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Project Valuation when There are Two Cashflow Streams

by

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Abstract

Some authors (Lewellen, 1977, Shall, 1972, Butters *et al.*, 1987, Laughton and Jacoby, 1993, Jacoby and Laughton, 1992, Salahor, 1998) advocate the separate discounting of different cashflows when calculating net present value (NPV). However, some textbooks (Brealy and Myers, 1991, Copeland and Weston, 1992) focus on calculating NPV by discounting the expected net after tax cashflow using the weighted average cost of capital (WACC) as the discount rate. We show that discounting the expected net after tax cashflow of a project using the WACC yields an incorrect project NPV. A new method for calculating project NPV's using a separate cashflow discounting method is proposed and applied to calculating the NPV's of some North Sea oil projects.

Key Words: Separate discounting, CAPM, oil projects
JEL Classificication: Q4, G12, G31

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1. Introduction

In this paper, an alternative method to using the weighted average cost of capital (WACC) as the discount rate for calculating net present value (NPV) is presented for investments which have a cost cashflow and a revenue cashflow each of which has different systematic risk. It is assumed that the cashflows have an equal capital asset pricing model (CAPM) expected after tax revenue beta across projects and an equal CAPM expected after tax cost beta across projects. North Sea oil projects are an example of this type of investment project.

Under these assumptions, it is shown in Section 2 that the expected net cashflow CAPM beta of a project will not generally be equal to the expected net cashflow beta of the portfolio. When the WACC is obtained from the expected net cashflow beta of the portfolio, discounting the expected net cashflow of a project using the WACC as the discount rate will yield an incorrect estimate of project NPV. The correct project NPV is obtained when the expected after tax revenue of a project is discounted using the required rate of return of this expected cashflow and the expected after tax cost cashflow is discounted using the required rate of return of this cashflow.

In Section 2 it is shown that value weighted CAPM betas can be used to find the implicit required rate of return on the expected after tax revenue cashflow, given the assumption that the required rate of return on the expected after tax cost cashflow is known. In this paper, the required rate of return of the expected after tax cost cashflow is assumed to be equal to the risk free rate of interest, which is known.

The expected after tax cost cashflow is discounted using the risk free rate because examined development cost factors in the oil industry have low systematic risk (Emhjellen, 1999, pp.152-157).[1] The risk free rate of return was used to discount expected after tax cost cashflow in the oil field examples in Jacoby and Laughton (1992) and the gas field examples in Salahor (1998).

When the WACC for a portfolio of projects, the expected cashflows of the projects, the taxation structure and the risk free rate of interest are known, an implicit required rate of return on the expected after tax revenue can be calculated. The project NPV's may then be calculated using the implicit required rate of return of the expected after tax revenue of the portfolio in discounting the expected after tax revenue cashflow of each project and the risk free rate of interest in discounting the expected after tax cost cashflow of each project.

In Section 3, project NPV’s are calculated using data for proposed oil projects in the North Sea using the WACC discounting method and the separate cashflow discounting method outlined above. The project NPV's of the two discounting methods are found to be substantially different for many of the projects. The separate cashflow discounting method is therefore recommended because it uses the correct discount rate for the expected after tax revenue of a project, given the assumption of correct discounting of the expected after tax cost cashflow of a project.

2. Value weighted betas

From the CAPM (Sharpe, 1964; Lintner, 1965 and Mossin, 1966) it is known that the *ex ante* beta determines the expected required rate of return on equity of one asset, $E(R_i)$, compared to other assets, since in the CAPM model the other variables (R_f , the risk free rate of return and $E(R_m)$, the expected rate of return on the portfolio) are the same for all assets. In this paper it is assumed that the company is 100% equity financed so that the expected required rate of return (WACC) is equal to the expected required rate of return on equity (see footnote 6).

The value additivity principle (Mossin, 1969, Shall 1972) implies that the beta of an asset (β_i) is a value weighted average of the individual cashflow betas. If the weights obtained using the present value (PV) of the expected after tax cost cashflow and the PV of the expected after tax revenue cashflow of a project expressed as ratios of the project NPV, are not the same as the PV weights of the portfolio formed in the same manner, the expected net cashflow CAPM beta of a project will not generally be equal to the expected net cashflow CAPM beta of the portfolio. This may be shown as follows.

We begin with the CAPM relationships:

$$E(R_i) = R_f + \beta_i (E(R_m) - R_f), \tag{2.1}$$

and

$$E(R_p) = R_f + \beta_p(E(R_m) - R_f). \tag{2.2}$$

In equation 2.2, β_p is the CAPM beta of the portfolio and $E(R_p)$ is the expected required rate of return on the portfolio .

From the value additivity principle, the present value of a company must be equal to the sum of the present values of its individual assets

$$V_p = \sum_{i=1}^{N} V_i . \tag{2.3}$$

In equation 2.3 V_i is the PV of the expected net cashflows of asset i discounted using $E(R_i)$ as the discount rate, and N is the number of assets.

Copeland and Weston (1992, p. 201) show that β_p can be written as a weighted average of the individual asset betas. If the weights are written as $w_i = \frac{V_i}{V_p}$, $\left(\sum_{i=1}^{N} w_i = 1\right)$ we have

$$\beta_p = \sum_{i=1}^{N} w_i \beta_i , \tag{2.4}$$

and the expected rate of return of the portfolio may be written as

$$E(R_p) = \sum_{i=1}^{N} w_i E(R_i), \tag{2.5}$$

Copeland and Weston (1992, p.173).

From the value additivity principle, the net present value of an asset is the sum of the present values of the component cashflows of the asset:

$$V_i = \sum_{j=1}^{M_i} X_{ij} , \tag{2.6}$$

where X_{ij} is the present value of the jth component of the cashflow of asset i discounted using the expected required rate of return of the jth component of asset i, $E(R_{ij})$, and M_i is the number of cashflow components of asset i. The expected rate of return on equity of an asset is equal to the sum of the value weighted expected rates of return of the individual component cashflows

$$E(R_i) = \sum_{j=1}^{M_i} w_{ij} E(R_{ij}). \tag{2.7}$$

In equation 2.7, $w_{ij} = \frac{X_{ij}}{V_i}$ and $\sum_{j=1}^{M_i} w_{ij} = 1$. From the CAPM relationship, the expected required rate of return of the jth component of the cashflow of asset i may be written:

$$E(R_{ij}) = R_f + \beta_{ij}(E(R_m) - R_f). \tag{2.8}$$

Substituting equation 2.8 into equation 2.7, β_i may be written as

$$\beta_i = \sum_{j=1}^{M_i} w_{ij}\beta_{ij}. \tag{2.9}$$

From equations 2.4 and 2.9, β_p may be written as

$$\beta_p = \sum_{i=1}^{N} w_i \sum_{j=1}^{M_i} w_{ij}\beta_{ij}. \tag{2.10}$$

The case of interest in this paper is where there are only two distinct cashflows generated by an asset. The expected after tax revenue cashflow and the expected after tax cost cashflow, where each of the expected after tax cashflow streams has a different beta, but the beta for each expected after tax cashflow is equal across assets. Thus, j=1,2, and to simplify notation, let $\beta_{i1} = \beta_D$ be the expected after tax revenue cashflow beta for asset i and $\beta_{i2} = \beta_C$ be the expected after tax cost cashflow beta for asset i. The value of the portfolio given by 2.3 may be written as

$$V_p = \sum_{i=1}^{N} V_i^D + \sum_{i=1}^{N} V_i^C. \tag{2.11}$$

In equation 2.11, V_i^D denotes the PV of the expected after tax revenue cashflow of asset i of the portfolio, and $V_i^C \leq 0$ denotes the PV of the expected after tax cost cashflow of asset i of the portfolio, (i=1,..N). The beta for asset i may be written as

$$\beta_i = w_i^D \beta_D + w_i^C \beta_C, \tag{2.12}$$

where $w_i^D = \frac{V_i^D}{V_i^D + V_i^C}$ and $w_i^C = \frac{V_i^C}{V_i^D + V_i^C}$,

and the beta of the portfolio may be written as

$$\beta_p = \sum_{i=1}^{N} w_i \left(w_i^D \beta_D + w_i^C \beta_C \right) . \qquad (2.13)$$

Multiplying out, we obtain

$$\beta_p = \sum_{i=1}^{N} w_i w_i^D \beta_D + \sum_{i=1}^{N} w_i w_i^C \beta_C . \qquad (2.14)$$

In equation 2.14,

$$\sum_{i=1}^{N} w_i w_i^D = \frac{\sum_{i=1}^{N} V_i^D}{\sum_{i=1}^{N} \left(V_i^D + V_i^C \right)} = \frac{V_p^D}{\left(V_p^D + V_p^C \right)} \quad \text{and} \quad \sum_{i=1}^{N} w_i w_i^C = \frac{V_p^C}{\left(V_p^D + V_p^C \right)} ,$$

where V_p^D is the present value of the expected after tax revenue cashflow of the portfolio, and V_p^C is the present value of the expected after tax cost cashflow of the portfolio.

From the CAPM and the value additivity principle, the use of the portfolio expected required rate of return in discounting the net expected after tax cashflow generated by the individual assets is only permissible when $\beta_p = \beta_i$. From equations 2.12 and 2.14 this implies that:

$$\beta_D \left(\frac{V_p^D}{V_p^D + V_p^C} \right) + \beta_C \left(\frac{V_p^C}{V_p^D + V_p^C} \right) = \beta_D \left(\frac{V_i^D}{V_i^D + V_i^C} \right) + \beta_C \left(\frac{V_i^C}{V_i^D + V_i^C} \right) \qquad (2.15)$$

Equation 2.15 shows that if the beta of the expected after tax cost cashflow is constant across assets and the beta of the expected after tax revenue cashflow is constant across assets, but these betas are unequal, the expected required rate of return of the portfolio cannot generally be used to discount the expected after tax net cashflow of an individual asset. The reason for this is that the weights formed by the present values of expected after tax revenue and expected after tax cost of a project relative to the NPV of the project will not generally be equal to the weights formed using the present values of the expected after tax revenue and expected after tax cost of the portfolio to the net present value of the portfolio. Thus, the two expected after tax cashflows (cost and revenue) of an asset must be discounted separately, using their respective expected required rates of return in order to calculate the correct NPV of the asset.

Value additivity in a multiperiod context

In a one period context, the following assumptions are required in order to calculate the NPV of the portfolio of oil projects and the individual projects:

1) The expected after tax revenue and cost cashflows for the individual projects and for the aggregate company portfolio are known.
2) The expected required rate of return for the expected after tax portfolio net cashflow is known.
3) The risk free rate of return, R_f and the expected rate of return on the market portfolio, $E(R_m)$ are known.
4) The expected required rate of return for the expected after tax cost cashflow is known and equal across projects (that is, the *ex ante* after tax cost cashflow beta, β_C, is equal across projects).
5) The expected required rate of return for the expected after tax revenue cashflow is known and is equal across projects (that is, the *ex ante* after tax revenue cashflow beta, β_D, is equal across projects).

In order to calculate the NPV's of multiperiod expected after tax cashflows using the discounted cashflow method based on WACC, Fama (1977) has shown that the expected required rates of return have to be constant for all periods. It is therefore assumed that the expected required rate of return on each type of project is constant for all periods (1 through T), and that the expected after tax cost and revenue cashflows of the portfolio and the projects are known or can be estimated.

In addition, it is required that assumptions 1 through 5 hold for all periods. From the value additivity principle, the NPV of a portfolio of multiperiod projects is the sum of the present values of the expected after tax cost and revenue cashflows

$$\sum_{t=0}^{T} \frac{-C_t + D_t}{(1+E(R_N))^t} = \sum_{t=0}^{T} \frac{D_t}{(1+E(R_D))^t} + \sum_{t=0}^{T} \frac{-C_t}{(1+E(R_C))^t} \ . \tag{2.22}$$

In equation 2.22, $E(R_N)$ is the expected required rate of return on the expected net after tax cashflow, $E(R_D)$ is the expected required rate of return on the expected after tax revenue cashflow, $E(R_C)$ is the expected required rate of return on the expected after tax cost cashflow, D_t is the expected after tax revenue cashflow in period t and C_t is the expected after tax cost cashflow in period t.

$E(R_N)$, $E(R_D)$ and $E(R_C)$ are assumed constant through time. From assumption 4, $E(R_C)$ is known. From assumption 2, $E(R_N)$ is known and from assumption 1, the expected after tax cost and revenue cashflows are known or can be estimated. The only unknown parameter for the company portfolio in equation 2.22 is $E(R_D)$, which is the implicit required rate of return on expected after tax revenue (IRRR) for the company portfolio.

Equation 2.22 may be written as

$$\sum_{i=1}^{N} w_i w_i^C = \frac{V_p^C}{\left(V_p^D + V_p^C\right)}, \qquad (2.23)$$

where i and IRRR are the respective expected required rates of return on C_t and D_t, and r is the expected required rate of return on the expected net after tax cashflow of the portfolio.

Rearranging and writing the left hand side of equation 2.23 as NPV,

$$\phi(\mathrm{IRRR}) = \mathrm{NPV} + \sum_{t=0}^{T} C_t (1+i)^{-t} - \sum_{t=0}^{T} D_t (1+\mathrm{IRRR})^{-t} \ . \qquad (2.24)$$

By finding the zero's of the polynomial given by equation 2.24 it is possible to determine the values of IRRR which satisfy ϕ(IRRR)=0. Since NPV, D_t, C_t and i are assumed to be known, it can be demonstrated that providing $\phi(0)<0$ and $\phi(1)>0$, there exists only one feasible solution to the polynomial.

Replacing the first two terms of the right hand side of equation 2.24 with a positive constant (b), we obtain

$$\phi(\mathrm{IRRR}) = b - \sum_{t=0}^{T} D_t (1+\mathrm{IRRR})^{-t} \ . \qquad (2.25)$$

Taking the first and second derivatives of 2.25 with respect to IRRR:

$$\phi'(\mathrm{IRRR}) = \sum_{t=0}^{T} t D_t (1+\mathrm{IRRR})^{-(t+1)} > 0 \ , \qquad (2.26)$$

$$\phi''(\mathrm{IRRR}) = \sum_{t=0}^{T} -(t+1) t D_t (1+\mathrm{IRRR})^{-(t+2)} < 0 \qquad (2.27)$$

Equation 2.26 is a positive, continuously decreasing function of IRRR, with limit 0 as IRRR$\rightarrow \infty$. Thus, ϕ(IRRR) is a strictly increasing, strictly concave function, so that if $\phi(0)<0$ and $\phi(1)>0$, which is the case for a portfolio of oil projects, there is a unique solution to ϕ(IRRR)=0, which we shall label as IRRR*.

3. Empirical results- valuation based on the WACC discounting method and the separate cashflow discounting method

The project data

The oil project data were provided by The Norwegian State Oil Company (STATOIL). The projects are all oil exploration projects in the North Sea and are subject to the Norwegian tax regime. The Norwegian government invites oil companies to apply for interests in exploration areas. Several oil companies might have an interest in projects in a particular exploration area depending on government allocations or purchases of ownership interests. Sales and purchases of oil projects are subject to the approval of the Norwegian government. A normal ownership structure of an oil project is from 1 to 5 companies participating in the planning and development of the project. One firm however, is responsible for the actual development of the project.

Data for 14 projects, A through N, are listed in Emhjellen and Alaouze(1999). Since the implicit required rate of return on the expected after tax revenue of the portfolio is required, the assumption is made that the WACC of the portfolio is known and that the oil projects belong to a hypothetical company. Annual data on expected investment cost cashflow, expected operating cost cashflow, and expected production as of August 1994 were provided for each of the projects. Project lives range from 5 years to 21 years. The project data provide the basis for the estimation of the expected after tax cashflows and the calculation of the net present values of the company portfolio and the individual oil projects.

Assumptions used in calculating project NPV

The expected after tax cashflows used in calculating present values are based on the Norwegian tax regime with no time lag for tax payments. Taxes are paid in the year they occur, and tax benefits are received in the year they occur. The assumptions are as follows:

1) A 50% special tax is applied to the offshore oil industry. The actual amount of special tax is determined by the tax base, which is calculated as follows:

Tax base = Revenue (d) - Operating cost (OC) - Depreciation tax shield (DTS) - Interest payments (IP) - Additional depreciation allowance (ADA). The special tax is equal to the tax base multiplied by 0.5.

2) An ordinary corporate tax rate of 28% is applied to $(d - OC - DTS - IP)$.

3) Total tax = special tax +ordinary tax

With the separate cashflow discounting method of NPV calculation, revenue cashflow tax and cost cashflow tax are given in 4) and 5) respectively.

4) Revenue cashflow tax = d $\times$ (0.5+0.28).

5) Cost cashflow tax = total tax - revenue cashflow tax.

6) The depreciation amount is 6 year linear depreciation applied to investment. This tax benefit can be claimed against other offshore oil revenue of the company. When calculating the expected after tax cost and revenue cashflows, the tax effect of the depreciation amount is treated as a reduction in costs.

7) The additional depreciation allowance (ADA) (applicable to offshore oil development) is 30% of investment treated as 6 year linear depreciation. When calculating the expected after tax cost and revenue cashflows, the tax effect of this depreciation amount is treated as a reduction in costs.

8) All calculations are in US dollars with a fixed exchange rate of 1 US dollar equal to 7 Norwegian kroner (NOK).[2]

9) Expected nominal cashflows (expected cost and expected revenue) were calculated assuming a constant expected inflation rate of 3.5%. An inflation rate of 3.5% was used by Statoil in calculating the real cost cashflows. All discounting was done on nominal expected cost and nominal expected revenue.

10) The nominal risk free rate of return is assumed equal to 6.5%.[3]

11) The expected oil price is assumed constant at a real $16US per barrel.[4]

12) The market risk premium ($E(R_m)$-R_f) is assumed equal to 6%.[5]

13) The company is 100% equity financed. This implies that the WACC reduces to the expected required rate of return on equity.[6]

14) The WACC for the portfolio of oil projects is assumed to be 10.5%.[7]

Results

A computer cashflow model was created using the Excel spreadsheet (version 5.0). Discounting based on one year periods was chosen for the model because the data are annual.

The project NPV's, the portfolio NPV (obtained using the WACC as the discount rate) and IRRR* (equal to 0.09843) were calculated, and it was found that the NPV of Project I was negative. When the expected after tax cost and revenue cashflows of the projects were discounted separately using IRRR* as the discount rate for revenue and 0.065 as the discount rate for costs, the NPV of project I was still negative.

For these reasons, project I was removed from the portfolio and the project NPV's, the portfolio NPV and IRRR* were recalculated. Project NPV's were also recalculated using the new IRRR* of 0.09873 as the discount rate for expected after tax revenue and 0.065 as the discount rate for expected after tax costs and the results shown in columns two and three of Table 1. The difference between the NPV's of each project obtained using the two discounting approaches is shown in column four of Table 1.

Table 1 shows that undervaluations of project NPV obtained using the WACC as the discount rate range from 19.6 million dollars (project N) to 0.3 million dollars for project H. Overvaluations range from 44.5 million dollars (project D) to 3.7 million dollars (project E). As assumed, the portfolio NPV is the same with both NPV calculation methods (1251.3 million dollars).

The separate cashflow discounting approach allows further refinement of the calculation of project NPV's. Because oil production falls with the life of a well, period t is included in the data for a project if the expected nominal after tax revenue (D_t) exceeds the expected nominal after tax operating cost (C_t^0),

$$D_t > C_t^0. \tag{3.1}$$

When the risk free rate of return (i) is less than the implicit rate of return on after tax revenue (IRRR*), it is possible for 3.1 to hold, but

$$D_t(1 + \mathrm{IRRR}^*)^{-t} < C_t^0(1 + i)^{-t} \ , \tag{3.2}$$

in which case the period t NPV for a project is negative.

Because oil projects have a large part of the cost (investment) early and production decreases toward the end of the production period, it was found, using the separate cashflow discounting method, that negative cashflow NPV's occurred, as expected, at the end of the production period. Projects B and D had two years of negative tail period NPV's, projects E, J, M and N had three years of negative tail period NPV's, while project C had four years of negative tail period NPV's. The other projects had positive NPV's in all production periods.

The periods with negative NPV's have an option value that is not accounted for by the separate cashflow discounting method. The value of various options related to a project cannot be calculated by the WACC discounting method or the separate discounting method. The calculation of option value estimates requires dynamic programming, option pricing or a simple decision tree approach (Dixit and Pindyck, 1994).

Using the WACC as the discount rate has led to overvaluation of the portfolio and the operation of some projects (at least for NPV calculation purposes) beyond their profitable life. After eliminating the periods with negative NPV's from the project data, the portfolio NPV must be recalculated using the WACC as the discount factor and the IRRR* recalculated.

Project NPV's calculated using the WACC as the discount rate, the portfolio NPV, IRRR* and the project NPV's calculated using the separate cashflow discounting method are shown in columns five and six of Table 1. The portfolio NPV is 1221 million, down 30.3 million dollars as a result of the removal of nominal expected cashflows that would contribute positively to NPV with the WACC method of calculation. The WACC project NPV's have only changed for those projects where end period expected cashflows were removed.

The separate discounting of the expected after tax cost and revenue cashflows by their respective discount rates (0.065 and 0.10099) has resulted in changes to the NPV's of all projects. Column seven of Table 1 shows the difference between the NPV's for each project obtained using the two discounting methods. Overvaluations range from 49.5 million dollars (project D) to 0.5 million dollars (project H), and undervaluations range from -18.9 million dollars (project N) to -0.1 million dollars (project F).

The two NPV methods also give a different ranking of the projects. The rankings of the following projects have changed: project D has changed from 2 to 5, project M from 3 to 2, project J from 4 to 3, project B from 5 to 4, project E from 6 to 8 and project G from 8 to 6.

4. Conclusions

It was shown that the separate discounting method described above results in better estimates of project NPV's than the WACC method when there are two distinct cashflows associated with a project, each of which has different systematic risk but the same systematic risk for each type of cashflow, across projects in a portfolio. In terms of absolute and percentage NPV errors, the NPV differences for the sample oil projects are substantial.

In addition, periods with negative NPV's were identified for some projects that did not occur using the WACC as the discount rate. This has implications for NPV calculations, and may also be of strategic importance in negotiations for the purchase and sale of oil-projects, as well as in negotiations for the lease of oil production ships or oil production rigs.

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Footnotes:

[1]Using deflated series on wages (onshore and offshore), steel prices and the Total Index of the Oslo stock exchange, Emhjellen (1999, pp.154-156) estimated the *ex-post* offshore wage cost beta to be 0.0052, the *ex-post* onshore wage cost beta to be 0.00747 and the *ex-post* steel cost beta to be 0.1609. An estimate of 0.1092 for the *ex-post* investment cost beta was obtained by forming a PV weighted average of the estimated *ex-post* onshore wage cost beta (to represent labour costs) and the estimated *ex-post* steel cost beta (to represent material costs). In forming the weighted average it was assumed that the PV of labour costs is 60% of the after tax cost cashflow and that the PV of steel costs is 40% of the after tax cost cashflow. The weights used were obtained from the cost structure of project N (see Section 3). Assuming a risk free rate of return of 6.5% and a market risk premium of 6% for Norway, the required rate of return on the expected after tax investment cost cash flow was estimated at 7.16%, which is close to the assumed risk free rate of return (6.5%).

[2]The average exchange rate for June 1994 was 1$US=7.08NOK, and the average July 1994 exchange rate was 1$US=6.87NOK (Statistics Norway, "Monthly Bulletin of Statistics" 1995, Table 92).

[3]The effective yield on Norwegian government securities (9-12 months) in August 1994 was 6.39%, up from 5.94% in July. (Central Bank of Norway).

[4]With the exception of the four month period (December 1993 through March 1994) when the average monthly Brent Blend oil price per barrel was below $14, the oil price has been in the range $15 to $18 per barrel (as of August 1994). Statistics Norway, "Monthly Bulletin of Statistics" 1995, Table 93.

[5]The average historical spread between the Treasury bill rate (real) and the real rate of return on common stocks in the US was 8.4% for the period from 1926 to 1988 (Ibbotson Associates Inc., 1989). The market risk premium for Norway was estimated to be 6% by Johnsen (1991). Therefore this was assumed to be the market risk premium.

[6]The weighted average cost of capital for an asset is given by: $WACC=E(R_e)(E/(E+D))+E(R_d)(D/(D+E)(1-\gamma)$, where: $E(R_d)$ is the expected required rate of return on debt (estimated from the rate paid on new issues of long term company bonds), γ is the marginal corporate tax rate, E is the market value of equity (estimated by the total number of outstanding shares times the share price), D is the market value of debt (estimated by the market value of company securities minus the market value of common stock) and $E(R_e)$ is the expected required rate of return on equity (estimated using the risk free rate of return plus the market risk premium for the asset given by the CAPM model). When D =0, the WACC reduces to the expected required rate of return on equity (Copeland and Weston 1992, Brealy and Myers 1991).

[7]The value of 10.5% for the WACC is close to that estimated for the Norwegian oil companies Saga Petroleum and Norsk Hydro using the unlevered CAPM betas of the companies. There is a simple relationship between the levered and unlevered betas for an asset given the assumptions of the CAPM: $\beta^L=\beta^U[1+(1-\gamma)D/E]$, where β^L is the levered beta, β^U is the unlevered beta and γ , D and E are as defined in footnote 6 (Butters *et al*. 1987, p351).

The levered betas of the Norwegian oil companies Saga Petroleum and Norsk Hydro were, as given in the Norwegian financial newspaper "Dagens Næringliv" on the 30th of December 1994, 0.9 and 1.0 respectively. The unlevered beta of Saga Petroleum was calculated to be 0.7121, and the unlevered beta of Norsk Hydro was calculated to be 0.7080.

The unlevered betas were estimated using the formula given above. A market debt/equity ratio of 119.92% (based on share price, number of outstanding shares and outstanding debt at the end of 1994), and a marginal tax rate of 78% were used to calculate the unlevered beta for Saga petroleum. A market debt/equity ratio of 80.08% (based on share price, number of outstanding shares and outstanding debt as of at the end of 1994), and an average marginal tax rate of 48.5% were used to calculate the unlevered beta for Norsk Hydro. The average marginal tax rate of Norsk Hydro was calculated by allocating the company debt according to the operating results of its four divisions for the year 1994: 55% of the interest on debt (for three of the divisions) was claimed against the onshore marginal tax rate (28%), and 45% of the interest on debt was claimed against the offshore marginal tax rate (78%).

Substituting an unlevered beta of 0.71 (β=0.71) into the CAPM formula for the expected required rate of return on equity ($E(R_e)=R_f+\beta(E(R_m)-R_f)$), gives a WACC of 10.76% (10.76%=6.5%+0.71$\times$6%). Thus, the WACC assumed for the portfolio of oil projects is very close to the WACC of the two representative oil companies when calculated using the unlevered betas of these companies.

Table 1: NPV results

	Discounting Method			Discounting Method		
Project	**WACC**	**Separate**	**Difference**	**WACC**	**Separate**	**Difference**
A	63.3	68.6	-5.3	63.3	66.2	-2.9
B	107.5	103.0	4.5	103.3	100.7	2.6
C	50.3	41.2	9.1	45.7	44.2	1.5
D	139.4	95.0	44.5	136.3	86.8	49.5
E	65.7	62.1	3.7	64.2	61.1	3.1
F	26.7	27.9	-1.2	26.7	26.7	-0.1
G	58.1	70.3	-12.2	58.1	67.6	-9.5
H	30.5	30.8	-0.3	30.5	30.0	0.5
J	119.5	125.6	-6.1	112.4	120.1	-7.7
K	13.2	14.6	-1.4	13.2	14.2	-1.0
L	17.0	23.3	-6.4	17.0	22.7	-5.8
M	137.3	146.6	-9.3	134.1	145.6	-11.5
N	422.7	442.4	-19.6	416.3	435.2	-18.9
Portfolio	1251.3	1251.3	0.0	1221.0	1221.0	0.0
IRRR*	0.09873			0.10099		

Note: Columns 2, 3 and 4 refer to project data with project I removed, columns 5, 6 and 7 refer to project data with project I and periods with negative expected cash flow NPV's (obtained with separate discounting) removed. The NPV's calculated using separate discounting were obtained using a nominal discount rate of 0.065 in discounting the expected after tax cost cashflows and a nominal discount rate of IRRR* in discounting the expected after tax revenue cashflows.