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Dynamic Cournot-Competitive Harvesting of a Common Pool Resource

by

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Abstract

A variant of the classical harvesting game, where a number of competing agents simultaneously harvest a common-pool natural resource, is analysed. The harvesting at each time is a Cournot competition. The critical assumption made in the present analysis is that most (or all) individual participants maintain a perspective which is *wholly myopic.* Three cases are analysed: 1) The case of aggressive myopic Cournot competition, 2) co-operation and 3) competition when there is an incumbent player. The development in the number of active participants is outlined, and stability criteria for a dynamic Cournot-competitive game are given. The exact deadweight loss due to lack of co-operation is calculated.

Key words: Dynamic game-theory, Cournot competition, Natural resource exploitation. JEL: C72, Q22.

INTRODUCTION[1]

This paper analyses a variant of the classical fisheries harvesting game, where a number of competing agents (fishing nations, fleets or vessels) simultaneously harvest a common-pool fish stock, and do so over an extended period of time. The harvesting at each time is a Cournot competition. That is to say, each nation, in choosing its current harvest rate, takes into account that the current harvest-landings price depends on the total harvest at the given time. Consequently landings price depends on the simultaneous actions of all the nations. The focus is on the harvesting sector in each nation.

Each nation will also take into account its current cost of harvest, which depends on the current fish stock biomass. However, the future stock biomass depends upon the prior history of harvesting, and this circumstance links the evolution of future harvest costs to current harvest rates.

The critical assumption made in the present analysis is that most (or all) individual participants in the fishery maintain a perspective which is *wholly myopic*. That is, in choosing its current harvest rate the myopic agent does *not* take into account this impact of the current harvest on the future biomass trajectory[2]. Such myopic behavior may profoundly affect the long run evolution of the harvesting game. Myopic fisheries are also studied by Hämäläinen et al. (1986 and 1990) among others.

We believe that this circumstance of myopic decision making often characterises traditional artesanal fisheries, especially in developing countries. Very often there will be no entity with the resources (or even the desire) to attempt a serious quantifica-

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[2]A relevant term for such agents in a continuous time model may be *stroboscopic* players. This would be agents who control the system completely during very short time intervals.

tion of the relation between harvesting intensity and subsequent fish-stock dynamics. If this relation is indeed significant, but is ignored, then the ongoing fishery may be seriously sub-optimal. Ignoring the specific biological characteristics of stock demography can have profound implications for the evolution of the fishery, affecting evolving stock biomass and harvesting fleet structure, determining which agents will remain viable participants in the ongoing fishery and which will be eliminated in the continuing competition. An important result in the paper is that we calculate the exact deadweight loss due to competition compared to co-operation.

In our study we examine an idealized fishery model, where the fish stock dynamics evolve according to a standard surplus production model, as conditioned by total harvest. Three cases in particular will be analysed: 1) The case of aggressive myopic Cournot competition, where all agents lack, or choose to ignore, information about the dynamic stock demographics. This is typically the case in many fisheries in developing countries or for distant water fishing nations. 2) Co-operation where the aim is to maximize the total net revenue from the fishery. 3) Competition where there is an incumbent agent, for example a nation that is dedicated to keep the fishery sustainable and therefore takes account of the stock biomass growth function while the others still do not; for example one coastal state versus a number of distant water fishing nations. Another example may be a stock that is shared between two nations where one nation wants to take stock dynamics into account and do dynamic optimisation whereas the other does not. A case in point may be Namibia and Angola; or even Norway and Russia.

In each case all participants have full information about the market demand function, which determines the unit market price as a function of the total harvest. Each participant also knows the current fish biomass level and the unit cost of harvesting, as a function of biomass, for all of the other agents as well.

As emphasized, an important feature of the model used here is that the *ex ves-*

sel landings price at any given time is a function of the combined harvest rate of all participants. This circumstance, which introduces technical complications (due to non-linearities in the objective functions) is treated only lightly in the fisheries harvesting literature, although it is obviously an important aspect of real world fisheries. To our knowledge its previous considerations for a harvesting game is that of Takayama and Simaan (1984), Reinganum and Stokey (1985), Dockner et al. (1989) and Datta and Mirman (1999). Takayama and Simaan (1984) analyse a dynamic game involving two countries and a single resource. Reinganum and Stokey (1985) look at a non-cooperative dynamic game involving n firms who are engaged in the extraction of a *nonrenewable* common property resource. Dockner et al. (1989) analysed a duopoly situation with complete information, and Datta and Mirman (1999) use discrete dynamics to analyse the effect of biological externalities when there are many countries harvesting two fish stocks.

The outline of the present paper is as follows. The general model is presented in the next section. Then we look at the two cases i) myopic competition, ii) co-operation and iii) the case where one country is dedicated to maintain sustainability. Several numerical examples are given. Finally the paper is concluded by a summary section.

GENERAL MODEL

Assume that all agents are selling their harvest on the same market but they differ with respect to their efficiency through the cost function. Let x denote the common fish stock and h_k denote agent k's harvest. The total harvest of all agents is

$$h = \sum_{k=1}^{n} h_k$$

where n is the number of *active* agents[3], that is, by definition, agents with positive harvest. It is assumed that there are no capacity constraints on individual agents.[4] This is because emphasis is put on the effects of competition, and capacity constraints will reduce these effects.

For simplicity, assume a linear market demand function which can be written on inverse form as

$$p(h) = Q - qh.$$

where $p(\cdot)$ is the market price, and Q and q are market parameters. Each agent has an individual cost function given as

$$\gamma_k(h_k, x) = c_k(x)h_k$$

where x is the current size of the common pool resource. This implies that agent k's net revenue, or utility function, in current value is given by

$$\pi_k = (Q - qh)h_k - c_k(x)h_k$$

which can be rewritten

$$\pi_k = [p_k(x) - qh]\, h_k, \quad h_k \geq 0, \tag{1}$$

where $p_k(x) = Q - c_k(x)$ reflects this agent's efficiency.[5] The number of potential agents is defined by the following:

Definition 1 *The number of potential agents $M = M(x) < \infty$ at some stock level x is defined as all agents with $p_k > 0$.*

[3]In the following n is always used to denote the number of *active* agents.

[4]This assumption will be relaxed later.

[5]The results in this paper hold for a general x-dependence in Q and q.

The dynamics of the common stock is given by

$$\frac{dx}{dt} = f(x) - h \tag{2}$$

where $f(x)$ is the biological growth function. A non-trivial steady state exists if $h = f(x)$ for $x > 0$. Criteria for stability of the steady state, if it exists, will be investigated later.

For a myopic agent, k, the objective is simply to maximize π_k at any point in time given the prevailing stock x. In the case of an incumbent agent her objective is to maximize[6]

$$\int_0^\infty e^{-\delta_k t}\pi_k dt$$

where δ_k denotes agent k's discount rate and t denotes time. Notice that different agents may have different discount rates which may be quite realistic when the agents are countries with different economic conditions.

MYOPIC COMPETITION

In this section it is assumed that none of the agents use any of the information they may have about the biological growth function. In other words, they do not perform any dynamic optimisation due to the common pool characteristic of the fishery. The reason for such myopic behaviour may be fourfold. First, as long as there is more than one agent, there is always an element of "the tragedy of the commons". The rationale is: whatever I do not harvest may be harvested by others and therefore I do not have any incentives to save fish for tomorrow. Under such circumstances no one have incentives to take the population dynamics of the stock into account. Second, the agents may simply not have, or do not believe in, information about population

[6] Functional dependence of the variables is often skipped to improve the visual appearance of the equations.

dynamics. The biology of many fish stocks around the world is poorly understood and many have not even been investigated. Even in the cases of fish stocks that have been thoroughly studied, biologists from time to time admit that their models where wrong and the stock dynamics has to be reconsidered from the beginning. Under such circumstances it is no wonder that the possible participants in the fishery do not believe strongly in the biological models. Third, it is widely recognised that commercial fishermen often operate with short time horizons when they make their decisions. This has been studied by Harms and Sylvia (2001). Fourth, with distant water fleets (DWF) roaming the oceans seeking targets-of-opportunity just to reap off any profits that might exist without interest in long-term conservation of the stock, these will have no incentives to take the population dynamics into account.

The agents are ranked according to their economic efficiencies:

$$p_1(x) \geq p_2(x) \geq ... \geq p_M(x),$$

and it is assumed, for simplicity, that the ranking remains constant at various stock levels. Each agent will choose $h_k = 0$ whenever $p_k - qh < 0$. As the agents do not care about the dynamics, agent k chooses the most myopic decision rule which is to maximize his π_k for all possible values of h_{-k}, where h_{-k} is defined as the other agents' harvest by $h_{-k} \equiv h - h_k$.

Definition 2 *The general notation* $L_{-k} = \sum_{j \neq k}^{n} L_j = n \langle L \rangle - L_k$ *is used, where* $\langle . \rangle$ *denotes arithmetic mean. Further,* $\langle L \rangle_{-k} = \frac{L_{-k}}{n-1}$ *is defined as the average of all the other agents except* k.

In order to maximize π_k agent k will solve

$$\frac{\partial \pi_k}{\partial h_k} = p_k - qh - qh_k = 0 \tag{3}$$

which implies

$$\frac{p_k}{q} = h + h_k, \; k = 1..n, \tag{4}$$

without caring about the dynamics or the future. Expression (4) represents a system of n equations and n unknowns, namely the h_ks. By summing over all active agents total harvest can be found:

$$h = \frac{n \langle p \rangle}{(n+1)q}. \tag{5}$$

By substituting this into (4) again the harvest of one active agent appears as:

$$h_k = \frac{np_k - p_{-k}}{(n+1)q}, \tag{6}$$

and the profit of one active agent is (from (4)):

$$\pi_k = qh_k^2.$$

This is the Cournot-solution for the case where all agents have information about each others efficiencies and about the market, but they do not take the biological growth function into account.

The number of active agents can now be found by the following:

Proposition 1 (Activity Principle) *The number of active agents n is the maximal integer value of $n \leq M$ satisfying $np_n > p_{-n}$ where M is the number of potential agents. For all active agents $p_k > \frac{n-1}{n} \langle p \rangle_{-k}$ must apply.*

The proof of Proposition 1 is straightforward. It is a direct consequence of the definition of active, (5) and (6). When applying this proposition in practice we start with the least efficient of the potential agents and work our way down until the criterion is satisfied.

Proposition 1 has some quite interesting implications. Note, e.g., that if there are only two potential agents, $\frac{n-1}{n} = \frac{1}{2}$, and both agents are active unless one of the agents is at least twice as efficient as the other one. With three agents, ranked according to efficiency, Agent 3 is active only if $p_3 > (p_1 + p_2)/3$, and Agent 2 is active only if $p_2 > (p_1 + p_3)/3$. As n approaches infinity, $\frac{n-1}{n}$ approaches one from below. In other words, no matter how many agents there are, a sufficient criterion to be active is that you are as efficient as the average of the other active agents. This may perhaps not be too surprising, but with few agents you can be much less efficient than that, and with only two agents, it is sufficient to be more than half as efficient as the other agent. Note that the number of active agents varies with the stock size, x, as the parameters p_k are functions of x. In other words, the number of active agents is a function of the current stock size, $n(x)$, at any point in time.

Proposition 1 implies that no active agent can be too far away from the average of the other active agents. The following corollary derived from Proposition 1 will be used later.

Corollary 1 *No active agent is more than twice as efficient as any of the other active agents.*

Proof: If such a agent exists, then Agent 1 must be more than twice as efficient as Agent n. From Proposition 1

$$np_n > p_{-n} = p_1 + p_2 + \cdots + p_{n-1} \geq p_1 + (n-2)p_n \Rightarrow 2p_n > p_1 \blacksquare$$

In other words, Agent 1 (the most efficient) can not be twice as efficient as Agent n (the least efficient). This yields boundaries on the efficiency of one active agent relative to the average of the other active agents. The boundaries are given by the next corollary:

Corollary 2 *The efficiency of any of the active agents relative to the average efficiency of the other active agents is bounded by*

$$1 - \frac{1}{n} < y_k \equiv \frac{p_k}{\langle p \rangle_{-k}} < 2.$$

This corollary follows directly from Proposition 1. In the following y_k will be referred to as k's relative efficiency. The net revenue of one active agent is given by

$$\pi_k = qh_k^2 = q\left(\frac{np_k - p_{-k}}{(n+1)q}\right)^2 = \frac{1}{q}\left[p_k - \frac{n}{n+1}\langle p\rangle\right]^2.$$

By summing over all active agents we get the total net revenue from the fishery:

$$\pi = q\sum_n h_k^2 = \frac{n}{q}\left[\langle p^2\rangle - \frac{(n+2)}{(n+1)^2}n\cdot\langle p\rangle^2\right]. \tag{7}$$

Dynamic aspect

Although none of the agents does any dynamic optimisation, the development of the resource is nevertheless governed by equation (2). Using (5) this can be rewritten

$$\frac{dx}{dt} = F(x) \equiv f(x) - \frac{n\cdot\langle p\rangle}{q(n+1)}. \tag{8}$$

A steady state, x^*, is characterised by $h(x^*) = f(x^*)$. The stability of the steady state depends upon the number of active agents and the biological and economic parameters. This is summarised in the following proposition.

Proposition 2 (Local stability) *A steady state, x^*, is locally asymptotic stable if*

$$\frac{f'}{f} - \frac{\langle p\rangle'}{\langle p\rangle} + \frac{\left[\left(1+\frac{1}{n}\right)q\right]'}{\left(1+\frac{1}{n}\right)q} < 0. \tag{9}$$

Proof: This proposition follows from linear stability, that is $F(x^*) = 0$ and $F'(x^*) < 0$.■

As the agents activities are based purely on economic criteria, the existence of a steady state is not guaranteed. From (8) it is seen that a steady state exists whenever the biological surplus production equals total myopic harvest. A steady state is guaranteed if Agent 1's cost structure is such that costs approach infinity when the stock biomass approaches zero; typical for trawl fisheries. If n is constant, as it usually is close to a steady state, this reduces to $\frac{f'}{f} < \frac{\langle p \rangle'}{\langle p \rangle}$, see (9). The interpretation of this is that the relative change in average efficiency must be greater than the relative change in surplus production in a stable steady state.

The numerical example below is meant to illustrate the development of the stock over time and how the number of active agents depends upon the stock when the efficiency parameters are highly sensitive to the stock size. This is typically the case in search (bottom trawl) fisheries.

Example 1 *In this example costs depend strongly upon the stock. The agents are characterised by* $p_k = 3 - k\frac{0.05}{x^2}$. *The slope of the demand curve is* $q = 7$, *and the surplus production function is* $f(x) = x(1-x)$. *The development of the stock and the number of active agents are depicted in Figures 1 and 2.*

Monopoly

In this section we compare the competitive situation described above with the case where one arbitrary active agent is given monopoly rights in harvesting, that is sole-owner privileges. In particular, it is interesting to compare the net revenue of the total competitive game with the net revenue of one agent with monopoly rights. This agent is denoted by m. The following definitions have been used:

$$\begin{aligned} A_m &\equiv \langle p \rangle_{-m} \qquad B_m^2 \equiv \left\langle p^2 \right\rangle_{-m} = \frac{1}{n-1} \sum_{k \neq m}^{n} p_k^2 \\ C_m^2 &\equiv \frac{B_m^2}{A_m^2} \qquad y_m \equiv \frac{p_m}{A_m} \in \left(1 - \frac{1}{n}, 2\right). \end{aligned}$$

A_m is the arithmetic mean and B_m^2 is the quadratic mean (root mean square) of the other agents' efficiencies. The term y_m is Agent m's efficiency relative to the average

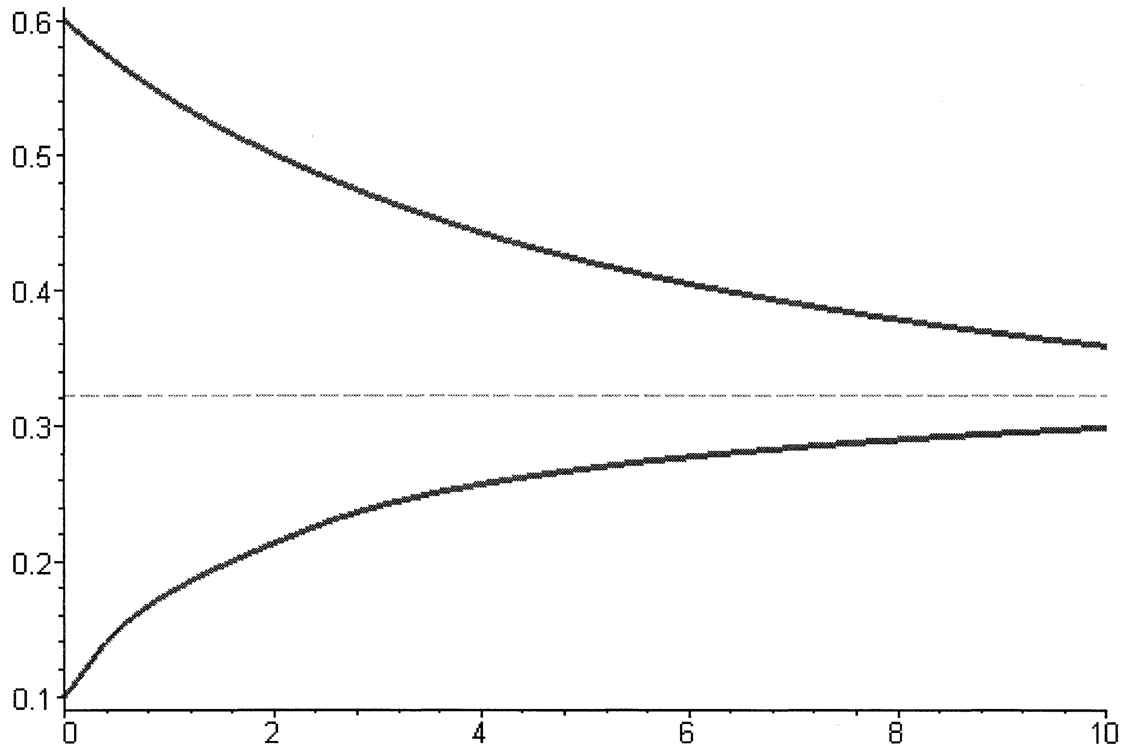

FIG. 1. *Illustration of Example 1. Stock development over time with two different initial stocks (0.6 and 0.1 respectively). Steady state is 0.32.*

of the others, and the boundaries on y_m are derived from Corollary 2. We also use the relationships

$$n\left<p\right> = \left[y_m + n - 1\right] A_m \qquad n\left<p^2\right> = \left[y_m^2 + (n-1)\, C_m^2\right] A_m^2.$$

With this notation the total net revenue from a fishery with n agents can now be rewritten

$$\begin{aligned}\pi &= \frac{A_m^2}{(n+1)^2 q}\left[\left(n^2+n-1\right) y_m^2 - 2\,(n-1)\,(n+2)\, y_m + \right. \\ &\quad \left. (n-1)\left\{(n+1)^2\, C_m^2 - (n-1)\,(n+2)\right\}\right] \qquad (10)\end{aligned}$$

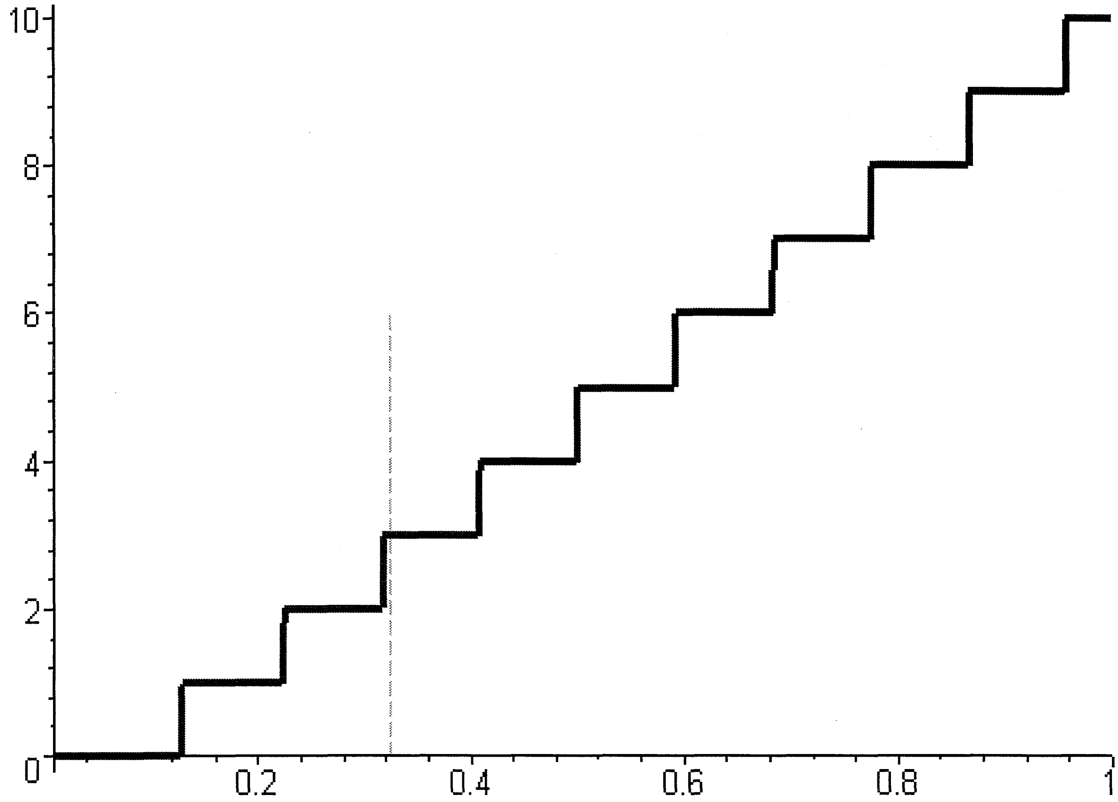

FIG. 2. *Illustration of Example 1. The number of agents as a function of the stock. The steady state stock level if 0.32 and the steady state number of agents is four.*

By comparison, if one of the agents has a monopoly in the resource, her net revenue will be

$$\pi_m = \frac{p_m^2}{4q} = \frac{A_m^2}{4q} \cdot y_m^2. \tag{11}$$

Whether one of the agents can generate more or less profits than the competitive fishery depends on her relative efficiency y_m. It turns out that the number of four active agents becomes important. This can be stated more precisely as follows:

Proposition 3 *For* $n \leq 4$

$$y_k < \frac{2n+6}{3n+5} \equiv \eta$$

is a sufficient criterion for an active agent, k, not to be suited as monopolist, i.e. not to generate more profit alone than the competitive pool can do.

For $n > 4$ this criterion will conflict with the Activity principle ($y_k > \frac{n-1}{n}$). What this proposition says is that the efficiency of an active agent can not be too far below the average of the others if she shall be able to generate more net revenue as a monopolist than the competitive fishery can. The proposition is valid for any $n > 1$, but it is not binding for $n > 4$. This is because the restriction given by the Activity principle is stronger than the restriction above when there are more than four agents. The proof can be sketched as follows: Assuming that (10) is no greater than (11) and using the well-known[7] inequality $C_m^2 \geq 1$, implies

$$\begin{aligned} & (3n^2+2n-5)y_m^2 - 8(n-1)(n+2)y_m + 4(n-1)(n+3) \\ = \quad & (n-1)(3n+5)(y_m-2)(y_m-\eta) \leq 0 \Rightarrow y_m \geq \eta. \end{aligned}$$

In other words, the criterion $y_k < \eta$ is sufficient to state that agent k can not produce more net revenue as a monopolist than the pool. The criterion $y_k < \eta$ is impossible to fulfil for more than 5 agents. From Corollary 2 we have $y_k > 1 - \frac{1}{n}$. A necessary condition for Proposition 3 to be meaningful is that $\eta > 1 - \frac{1}{n}$ which implies $n \leq 4$. The two next examples show that Proposition 3 has non-trivial implications. In these examples the p_ks are independent of x. This is typically the case in schooling fisheries.

Example 2 *Four potential agents are characterised by* $[p_1, p_2, p_3, p_4] = [3, 2, 2, 1]$ *and* $q = 1$. *Only the first three are active as Agent 1 is more than twice as efficient as Agent 4. Agent 3 is active as* $3p_3 = 6 > p_{-3} = 5$. *From (6) we get* $h_1 = \frac{5}{4}$ *and* $h_2 = h_3 = \frac{1}{4}$

[7]See, e.g., proposition 6.12 in Folland (1984).

and $h = \frac{7}{4}$. *From (1)* $\pi_1 = \frac{25}{16}$ *and* $\pi_2 = \pi_3 = \frac{1}{16}$ *and* $\pi = \frac{27}{16}$. *As a monopolist Agent 1 would generate* $\pi_1 = \frac{9}{4}$ *which is more than with competition whereas agents 2 and 3 would generate* $\pi_2 = \pi_3 = 1$ *which is less than with competition. This can be seen directly from Proposition 3.*

Example 3 *Eight potential agents are characterised by* $[p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8] = [1.25, 1.20, 1.15, 1.10, 1.05, 1, 1, 1]$ *and* $q = 1$. *Now* $y = \frac{28}{31} > \eta = \frac{22}{29}$, *and hence no agents can be excluded as unsuited as monopolists. The total profit with competition,* $\pi = 0.185$ *is less than what agent 8, the least efficient one, would generate as a monopolist, which is* $\pi_8 = 0.25$. *From the Activity principle it is found that all eight are active agents.*

It is seen from the last example that the propositions derived so far are not sufficiently strong to cover the case with many active agents. The next proposition, however, which is based on a priori knowledge about the potential agents' efficiencies only, give both necessary and sufficient conditions for when an active agent is suited as monopolist.

Proposition 4 *A necessary and sufficient condition for an active agent to generate more net revenue than the total net revenue generated by the competitive pool (be suited as a monopolist), is*

$$C_m^2 < 1 + \frac{1}{3n+5} \quad \text{and} \quad \underline{y_m} < y_m < \overline{y_m}$$

where

$$\underline{y_m} = \frac{4n+8}{3n+5} - \xi_m, \quad \overline{y_m} = \frac{4n+8}{3n+5} + \xi_m$$

and the real value

$$\xi_m = \frac{2n+2}{3n+5}\sqrt{3n+6-(3n+5)\,C_m^2}.$$

If ξ_m is imaginary, then $C_m^2 > 1 + \frac{1}{3n+5}$, and the agent is unsuited as monopolist. Note that Proposition 4 contains Proposition 3 as a special case as $C_m^2 \geq 1$ no matter how many potential agents there are. For $n > 4$ only Proposition 4 applies.

The proof for Proposition 4 follows directly from equations (10) and (11). The difference between the competitive pool's profit and the monopolist's profit is

$$\Delta \equiv \pi - \pi_m = \frac{(n-1)(3n+5)A_m^2}{4(n+1)^2 q} \left\{ \left(y_m - 4\frac{n+2}{3n+5} \right)^2 - \xi_m^2 \right\}$$

which gives the proposition directly. Proposition 4 is applied in the next example. This example is representative of a fishery with many individual vessels of different size, for example some large purse seiners and many small coastal seiners.

Example 4 *Consider a game with 1101 potential agents whose efficiencies are given by* $p_k = 1.1 - 0.001 \cdot (k-1)$. *From the Activity Principle it is found that there are 46 active agents. From Proposition 4* $\left[\underline{y_m}, \overline{y_m}\right] = [0.69, 1.99]$. *Hence all the 46 active agents are suited as monopolists.*

It is seen from the definitions that ξ_m decreases and $\underline{y_m}$ increases with C_m^2 as long as they are real values. This means that there exists a value of C_m^2 that makes $\underline{y_m} = 1$. If C_m^2 is greater than this value, then either $\underline{y_m} > 1$ or ξ_m takes an imaginary value. In both cases there will exist active agents who are unsuited as monopolists. By solving the equation $\underline{y_m} = 1$, the following corollary is found:

Corollary 3 *If an active agent, m, is less efficient than the average of his competitors and* $C_m^2 \geq 1 + \frac{n-1}{4(n+1)^2}$, *then this agent is unsuited as monopolist.*

The above corollary represents a sufficient condition for unsuitability as monopolist no matter how many agents there are.

It is relatively easy to construct examples that show that it under certain circumstances is possible even for an inactive agent to be suited as a monopolist.

Example 5 *Let there be 8 potential agents. There are 7 equal agents with efficiencies* $p_1 = \cdot\cdot = p_7 = 1$*, and one agent with 20 % lower efficiency,* $p_8 = 0.8$*. Then by the Activity Principle it is seen that this agent is inactive. However, by inserting into the expressions for net revenue it is seen that agent 8 generates more revenue alone (as a monopolist) than the competitive pool together.*

CO-OPERATION

In this section it is assumed that the agents co-operate, but they still act myopically. In the case that the most efficient agent has the capacity to take the total optimal harvest, it will be left to this agent to do so, and the optimisation problem is similar to the sole owner or monopolist referred to above. As Agent 1 is the most efficient agent, the optimal harvest is $h = p_1/2q$ and the total net revenue is $\frac{p_1^2}{4q}$. This is a rather trivial case, and the distribution of the net revenue is the only question that remains to be negotiated.

In the case that the agents do have capacity constraints, they will fill up their capacities according to their efficiency. Let each agent's fixed capacity be denoted $K_1,\ K_2, \ldots$ Further, let us assume that n agents are needed, that is

$$h = \sum_{m=1}^{n} h_m, \quad h_n \leq K_n, \quad n \in \{1, \ldots, M\}.$$

The objective is to maximize

$$\pi = \sum_{m=1}^{n} \pi_m = \sum_{m=1}^{n} (p_m - qh)\, h_m = \sum_{m=1}^{n} p_m h_m - qh^2.$$

The solution to this problem is to let the agents fill up their capacities according to their efficiencies. Total harvest will then equal the sum of the capacities except for the last active agent who may not fill up her capacity. The problem can be rewritten

$$\begin{aligned} \max_{h_1,\ldots,h_n} (\pi), \quad h_m &\in [0, K_m], \quad m \in \{1, \ldots, n\}, \\ h_k &= 0 \Rightarrow h_m = 0,\ m \geq k, \qquad h_n = K_n \Rightarrow h_m = K_m,\ m \leq n. \end{aligned}$$

Let κ_n be used to denote the sum of the n first capacities ($\kappa_0 = 0$). The solution with respect to total harvest and the total number of agents can then be summarized in the following proposition:

Proposition 5 *In the case that the agents co-operate, but still act myopically, the number of active agents, $n \in \{1, 2, 3, ...\}$, is determined by the capacity constraints*

$$\kappa_{n-1} < \frac{p_n}{2q} < \kappa_n, \tag{12}$$

and total harvest is

$$h = \frac{p_n}{2q}.$$

The double inequalities in (12) are mutually exclusive. If no solution exists, either all potential agents are active with full capacity or there are no active agents at all. Proposition 5 follows directly from Kuhn-Tucker's sufficient conditions. A sketch of the derivation of these results is given in appendix.

The deadweight loss due to myopic competition compared with myopic co-operation is defined as the difference in producers' surplus as we are ignoring the consumers' surplus in this paper and concentrate on the harvesting sector. The distribution of the surplus in the case of co-operation is not a topic in this paper.

The relative deadweight loss is defined as $D \equiv 1 - \frac{\pi_{comp}}{\pi_{co-op}}$ where π_{comp} is net revenue with competition and π_{co-op} is net revenue with co-operation. The deadweight loss is highest in the case where capacity constraints are not binding, that is $\pi_{co-op} = \pi_1$ and Agent 1 acts as a monopolist. This yields from (11) and (7)

$$D = 1 - 4n \cdot \left(\frac{\langle p \rangle}{p_1}\right)^2 \left[\frac{\langle p^2 \rangle}{\langle p \rangle^2} + \frac{1}{(n+1)^2} - 1\right] \geq 1 - \frac{n}{(n+1)^2}.$$

The right-hand side of the inequality (lower bound) is derived from noticing that $\frac{\langle p \rangle}{p_1} \geq \frac{1}{2}$ from Corollary 1 and $\frac{\langle p^2 \rangle}{\langle p \rangle^2} \geq 1$ from $C_m^2 \geq 1$, cfr. Footnote 5. An interesting special case is when all active agents are equal, and in this case $D = 1 - \frac{4n}{(n+1)^2}$. The deadweight losses for 2, 3 and 4 active agent are 11 %, 25 % and 36 % respectively.

The deadweight loss referred to above is due to competition only. Capacity constraints in this context is a technological matter that contributes to reduce the deadweight loss.

AN INCUMBENT AGENT

This section looks at the case where one agent, m, is incumbent and performs dynamic optimisation whereas the others do not. In this case Agent m must take strategic considerations into her decisions. Incumbent in this context implies that this agent takes on an obligation to achieve a steady state if possible.

This may, e.g., be the case when there is one home-fleet and several possible "invaders" who also have access to the resource. These invaders may represent distant water fleets (DWF) who roam the oceans seeking targets-of-opportunity with intent to reap the profits wherever it occurs without interest in long-term viability of the stock. As the DWFs come from far away they typically, but not necessarily, have higher costs than the incumbent nation. This may either be higher fixed costs associated with moving the fleet or opportunity costs. It is assumed that all agents know each others efficiencies, and that the DWFs have the same myopic utility functions as in the previous sections. It is further assumed that the incumbent agent, m, is one of the active agents.

The other agents, apart from m, know that m is non-myopic, as this can be observed, and Agent m knows that the others are myopic. Let us assume that there are $n-1$ active myopic agents, and each of them does the same optimisation as earlier given by (3). Adding all the active myopic agents yields

$$n \langle p \rangle - p_m - (n-1)qh - q(h - h_m) = 0 \Leftrightarrow h = \frac{\langle p \rangle}{q} - \frac{p_m}{nq} + \frac{h_m}{n}. \qquad (13)$$

This represents total harvest as a function of the stock, x, for all values of $\frac{h_m}{n}$. The dynamic optimisation problem for the incumbent, non-myopic agent is now

$$\max_{h_m} \int_0^\infty e^{-\delta_m t} \left\{ \left[\left(1 + \frac{1}{n} \right) p_m - \langle p \rangle \right] h_m - \frac{q}{n} h_m^2 \right\} dt$$

subject to

$$\frac{dx}{dt} = f(x) - \frac{\langle p \rangle}{q} + \frac{p_m}{nq} - \frac{h_m}{n}.$$

In the section "Myopic competition" it was shown that total harvest is given by (5) when all behave myopically. If one of the agents behaves non-myopically, total harvest is

$$h = \frac{p_{-m}}{nq} + \frac{h_m}{n}$$

Depending on the efficiencies of the myopic agents, the non-myopic agent may be faced with three different situations:

Case 1: Steady state is not possible because the total harvest of the myopic agents is greater than the surplus production, that is $h_{-m} > f(x)$ for all x where $f(x) > 0$.

Case 2: Steady state is not possible if all, including m, behave myopically. Steady state may, however, be achieved if Agent m reduces her harvest or does not harvest at all.

Case 3: Steady state is achieved even if all, including m, behave myopically.

Case 1 is rather trivial as the stock will inevitably go extinct, and the incumbent agent may just as well play along and mine the resource in an optimal manner. In Case 2 the incumbent agent is obliged to provide a steady state. If her discount rate is low, this may also be the optimal policy, and with zero discounting she will certainly benefit from a situation with a steady state. Case 3 is in many ways the most interesting case. In this case the obligation to provide a steady state is not a binding constraint. Therefore, if the incumbent agent deviates from myopic behaviour, it means that she is benefitting from it. This can be restated as follows:

Corollary 4 *The incumbent agent benefits from non-myopic behaviour if there exists a steady-state also when all agents, including the incumbent, behave myopically.*

This corollary simply follows from the optimisation. The alternative may have been optimal mining of the resource on the part of the incumbent, but this possibility is excluded by the definition of incumbent. Let us now make the following definition:

Definition 3 *Sustainable rent for the incumbent agent, $S(x)$, is defined as Agent m's net revenue for any stock level as long as the stock does not change, in other words when total harvest is equal to surplus production ($h = f$).*

Agent m's net price is then $p_m - qf > 0$ in order for m to be active. From (13) $h_m = nf - \frac{p_{-m}}{q}$. Hence,

$$S(x) = (p_m - qf)\left(nf - \frac{p_{-m}}{q}\right).$$

The sustainable rent is positive if and only if

$$\frac{p_{-m}}{nq} < f < \frac{p_m}{q}.$$

This interval exists if Agent m is an active agent and not the only active. As $S(x)$ is continuous, it must also have a maximum in this interval. This leads to the following proposition:

Proposition 6 *In Case 3, Agent m's sustainable rent, $S(x)$, is greater than Agent m's corresponding myopic net revenue in the myopic steady state.*

Proof: In the myopic steady state total harvest is given by (5) which must equal $f(x)$. Agent m's net revenue is given by

$$\pi_m = \frac{1}{q}(p_m - qf)^2.$$

Evaluating $S(x)$ for the same stock level yields

$$S(x) - \pi_m = \frac{1}{q}(p_m - qf)\left[nqf - p_{-m} - (p_m - qf)\right] = (n+1)\left(1 - \frac{1}{n}\right)(p_m - qf)\, f > 0 \;\blacksquare$$

The implication of this proposition is that at the myopic steady state the incumbent agent, m, can benefit from deviating from myopic behaviour but still stay at the same steady state. However, she can do even better than that by moving away from this steady state and towards the optimal dynamic steady state.

Next we wish to find the optimal dynamic behaviour of the incumbent agent. Following the approach outlined in Sandal and Steinshamn (2001) it is possible to find the optimal harvest for the incumbent agent as a feedback control law (function of the state variable). If the incumbent has a discount rate of zero, the optimal feedback harvest can be given explicitly as

$$h_m(x) = \max\left[0, nf - \frac{p_{-m}}{q} - \text{sign}(x^* - x)\sqrt{(S^* - S)\cdot n/q}\right] \tag{14}$$

where $S^* = S(x^*) = \max S(x)$. The expression for the feedback rule follows from the fact that the optimal Hamiltonian is constant with zero discounting. When applying this rule it is imperative to keep in mind that $n = n(x)$ is a step function determined by the Activity Principle (Proposition 1). As we know that the maximum of S is an interior solution, we can use the maximum principle to eliminate the costate variable and find the feedback rule given by (14). A proof of Catching-Up (CU) optimality in the case of zero discounting, applying a version of Mangasarin's Sufficiency Theorem[8], is given in Sandal and Steinshamn (2001). This feedback rule will be used in the numerical examples in the next section.

[8]Theorem 13 in Seierstad and Sydsæter (1987).

Numerical illustration of the case with an incumbent agent

In this section a numerical illustration of the case with one incumbent agent will be given, and this will be compared to the case where all agents act myopically. It is assumed that the discount rate is zero and that the incumbent agent has dedicated herself to aim for a sustainable fishery if possible. As long as the discount rate is small compared to the intrinsic growth rate of the fish stock, discounting will only have minor effects on the optimal control, see Sandal and Steinshamn (1997).

In this example there are five potential agents, and the numerical specification is as follows:

$$p_1 = 1 - \frac{0.28}{x},\ p_2 = 1 - \frac{0.30}{x},\ p_3 = 1 - \frac{0.32}{x},\ p_4 = 1 - \frac{0.35}{x},\ p_5 = 1 - \frac{0.40}{x},\ q = 1,$$

and the biological growth function is the rescaled logistic function

$$f(x) = x(1 - x).$$

The rescaling implies that the carrying capacity is one, the intrinsic growth rate is one and maximum sustainable yield is 0.25; see Clark (1990). In other words, the stock is measured relative to the carrying capacity, and time units are measured relative to the biological clock; i.e. as the inverse of the intrinsic growth rate.

Agent 1 is the incumbent, and it is interesting to compare the case where all three behave myopically with the case where Agent 1 behaves non-myopically. The development in the number of active agents when all behave myopically is illustrated in Figure 3.

Figure 4 shows surplus production and the sole owner outcome with Agent 1 as the sole owner, total harvest and Agent 1's harvest when all behave myopically, and total harvest and Agent 1's harvest when Agent 1 is incumbent. Note that the steady

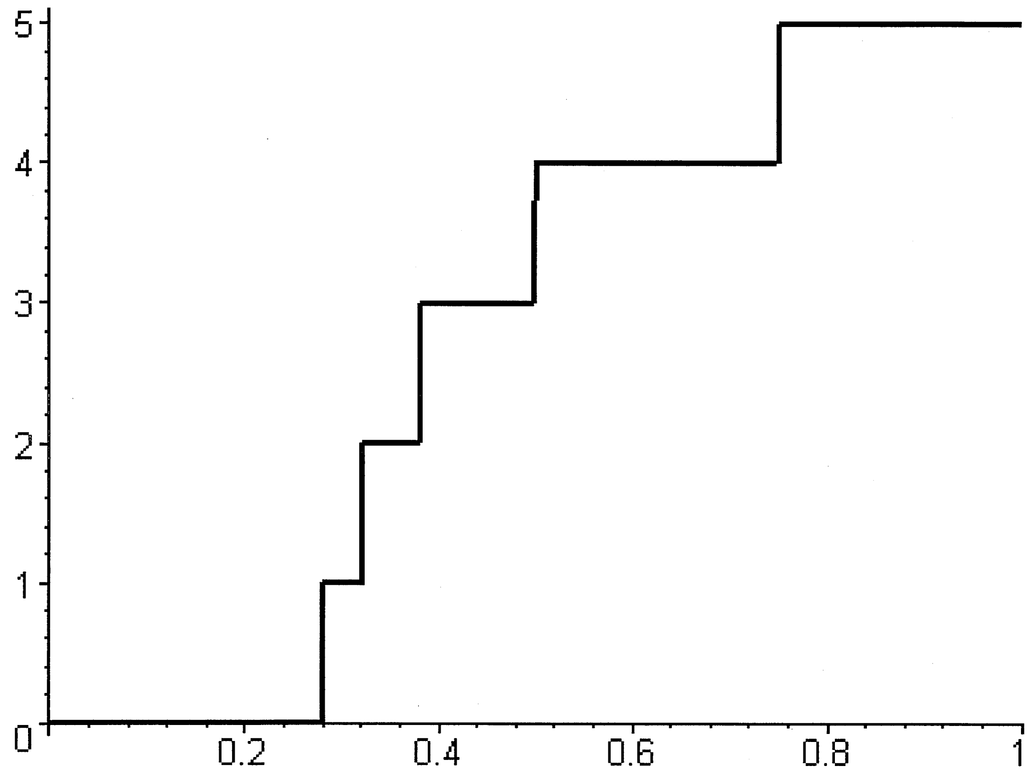

FIG. 3. *The development of the number of active agents as a function of the stock when everybody behaves myopically.*

states, both when all are myopic and when Agent 1 is incumbent, are to the left of msy (maximum sustainable yield), whereas with a sole owner the optimal steady state is always to the right of msy when costs decrease with x and there is a reasonably small discounting. An interesting, and somewhat counterintuitive, result is that the steady state is characterised by a less conservative policy (lower standing stock) when Agent 1 is incumbent than when all behave myopically. However, for lower stock levels the policy when all behave myopically is less conservative than when Agent 1 is non-myopic and behaves strategically.

Agent 1's incumbent harvest is a stepwise function where the steps occur at the

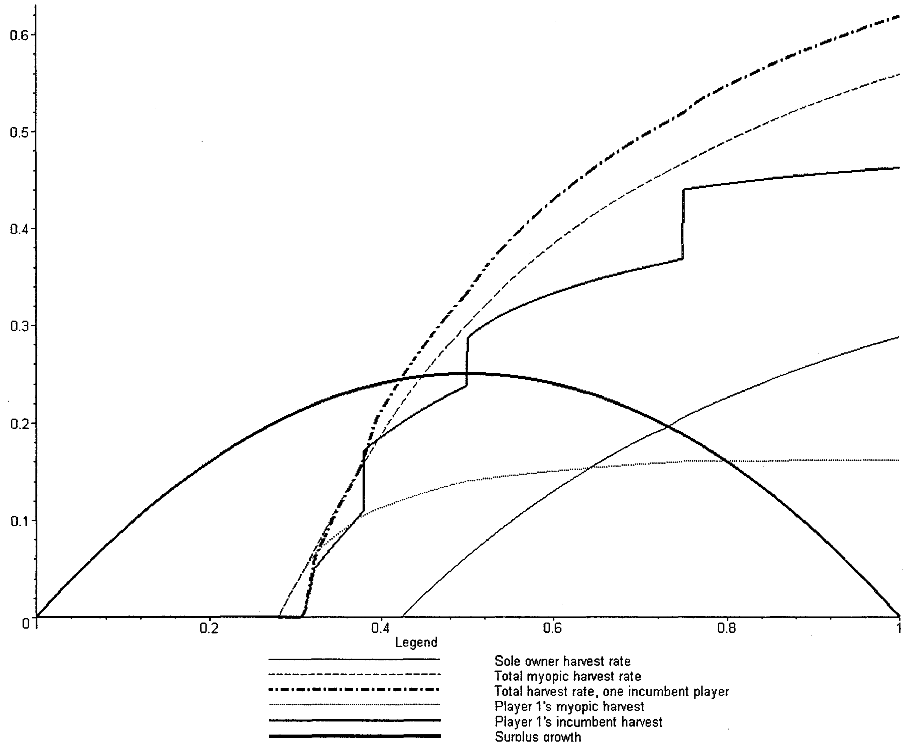

FIG. 4. *Surplus production, sole owner's harvest, total harvest when all are myopic, total harvest when agent 1 is incumbent, Agent 1's harvest when she is myopic and Agent 1's harvest when she is incumbent.*

entrance of new agents. At low stock levels the incumbent is alone and can benefit from a small harvest both in that she gets a high price and that the stock builds up fast implying lower costs. As new agents enter, Agent 1 will increase her harvest in order to meet the competition and reduce the harvest of the newcomer. This will reduce the price but, at the same time, the build-up of the stock is slower which will postpone the entrance of even more participants. Eventually the fishery goes into a steady state, in this example with three active agents. Figure 4 also shows by how much Agent 1 increases her harvest each time a new agent enters, and it is seen that this is quite substantial.

SUMMARY AND CONCLUSIONS

In this paper we have investigated a fishery game where the market price depends on total harvest and the participants act myopically in the sense that they do not take stock dynamics into account in their decisions. The model has also been extended to include the case where one participant takes the stock dynamics into account whereas the others do not. The number of active participants out of the potential has been endogenously determined. One of the main conclusions from this analysis is that both the stock development and the number of active participants are heavily dependent upon the individual efficiency parameters. With two agents one must be twice as efficient as the other in order to exclude the other. With a large number of agents, on the other hand, one must be as efficient as the average of the rest in order to be active. This implies that the participants must have similar efficiencies in order to accommodate a large number of active agents.

With one incumbent agent, the incumbent benefits from not being myopic if there also exists a steady state when everybody behave myopically. However, the steady state when all behave myopically may be less conservative (lower standing stock) than when one of the participants is non-myopic.

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