case 2. This case leads to the equilibrium biomass $x^* = K(1 - \delta/r)/2$. The term $\delta/r$, i.e., the ratio of the discount factor to the growth rate, is what is referred to as the bionomic growth ratio (Clark 1990). Setting $\delta = 0$ yields the MSY policy as usual. The optimal equilibrium harvest $h^*$ is obtained by simply substituting the $x^*$ into the equilibrium stock-harvest relation above. The second case seems to be embedded in case one if $\delta = \sqrt{r^2 - 2ar / K}$. It is important to notice the link between $r$ and $\delta$.

The reader would have noticed that in case 1, the equilibria arise because the marginal revenue or utility $a - 2h = 0$ is zero "economic optimality". This yields two equilibrium Biomass levels both of which are shown to be stable (Clark 1990, p184). In the second case, the equilibrium is because, $r(1 - 2x/K) = 0$. This can be seen as a biological equilibrium because that is where the growth is maximum.

## The twin experiments

To test the performance of the data assimilation method, artificial data are generated from the model itself using known values of the parameters and initial conditions. This will avoid any inconsistencies between the data and the model. The data are generated by $\hat{x}_n = x_n(1 + \varepsilon_n)$ where $\varepsilon_n$ ~ $U(b,c)$ and $U$ denotes uniform. That is, the model is solved and a given percentage of random noise is added to the model predictions. Note that the noise is assumed to be measurement errors not process errors. The goal of the experiments is to show that for a reasonable amount of noise, the VAM is capable of retrieving independent model parameters well. To illustrate the technique we use the following hypothetical parameter values: $r$ = 0.35 per unit time and $K$ = 6000 tons. From which the value of $a$ is deduced. This is done by using the example in case 1 with the term under the square root equal to zero. This yields: $x^* = K/2$ = 3000 tons, $h^* = rK/4$ = 525 tons and $a = 2h^*$ = 1050 tons. The initial conditions are $x(0)$ = 1200 tons and $h(0)$ = 120 tons.

Several experiments were performed using clean and noisy data. For the noisy data, 100 simulations were conducted by resampling from the same distribution and the means of the recovered parameters are then calculated. The results are shown in Table 1 for different levels of noise. The noise level was increased in steps of 5 % to a maximum of 20 %. Estimates of the parameters and their bias are provided to help readers to easily judge the performance of the algorithm. The bias is defined as the difference between the true parameter value and the estimated value. The bias is quite insignificant compared to the noise in the data. It in general increases with increasing level of noise but the increase is less than proportionate. The parameters of the model dynamics are recovered reasonably well as shown in Table 1. The real test of any assimilation scheme is when it is applied to real data. We do not use real data because most real world fisheries have traditionally not been optimally managed and thus the available data may be inconsistent with the model developed in this paper. Note that different initial guesses for the parameters are used to check for convergence to absolute minimum. The results are not very sensitive to the initial guesses of the parameters.

## Summary and conclusion

This work to our best knowledge represents the first attempt to explore the highly advanced and attractive method of parameter estimation in resource economics. The basic idea is to find parameters of a dynamic fisheries model which yield model results that are as close as possible to the observed quantities.

In a few experiments we have demonstrated the utility of the adjoint technique in recovering model parameters such as the growth rate of a bioeconomic model. These parameters are vital in understanding the dynamics of the exploited species which can lead to a more accurate and realistic management policies. Estimates of the bias show that for a reasonable amount of noise, the recovered values are not too far away from the true values. To conclude, the paper has accomplished the following. A novel approach to dynamic model parameter estimation has been introduced with reasonable degree of success using a simple bioeconomic model. Its potential in handling nonlinear models is also demonstrated and thus it is important to explore more of the capabilities of the method in future.

# Appendix

## *Derivation of the bioeconomic model*

Consider the problem

$$max\int_0^{T_f} \pi(h)\,dt \tag{A.1}$$

subject to

$$\frac{dx}{dt} = f(x) - h \tag{A.2}$$

$$x(0) = x_0 \tag{A.3}$$

Defining the current value Hamiltonian function *H(x,h,m,t)*

$$H = \pi(h) + m(f - h) \tag{A.4}$$

where *m(t)* is the current value multiplier. Assuming an inner solution, the first order conditions are:

$$\frac{dH}{dh} = \pi_h - m = 0 \tag{A.5}$$

$$\frac{dm}{dt} = \delta\ m - H_x \tag{A.6}$$

From (A.5) we have

$$\frac{dm}{dt} = \pi_{hh}\dot{h} + \pi_{hx}\dot{x}\,. \tag{A.7}$$

Equating (A6) and (A7), and rearranging yields

$$\frac{dh}{dt} = \frac{(\delta - f_x)\pi_h}{\pi_{hh}} \tag{A.8}$$

Equations (A.2) and (A.8) constitute a 2-dimensional dynamical system.

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Table 1: Results of assimilation experiments for different level of noise. The true parameters are: $r$ = 0.35 per unit time, $K$ = 6000 tons and $a$ = 1050 tons. The criterion for convergence is $J / J_0 = 10^{-8}$ where $J$ is the initial value of the cost function.

| Parameters | Percent Noise (%) | | | | |
|---|---|---|---|---|---|
| | 0 | 5 | 10 | 15 | 20 |
| $\bar{r}$ | 0.3500 | 0.3505 | 0.3491 | 0.3545 | 0.3554 |
| $\bar{K}$ | 6000 | 5998 | 6114 | 6142 | 6228 |
| $\bar{a}$ | 1050 | 1053 | 1063 | 1077 | 1089 |
| $\lVert r - \bar{r} \rVert$ | 0.0000 | 0.0005 | 0.0009 | 0.0013 | 0.0019 |
| $\lVert K - \bar{K} \rVert$ | 0.0000 | 2.0000 | 114.00 | 142.00 | 228.00 |
| $\lVert a - \bar{a} \rVert$ | 0.0000 | 3.0000 | 13.000 | 27.000 | 39.000 |

Estimated model parameters and their bias. $\lVert . \rVert$ is the absolute value of the argument and bar is the estimated value. Number of simulated observations used is 100.