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Abstract

Shrinkage estimators have recently become popular in estimation of heterogeneous models on panel data. In this paper we show that the estimated covariance matrix in the posterior distribution of the shrinkage estimator fails to include the variability of the hyperparameters. Hence, standard confidence intervals for the parameters based on the "estimated posterior" distribution, are too narrow and thus the t-statistic is upward biased. The bootstrap method, which incorporates some of the variability in the hyperparameters, is an alternative method to obtain confidence intervals for the parameters. Our empirical example show that one has to be aware of the method used, since it can lead to significantly different economic conclusions.

Keywords: empirical Bayes estimator, t-statistics, delta method, bootstrap.

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1 Introduction

Shrinkage estimators, which may be implemented as empirical Bayes estimators, have recently become popular in estimation of demand models on panel data, but there has been little focus on the estimation of confidence intervals in such studies. Shrinkage estimators provide a trade-off between individual cross-section specific and pooled estimation, and has become popular since it allows some heterogeneity in the estimated parameters. For example, Maddala et al. (1997) used shrinkage estimators in the estimation of short-run and long-run price and income elasticities of residential demand for electricity and natural gas in the United States. Baltagi et al. (2003) estimated a dynamic panel data model of French regional gasoline consumption on a panel data set from 21 French regions. Baltagi and Griffin (1997) used a dynamic demand specification for gasoline and the Hu and Maddala (1994) shrinkage estimator, in the context of gasoline demand across 18 OECD countries.

In this paper we focus on confidence intervals and t-statistics of the estimated parameters from the iterative Maddala et al. (1997) shrinkage estimator. The method used to compute the t-statistics is generally the delta method based on the "estimated posterior" distribution, where the reported t-statistics often have relatively high values. Several papers in the economic literature discuss whether to use bootstrap methods or the delta method to obtain confidence intervals for nonlinear functions of parameters, like the long-run elasticities (Vinod and McCullough, 1994; Vinod, 1995; Li and Maddala, 1999; Kazimi and Brownstone, 1999). Less attention have been on why the bootstrap performs better than the delta method, and particulary for nonlinear functions of shrinkage parameters. We provide one possible explanation.

We demonstrate that the standard way of obtaining confidence intervals for the shrinkage parameter estimates based on the estimated posterior distribution, leads to narrow intervals and thus upward bias the t-statistics. The iterative shrinkage estimator preestimate a set of the prior unknown parameters (the hyperparameters) from the sample and threat them as fixed when the covariance matrix in the posterior distribution is computed. Hence, the variability in the hyperparameters are ignored in the estimated posterior distribution, and the delta method provides upward biased t-statistics. This pitfall is known in the theoretical literature on empirical Bayes methods, where possible solutions through use of bootstrap technique are suggested (Laird and Louis, 1987; Carlin and Gelfand, 1991). However, the pitfall and possible economic consequences have not been on the agenda in econometric studies using the shrinkage estimator.

2 The variability of the hyperparameters

Consider a general hierarchical model in two stages of the following form: At the first stage, given some parameter θ_i, the data vector $\mathbf{y}_i$ is independently distributed as $f_i(\mathbf{y}_i|\theta_i)$, for all $i = 1, 2, \ldots, N$ (number of cross-sections). At the second stage, the θ_i are supposed to be independently and identically distributed with distribution $\pi(\theta_i|\eta)$, where η indexes the family π. The posterior distribution of θ_i is denoted $f_i(\theta_i|\mathbf{y}_i, \eta)$. The empirical Bayes approach, which treat η as a fixed unknown, obtains inference on the estimated posterior distribution $f_i(\theta_i|\mathbf{y}_i, \hat{\eta})$, where $\hat{\eta}$ is some empirical Bayes estimate of η. Of particular interest is the expectation $\mathrm{E}(\theta_i|\mathbf{y}_i)$ and the variance $\mathrm{var}(\theta_i|\mathbf{y}_i)$. When the estimated posterior is used as a basis for inference about θ_i, $\mathrm{E}(\theta_i|\mathbf{y}_i)$ and $\mathrm{var}(\theta_i|\mathbf{y}_i)$ are approximated by $\mathrm{E}(\theta_i|\mathbf{y}_i, \hat{\eta})$ and $\mathrm{var}(\theta_i|\mathbf{y}_i, \hat{\eta})$. There is a substantial amount of literature which demonstrate that $\mathrm{E}(\theta_i|\mathbf{y}_i, \hat{\eta})$ often performs well as an estimator of θ_i (Morris, 1983). Unfortunately, confidence intervals for θ_i based on the estimated posterior are generally too narrow and fail to attain the nominal coverage probability (Carlin and Gelfand, 1991). The explanation for this problem is that the estimated posterior ignores the variability in $\hat{\eta}$, since the empirical Bayes estimator typically assume that η is fixed ($\eta = \hat{\eta}$). By the law of double expectation, $\text{var}(\theta_i|\mathbf{y})$ may be written

$$\text{var}(\theta_i|\mathbf{y}) = \text{E}_{\eta|\mathbf{y}}[\text{var}(\theta_i|\mathbf{y}_i, \eta)] + \text{var}_{\eta|\mathbf{y}}[\text{E}(\theta_i|\mathbf{y}_i, \eta)]. \tag{1}$$

Fixing η imply that the second term of equation (1) is assumed to be zero and $\text{var}(\theta_i|\mathbf{y})$ is underestimated. That is, $\text{var}(\theta_i|\mathbf{y}_i, \hat{\eta})$ only approximates the first term of equation (1).

The iterative shrinkage estimator is based on the hierarchical linear regression model in two stages of the following form

$$\boldsymbol{y}_i|\boldsymbol{X}_i, \boldsymbol{\beta}_i \sim \mathscr{N}(\boldsymbol{X}_i\boldsymbol{\beta}_i, \psi_i^2\boldsymbol{I}), \tag{2}$$

and

$$\boldsymbol{\beta}_i|\boldsymbol{\mu} \sim \mathscr{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \tag{3}$$

for all i, where $\boldsymbol{y}_i$ is a $T \times 1$ vector of the dependent variables, $\boldsymbol{X}_i$ is the $T \times (K+1)$ matrix of some K explanatory variables (that may include lags of the dependent variable), the first column of $\boldsymbol{X}_i$ is a vector of ones, $\boldsymbol{\beta}_i$ is some $(K+1) \times 1$ unknown parameter vector, $\boldsymbol{\mu}$ is some unknown and fixed $(K+1) \times 1$ vector, and T is the number of time-section observations. Further, $\boldsymbol{\Sigma}$ is some $(K+1) \times (K+1)$ covariance matrix, $\psi_i \geq 0$ is some scalar and $\boldsymbol{I}$ is the identity matrix. The parameters $\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$ and ψ_i are unknown and have to be specified. The posterior distribution is denoted as $\pi(\boldsymbol{\beta}_i|\boldsymbol{y}_i, \boldsymbol{X}_i, \boldsymbol{\mu})$ and is normally distributed with some expectation $\boldsymbol{\omega}_i$ given by

$$\boldsymbol{\omega}_i = \boldsymbol{\Omega}_i \left(\frac{1}{\psi_i} \boldsymbol{X}_i{}' \boldsymbol{X}_i \hat{\boldsymbol{\beta}}_i + \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \right), \tag{4}$$

and covariance $\boldsymbol{\Omega}_i$

$$\boldsymbol{\Omega}_i = \left(\frac{1}{\psi_i} \boldsymbol{X}_i{}' \boldsymbol{X}_i + \boldsymbol{\Sigma}^{-1} \right)^{-1}, \tag{5}$$

for all i. When $\boldsymbol{X}_i$ include lagged dependent variables, the normality of the posterior distribution holds only asymptotically (Maddala et al., 1997).

The iterative shrinkage estimator approximates the unspecified parameters $\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$ and ψ_i^2 by the sample-based estimates

$$\tilde{\boldsymbol{\mu}} = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\omega}_i, \tag{6}$$

$$\tilde{\boldsymbol{\Sigma}} = \boldsymbol{R} + \frac{1}{N-1}\sum_{i=1}^{N}\left(\boldsymbol{\omega}_i - \tilde{\boldsymbol{\mu}}\right)'\left(\boldsymbol{\omega}_i - \tilde{\boldsymbol{\mu}}\right), \tag{7}$$

and

$$\tilde{\psi}_i^2 = \frac{\left(\boldsymbol{y}_i - \boldsymbol{X}_i\boldsymbol{\omega}_i{}'\right)'\left(\boldsymbol{y}_i - \boldsymbol{X}_i\boldsymbol{\omega}_i{}'\right)}{T-(K+1)}, \tag{8}$$

for all i, respectively, where $\boldsymbol{R}$ is a diagonal matrix with small positive entries. Cross-section specific OLS estimates $\hat{\boldsymbol{\beta}}_i$ of $\boldsymbol{\beta}_i$ is typical used initially for $\boldsymbol{\omega}_i$ in the iterative procedure (See Maddala et al. (1997, pp. 93-94) for details. We use $\boldsymbol{R} = 10^{-7}\boldsymbol{I}$ in our empirical application.). According to a Monte-Carlo study by Hu and Maddala (1994) the iterative procedure gives better estimates in the mean squared sense for both the overall mean $\boldsymbol{\mu}$ and the heterogeneity matrix $\boldsymbol{\Sigma}$ than two-step procedures.

When the covariance matrix is computed in equation (5), $\boldsymbol{\mu}$ is treated as fixed. Hence, the estimated posterior distribution of the iterative shrinkage estimator fails to account for the variability of $\boldsymbol{\mu}$. Let g be some function of the parameters. Normally, inferences on $g(\boldsymbol{\beta}_i)$ include the posterior variance $\text{var}(g(\boldsymbol{\beta}_i)|\boldsymbol{y}_i, \boldsymbol{X}_i)$. Inferences on the shrinkage estimator are based on the estimated posterior $\pi(\boldsymbol{\beta}_i|\boldsymbol{y}_i, \boldsymbol{X}_i, \hat{\boldsymbol{\mu}})$. Hence, $\text{var}(g(\boldsymbol{\beta}_i)|\boldsymbol{y}_i, \boldsymbol{X}_i)$ is approximated by $\text{var}(g(\boldsymbol{\beta}_i)|\boldsymbol{y}_i, \boldsymbol{X}_i, \hat{\boldsymbol{\mu}})$, where $\hat{\boldsymbol{\mu}}$ is a point estimate of $\boldsymbol{\mu}$ (like equation 6). By the law of double expectation, $\text{var}(g(\boldsymbol{\beta}_i)|\boldsymbol{y}_i, \boldsymbol{X}_i)$ may be split into a summation of $\text{E}_{\boldsymbol{\mu}|\boldsymbol{y}_i,\boldsymbol{X}_i}[\text{var}(g(\boldsymbol{\beta}_i)|\boldsymbol{y}_i, \boldsymbol{X}_i, \boldsymbol{\mu})]$ and $\text{var}_{\boldsymbol{\mu}|\boldsymbol{y}_i,\boldsymbol{X}_i}[\text{E}(g(\boldsymbol{\beta}_i)|\boldsymbol{y}_i, \boldsymbol{X}_i, \boldsymbol{\mu})]$ as shown in general in equation (1). Since the shrinkage estimator treat $\boldsymbol{\mu}$ as fixed ($\boldsymbol{\mu} = \hat{\boldsymbol{\mu}}$), the second term is assumed to be zero. As the variability of $\hat{\boldsymbol{\mu}}$ is ignored, $\text{var}(g(\boldsymbol{\beta}_i)|\boldsymbol{y}_i, \boldsymbol{X}_i, \hat{\boldsymbol{\mu}})$ approximate only the first term and the covariance matrix will be underestimated. In particular, the elements along the diagonal of $\boldsymbol{\Omega}_i$ will be to small and the t-statistics of $g(\boldsymbol{\beta}_i)$ based on the estimated posterior is general upward biased.

There has been a discussion in the economic literature whether to use bootstrap methods or the delta method to obtain confidence intervals for nonlinear functions of parameters, like the long-run elasticities (Vinod and McCullough, 1994; Vinod, 1995; Li and Maddala, 1999; Kazimi and Brownstone, 1999). Less attention have been on why the bootstrap performs better than the delta method, and particularly for nonlinear functions of shrinkage parameters. It follows from the discussion above that the fundamental problem of the delta method used on the shrinkage estimator is that the estimated covariance matrix in the posterior distribution fails to include the variability in the hyperparameters, which is an explanation why the bootstrap method provides larger confidence intervals.

3 An empirical example

To illustrate the consequences of using the estimated posterior distribution or the bootstrap method as a basis for statistical inference on the iterative shrinkage estimator, this section estimates residential natural gas price, cross-price and income elasticity estimates from a panel data of 12 European countries covering the period from 1978 to 2002. Data and model i similar to the study of Maddala et al. (1997).

To estimate the price and income elasticities of natural gas demand in the short-run and long-run, we specify a dynamic loglinear demand model of the form

$$\begin{aligned} y_{t,i}^{NG} &= \beta_i^0 + \beta_i^y y_{t-1,i}^{NG} + \beta_i^{NG} p_{t,i}^{NG} + \beta_i^{LFO} p_{t,i}^{LFO} \\ &\quad + \beta_i^{EL} p_{t,i}^{EL} + \beta_i^m m_{t,i} + \beta_i^z z_{t,i} + \varepsilon_{t,i}, \qquad (9) \end{aligned}$$

for all $t = 1, 2, \ldots, T_i$ (year subscript) and $i = 1, 2, \ldots, 12$ (country subscript), respectively, where $y_{t,i}^{NG}$ = ln(residential natural gas consumption per capita),

$p_{t,i}^{NG}$ = ln(real residential natural gas price), $p_{t,i}^{LFO}$ = ln(real residential light fuel oil price), $p_{t,i}^{EL}$ = ln(real residential electrical price), $m_{t,i}$ = ln(real personal income per capita), $z_{t,i}$ = ln(heating degree days index), and $\varepsilon_{t,i} \sim \mathscr{N}(0, \psi_i^2)$ is some error term. The annual residential prices and quantities were obtained from IEA (2004) and the private income and consumer price index from IMF (2003). Annual weather data (heating degree days index) by country were taken from Klein Tank et al. (2002). The prices (total end-use prices inclusive taxes) and private consumption were deflated using the consumer price index (basis year = 1999) from the IFS, and provided in Euro per toe tonnes of oil equivalent (€/toe) and thousand Euro per capita (k€/cap), respectively. The natural gas demand was provided in tonnes of oil equivalent per thousands of capita (toe/kcap) and the heating degree days index is unit-free.

3.1 Empirical results

Residential energy consumption is characterized by very limited technological substitution possibilities between different energy carriers in the short-run, after investments in heating infrastructure has been undertaken. In the longer run it is also costly to switch between energy carriers due to high investment costs in heating infrastructure. Thus, one should expect a priori low cross-price and own-price elasticities. We will see that this is confirmed by our empirical results.

Following Maddala et al. (1997), country-specific OLS estimates is used as initial values of the iterative shrinkage estimator. Tables 1 and 2 contain the estimated country-specific OLS and shrinkage parameters of equation (9), with maximum, average, and minimum values of the estimated parameters. The t-statistics of the shrinkage parameters were obtained from the estimated posterior distribution using the delta method. A comparison of the t-statistics based on the delta method, found 31 of 84 parameters significant at the 5% level for the OLS estimator and 70 of 84 for the shrinkage estimator. This is in accordance with the results reported in other studies comparing the shrinkage with OLS.

Several studies note that shrinkage estimators give more reasonable results from an economic perspective than individual country estimates. Here, we compare the shrinkage estimates with individual country OLS elasticity estimates. The short-run elasticities are identical to the estimated parameters, while the long-run price and income elasticities are given by $\beta_i^{NG}/(1-\beta_i^y)$, $\beta_i^{LFO}/(1-\beta_i^y)$, $\beta_i^{EL}/(1-\beta_i^y)$ and $\beta_i^m/(1-\beta_i^y)$. Tables 3 and 4 contain the estimated country-specific OLS and shrinkage price and income elasticities in the short-run and long-run, with maximum, average, and minimum values of the elasticities. The t-statistics were obtained from the estimated posterior distribution using the delta method. The OLS elasticity estimates were found to have substantial variation across countries, and often had implausible signs and values, while the shrinkage found more plausible estimates with smaller variation. These results are similar to findings in earlier studies. Furthermore, the t-statistics based on the delta method indicate the 68 of 96 elasticities are significant at a 5% level for the shrinkage estimator, while only 24 of 96 are significant for the country-specific OLS estimator.

Based on the residuals of the shrinkage estimation, 10000 bootstrap samples were generated. The bootstrap samples were generated recursively due to the dynamic structure. On each bootstrap sample, the parameters of equation (9) were estimated using the iterative shrinkage algorithm. Table 5 present the 95% bootstrap percentile confidence intervals for the shrinkage elasticities. At the 5% significance level, the short-run and long-run income elasticities were found significant for 11 of 12 countries. On the other hand, none of the own-price and cross-price elasticities were found significant. Thus, only 22 of 96 elasticities were found to be significant at the 5% level. This is in contrast to the the t-statistics based on the estimated posterior distribution where 68 of 96 elasticities were found significant at the 5% level, which is an indication of magnitude of the upward bias of the estimated posterior distribution based t-statistics.

The empirical example shows that the way of obtaining confidence intervals for the shrinkage estimator can make a difference with respect to the economic conclusions. When the estimated posterior distribution and the delta method is used, most of the cross-price elasticities are found significant at the 1% level, indicating that light fuel oil and electricity are substitutes for natural gas. On the other hand, if bootstrap intervals are used no cross-price elasticities are significantly different from zero, indicating no substitution. Moreover, none of the own-price elasticities are significantly different from zero according to the bootstrap intervals at a 5% level, although the own-price elasticities of Netherlands are significantly different from zero at a 10% level. This may not be to surprising given that households in general cannot switch because of expensive investments in structure and large transformation costs. Only the income elasticities remain significantly different from zero at the 5% level, with the bootstrap method.

4 Summary and conclusions

Shrinkage estimators has recently become popular in estimation of heterogeneous models on panel data. The method used to compute the t-statistics is generally the delta method based on the estimated posterior distribution, where the reported t-statistics often have relatively high values. We demonstrate that confidence intervals for the iterative shrinkage estimator based on the estimated posterior distribution are too narrow and thus upward bias the t-statistics.

The shrinkage estimator preestimate the hyperparameters from the sample and threat them as fixed when the covariance matrix in the posterior distribution of the parameters is computed. Hence, the variability in the hyperparameters are ignored in the estimated posterior distribution, and the delta method consequently provide upward biased t-statistics. This pitfall is known in the theoretical literature on empirical Bayes methods, where possible solutions through use of bootstrap technique are suggested. However, the pitfall and its possible economic consequences have not been on the agenda in econometric studies using the shrinkage estimator.

Our case study of residential energy elasticities of demand in 12 European countries, using a dynamic loglinear demand model and the iterative shrinkage estimator, demonstrates that the bootstrap method provides wider confidence intervals from the shrinkage estimator than the standard delta method. At the 5% level, the bootstrap method found 22 of 96 elasticities significant. In contrast, the standard delta method found 68 of 96 elasticities significant. In particular, we find that there are significant own-price and cross-price effects using the standard method for several countries, while the bootstrap method does not provide a single significant own-price and cross-price elasticity.

As demonstrated here, the method used to obtain confidence intervals for the shrinkage estimator may make a significant difference for the economic conclusions that is derived from the results. It is not sufficient that the point estimate of the shrinkage estimator are "plausible". The bootstrap seems to better take into account the uncertainty in the shrinkage elasticity estimates than the delta method based on estimated posterior distribution. Thus, one should be aware of what method is used to obtain t-statistics and confidence intervals for the shrinkage estimator.

Appendix: Tables

Table 1: Country-specific OLS parameter estimates.[a]

Country	β_i^y	β_i^{NG}	β_i^{LFO}	β_i^{EL}	β_i^m	β_i^z	β_i^0
Austria	0.361	0.027	-0.008	0.484*	1.981*	0.294	-6.979*
	(1.7)	(0.22)	(-0.07)	(2.3)	(3.6)	(1.9)	(-3.4)
Belgium	0.356*	0.141	0.060	-0.733*	0.430	0.656*	3.909
	(4.3)	(1.04)	(1.53)	(-2.1)	(1.30)	(7.0)	(1.37)
Denmark	0.840*	-0.506	0.395	0.405	-0.813	0.775*	-3.571
	(13.4)	(-0.85)	(0.49)	(0.97)	(-1.23)	(3.4)	(-1.13)
Finland	0.547*	-0.635	0.393	-0.072	-0.853	0.798	0.621
	(2.9)	(-1.21)	(0.59)	(-0.07)	(-0.97)	(0.73)	(0.07)
France	0.521*	-0.152	0.524*	-1.254*	0.594	0.286	6.614
	(2.5)	(-0.37)	(2.6)	(-2.6)	(0.84)	(0.93)	(1.9)
Germany	0.649*	-0.217	0.128	4.920	3.957	1.314	-50.23
	(3.3)	(-0.16)	(0.18)	(1.7)	(2.0)	(0.86)	(-1.9)
Ireland	0.508*	-0.158	-0.617*	-0.183	1.099	0.771	2.166
	(3.5)	(-0.55)	(-2.1)	(-0.45)	(2.0)	(0.96)	(0.40)
Italy	0.610*	0.277*	-0.190	0.012	1.078*	0.294*	-2.218
	(3.4)	(2.1)	(-1.47)	(0.07)	(2.2)	(2.2)	(-1.8)
Netherlands	0.245*	-0.256*	0.049	0.185	-0.206	0.820*	1.389
	(2.4)	(-3.8)	(0.52)	(1.10)	(-1.64)	(7.1)	(1.29)
Spain	0.845*	0.183	-0.085	-0.003	1.154*	0.330	-3.886
	(11.7)	(0.92)	(-0.80)	(-0.03)	(3.0)	(1.69)	(-2.24)
Switzerland	0.615*	-0.622*	0.054	-0.639	1.790*	0.513*	2.831
	(4.5)	(-2.5)	(0.85)	(-2.0)	(2.3)	(2.5)	(0.89)
UK	0.577*	-0.056	-0.020	0.042	0.330*	0.493*	-0.372
	(5.6)	(-0.66)	(-0.73)	(0.41)	(3.2)	(5.1)	(-0.43)
Min	0.245	-0.635	-0.617	-1.254	-0.853	0.286	-50.23
Avg	0.556	-0.165	0.057	0.264	0.878	0.612	-4.143
Max	0.845	0.277	0.524	4.920	3.957	1.314	6.614

[a]Figures put in parenthesis denote the t-statistics and the symbol * denotes statistically significant at the 5% level.

Table 2: Country-specific shrinkage parameter estimates.[a]

Country	β_i^y	β_i^{NG}	β_i^{LFO}	β_i^{EL}	β_i^m	β_i^z	β_i^0
Austria	0.696*	-0.128*	0.060*	0.0001	0.753*	0.492*	-2.248*
	(61.2)	(-5.4)	(6.2)	(0.06)	(10.0)	(21.1)	(-14.0)
Belgium	0.595*	-0.058*	0.026*	-0.003	0.619*	0.529*	-1.402*
	(53.6)	(-3.1)	(3.4)	(-0.44)	(9.4)	(26.2)	(-9.5)
Denmark	0.704*	-0.103*	0.050*	-0.007	0.824*	0.470*	-2.382*
	(27.0)	(-3.3)	(4.0)	(-0.60)	(6.3)	(11.8)	(-7.4)
Finland	0.841*	-0.090	0.049	-0.030	1.219*	0.352*	-3.768*
	(57.0)	(-0.96)	(1.24)	(-1.20)	(5.8)	(5.3)	(-12.1)
France	0.717*	0.015	-0.0003	-0.038*	1.090*	0.386*	-2.765*
	(28.1)	(0.28)	(-0.01)	(-2.29)	(6.4)	(7.3)	(-7.6)
Germany	0.721*	0.006	0.003	-0.036	1.082*	0.389*	-2.781*
	(18.5)	(0.08)	(0.11)	(-1.58)	(4.6)	(5.4)	(-5.5)
Ireland	0.717*	-0.040	0.023	-0.024	0.981*	0.421*	-2.642*
	(52.3)	(-0.70)	(0.97)	(-1.54)	(6.8)	(9.3)	(-10.7)
Italy	0.645*	0.104*	-0.041*	-0.050*	1.070*	0.388*	-2.245*
	(85.0)	(5.4)	(-5.1)	(-8.7)	(19.1)	(22.3)	(-20.0)
Netherlands	0.353*	-0.156*	0.059*	0.055*	-0.221*	0.783*	1.192*
	(29.9)	(-9.1)	(8.5)	(9.6)	(-3.5)	(40.3)	(7.9)
Spain	0.748*	0.174*	-0.067*	-0.082*	1.484*	0.263*	-3.411*
	(175.9)	(6.4)	(-5.8)	(-11.2)	(23.2)	(13.0)	(-34.6)
Switzerland	0.864*	-0.142*	0.072*	-0.020*	1.177*	0.367*	-3.878*
	(44.2)	(-9.5)	(12.7)	(-3.1)	(14.6)	(14.9)	(-17.4)
UK	0.529*	-0.054*	0.022*	0.005*	0.451*	0.579*	-0.765*
	(90.4)	(-7.8)	(7.8)	(2.3)	(16.6)	(69.4)	(-11.1)
Min	0.353	-0.156	-0.067	-0.082	-0.221	0.263	-3.878
Avg	0.677	-0.039	0.021	-0.019	0.877	0.452	-2.258
Max	0.864	0.174	0.072	0.055	1.484	0.783	1.192

[a]Figures put in parenthesis denote the t-statistics and the symbol * denotes statistically significant at the 5% level.

Table 3: Country-specific OLS elasticity estimates.[a]

	Natural Gas		**L. Fuel Oil**		**Electricity**		**Income**	
Estimator	SR	LR	SR	LR	SR	LR	SR	LR
Austria	0.027	0.043	-0.008	-0.012	0.484*	0.758*	1.981*	3.099*
	(0.22)	(0.23)	(-0.07)	(-0.07)	(2.3)	(2.2)	(3.6)	(10.1)
Belgium	0.141	0.218	0.060	0.093	-0.733	-1.137*	0.430	0.667
	(1.04)	(1.07)	(1.53)	(1.49)	(-2.0)	(-2.1)	(1.30)	(1.31)
Denmark	-0.506	-3.171	0.395	2.476	0.405	2.536	-0.813	-5.096
	(-0.85)	(-0.90)	(0.49)	(0.49)	(0.97)	(0.83)	(-1.23)	(-0.89)
Finland	-0.635	-1.403	0.393	0.867	-0.072	-0.158	-0.853	-1.882
	(-1.21)	(-1.28)	(0.59)	(0.60)	(-0.07)	(-0.07)	(-0.97)	(-1.11)
France	-0.152	-0.317	0.524*	1.095	-1.254*	-2.619*	0.594	1.241
	(-0.37)	(-0.33)	(2.6)	(1.40)	(-2.6)	(-2.8)	(0.84)	(1.12)
Germany	-0.217	-0.618	0.128	0.366	4.920	14.002	3.957	11.262*
	(-0.16)	(-0.16)	(0.18)	(0.18)	(1.7)	(1.8)	(1.9)	(2.4)
Ireland	-0.158	-0.320	-0.617*	-1.254	-0.183	-0.371	1.099	2.233*
	(-0.55)	(-0.52)	(-2.1)	(-1.8)	(-0.45)	(-0.48)	(2.0)	(2.6)
Italy	0.277	0.712	-0.190	-0.487	0.012	0.030	1.078*	2.765*
	(2.08)	(2.58)	(-1.47)	(-1.52)	(0.07)	(0.07)	(2.2)	(6.5)
Netherlands	-0.256*	-0.340*	0.049	0.064	0.185	0.244	-0.206	-0.272
	(-3.8)	(-3.6)	(0.52)	(0.51)	(1.10)	(1.12)	(-1.64)	(-1.8)
Spain	0.183	1.179	-0.085	-0.546	-0.003	-0.016	1.154*	7.442*
	(0.92)	(0.88)	(-0.80)	(-0.74)	(-0.03)	(-0.03)	(3.0)	(3.8)
Switzerland	-0.622*	-1.614*	0.054	0.141	-0.639	-1.658*	1.790*	4.647*
	(-2.5)	(-3.0)	(0.86)	(0.77)	(-2.0)	(-2.3)	(2.3)	(4.3)
UK	-0.056	-0.133	-0.020	-0.046	0.042	0.100	0.330*	0.780*
	(-0.66)	(-0.64)	(-0.73)	(-0.77)	(0.41)	(0.41)	(3.2)	(4.9)
Min	-0.635	-3.171	-0.617	-1.254	-1.254	-2.619	-0.853	-5.096
Avg	-0.165	-0.480	0.057	0.230	0.264	0.976	0.878	2.241
Max	0.277	1.179	0.524	2.476	4.920	14.002	3.957	11.262

[a]Figures put in parenthesis denote the t-statistics and the symbol * denotes statistically significant different from zero at the 5% level.

Table 4: Country-specific shrinkage elasticity estimates.[a]

Estimator	Natural Gas SR	Natural Gas LR	L. Fuel Oil SR	L. Fuel Oil LR	Electricity SR	Electricity LR	Income SR	Income LR
Austria	-0.128*	-0.421*	0.060*	0.197*	0.001	0.002	0.753*	2.475*
	(-5.5)	(-6.7)	(6.2)	(7.8)	(0.06)	(0.06)	(10.0)	(7.3)
Belgium	-0.058*	-0.144*	0.026*	0.065*	-0.003	-0.007	0.619*	1.526*
	(-3.1)	(-3.3)	(3.5)	(3.8)	(-0.44)	(-0.43)	(9.4)	(7.5)
Denmark	-0.103*	-0.349*	0.050*	0.168*	-0.007	-0.023	0.824*	2.781*
	(-3.3)	(-4.5)	(4.0)	(5.9)	(-0.60)	(-0.57)	(6.4)	(4.0)
Finland	-0.090	-0.565	0.049	0.309	-0.030	-0.186	1.219*	7.690*
	(-0.96)	(-1.01)	(1.24)	(1.32)	(-1.20)	(-1.12)	(5.8)	(4.0)
France	0.015	0.053	0.001	-0.001	-0.038*	-0.135*	1.090*	3.853*
	(0.28)	(0.28)	(-0.01)	(-0.01)	(-2.29)	(-1.91)	(6.44)	(4.10)
Germany	0.006	0.023	0.003	0.012	-0.036	-0.130	1.082*	3.874*
	(0.08)	(0.08)	(0.11)	(0.11)	(-1.58)	(-1.35)	(4.6)	(2.9)
Ireland	-0.040	-0.141	0.023	0.082	-0.024	-0.086	0.981*	3.461*
	(-0.70)	(-0.72)	(0.97)	(1.01)	(-1.54)	(-1.45)	(6.8)	(5.2)
Italy	0.104*	0.293*	-0.041*	-0.116*	-0.050*	-0.141*	1.070*	3.016*
	(5.4)	(4.9)	(-5.1)	(-4.7)	(-8.7)	(-7.4)	(19.1)	(13.7)
Netherlands	-0.156*	-0.241*	0.059*	0.091*	0.055*	0.086*	-0.221*	-0.341*
	(-9.1)	(-10.7)	(8.5)	(9.9)	(9.6)	(11.5)	(-3.5)	(-3.7)
Spain	0.174*	0.691*	-0.067*	-0.266*	-0.082*	-0.325*	1.484*	5.893*
	(6.4)	(5.8)	(-5.8)	(-5.3)	(-11.2)	(-9.4)	(23.2)	(16.8)
Switzerland	-0.142*	-1.041*	0.072*	0.530*	-0.020*	-0.145*	1.177*	8.646*
	(-9.5)	(-17.7)	(12.7)	(13.0)	(-3.1)	(-2.16)	(14.6)	(4.7)
UK	-0.054*	-0.115*	0.022*	0.047*	0.005*	0.012*	0.451*	0.958*
	(-7.8)	(-8.4)	(7.8)	(8.4)	(2.3)	(2.4)	(16.6)	(13.8)
Min	-0.156	-1.041	-0.067	-0.266	-0.082	-0.325	-0.221	-0.341
Avg	-0.039	-0.163	0.021	0.093	-0.019	-0.090	0.877	3.653
Max	0.174	0.691	0.072	0.530	0.055	0.086	1.484	8.646

[a]Figures put in parenthesis denote the t-statistics and the symbol * denotes statistically significant different from zero at the 5% level.

Table 5: The 95% bootstrap percentile intervals of the elasticities based on 10000 bootstrap samples.[b]

Country	Natural gas price Short-run LL	Natural gas price Short-run UL	Natural gas price Long-run LL	Natural gas price Long-run UL	Light fuel oil price Short-run LL	Light fuel oil price Short-run UL	Light fuel oil price Long-run LL	Light fuel oil price Long-run UL
Austria	-0.35	0.07	-0.88	0.18	-0.06	0.16	-0.16	0.43
Belgium	-0.16	0.05	-0.35	0.10	-0.04	0.08	-0.08	0.18
Denmark	-0.32	0.16	-0.96	0.47	-0.11	0.16	-0.31	0.49
Finland	-0.39	0.29	-2.01	1.25	-0.21	0.20	-0.95	1.07
France	-0.18	0.22	-0.48	0.60	-0.12	0.10	-0.33	0.28
Germany	-0.21	0.21	-0.58	0.57	-0.12	0.11	-0.33	0.31
Ireland	-0.27	0.17	-0.71	0.46	-0.10	0.13	-0.28	0.36
Italy	-0.11	0.27	-0.28	0.63	-0.15	0.10	-0.37	0.27
Netherlands	-0.29	0.01	-0.45	0.02	-0.07	0.19	-0.10	0.30
Spain	-0.13	0.46	-0.41	1.57	-0.23	0.12	-0.80	0.37
Switzerland	-0.50	0.17	-2.30	0.69	-0.13	0.22	-0.44	1.30
UK	-0.16	0.05	-0.31	0.10	-0.04	0.09	-0.07	0.18

Country	Electricity price Short-run LL	Electricity price Short-run UL	Electricity price Long-run LL	Electricity price Long-run UL	Income Short-run LL	Income Short-run UL	Income Long-run LL	Income Long-run UL
Austria	-0.33	0.22	-0.77	0.57	0.62	1.38	1.63	3.23
Belgium	-0.15	0.14	-0.31	0.29	0.48	0.94	0.99	1.99
Denmark	-0.25	0.31	-0.71	0.91	0.61	2.02	1.90	5.34
Finland	-0.53	0.36	-2.49	1.42	1.05	2.77	4.08	13.36
France	-0.24	0.11	-0.62	0.29	0.78	1.80	1.82	4.85
Germany	-0.26	0.14	-0.69	0.38	0.67	1.89	1.50	5.21
Ireland	-0.32	0.17	-0.81	0.43	0.81	1.67	2.12	4.43
Italy	-0.21	0.14	-0.51	0.34	0.89	1.58	2.33	3.60
Netherlands	-0.20	0.32	-0.32	0.48	-0.51	0.06	-0.67	0.10
Spain	-0.26	0.10	-0.80	0.34	1.17	2.31	4.23	6.77
Switzerland	-0.69	0.42	-2.43	1.80	0.87	3.06	3.31	12.59
UK	-0.14	0.17	-0.28	0.32	0.38	0.63	0.73	1.20

[b]LL and UL denotes the lower and upper limits.

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