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**Oil price risk, prudent fiscal policy,  
and generational accounting**

by

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# Oil price risk, prudent fiscal policy, and generational accounting

By

Erling Steigum\*

## **Abstract**

This paper proposes a method for adjusting generational accounts for oil price risk. The analytical framework is an overlapping generations model of a small open economy with an exhaustible resource wealth owned by the government. Adopting a CRRA welfare function, the optimal tax policy involves intergenerational risk sharing and fiscal prudence in terms of precautionary public saving. It is the optimality of fiscal prudence that warrants a risk adjustment of the petroleum wealth. The risk adjustment factor is derived from a quadratic approximation of the first-order condition for social welfare maximization. The method is illustrated using Norway as an example.

*Key words:* Petroleum wealth, precautionary saving, fiscal prudence, generational accounting.

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## 1. Introduction

This paper derives implications of government asset income risk for optimal intergenerational tax policy and suggests a method for adjusting generational accounts for this source of risk. In particular, our analysis applies to oil-producing countries in which a significant source of government revenue is oil and natural gas production. In Norway, where the Government is the main stakeholder in the petroleum sector, coping with the large oil revenue risk has long been recognized as a very challenging problem for fiscal planning.

In recent years, the generational accounting method has been widely used around the world as a tool of measuring the sustainability of current fiscal and social policy; see for example Auerbach, Kotlikoff and Leibfritz (1999) and Raffelhüschen (1999).<sup>1</sup> This method utilizes the *intertemporal budget constraint* of the government to calculate the effects of maintaining the present tax and transfer policy into the indefinite future. In this way, the method permits a measurement of the extent to which future fiscal adjustments will be necessary to meet the constraint. The timing of the necessary fiscal adjustments will be important for economic efficiency as well as for the distribution of welfare among present and future generations.<sup>2</sup>

It is well known that the combination of population aging and pay-as-you-go pension finance renders the conventional norm of a zero budget deficit insufficient to prevent future tax hikes or spending cuts. Generational accounting is forward-looking, however, and therefore accounts for the increase in the implicit social security debt of the government (and other old-age related spending) due to expected future demographic change. Hence, it permits a more reliable assessment of the long-term fiscal policy stance than the conventional norm.

Generational accounts for Norway were first introduced by Auerbach et al. (1993). The government adopted generational accounting in 1994. It is now regularly used when preparing the annual national budgets and the four-year long term planning reports for the Parliament. Norway faces long-term fiscal challenges similar to those of other OECD-countries, such as population aging and increased social security spending when the baby boom cohorts retire. As indicated

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<sup>1</sup> The method was introduced by Auerbach, Gokhale and Kotlikoff (1991).

<sup>2</sup> For a small open economy, general equilibrium effects running through endogenous factor prices are likely to be less important than for large economies, see Fehr and Kotlikoff (1997). For a quantitative assessment of generational accounting by means of a social welfare function, see Raffelhüschen and Risa (1997).

above, the uncertain future petroleum revenues of the government represent an additional complication in assessing fiscal policy, highlighting an unresolved problem: How to adjust for the considerable oil revenue risk? This paper suggests a simple framework in which to account for this risk in a systematic way.<sup>3</sup>

Most governments around the world must live with the fact that future revenues from taxation and assets are uncertain. In theory, one could imagine optimal international risk-sharing arrangements, but this is not what we observe.<sup>4</sup> Government risk exposure is often larger for small countries with a narrow industrial base, such as many developing countries, than for larger industrialized countries. Developing countries that are heavily dependent on primary exports are particularly vulnerable to large external trade shocks. Quite often, their governments have considerable stakes in important export sectors, particularly oil exporting countries. Collier and Gunning (1999) have collected a considerable number of case studies of large trade shocks from Africa, Latin America and Asia. Examples include the oil shocks in Venezuela and Indonesia in the 1980s and the mining shocks in Zambia, Botswana and Bolivia. In all these cases the shocks affected government revenues adversely. An extreme example is Venezuela, where 90 percent of the export revenues and 60 percent of the government's total revenues used to come from the oil sector (Hausmann, 1999). During the turbulent 1980s, Venezuela's GDP per capita decreased by 18 percent.

The dramatic economic crises in Finland and Sweden in the early 1990s demonstrated that the governments of small, industrialized welfare states are also exposed to substantial risks due to rapid spillovers of macroeconomic risks to the tax-transfer system. Both countries ran fiscal surpluses during the preceding boom in the late 1980s. In the midst of the crisis in 1993, the public sector budget deficits had grown to 13 percent in Sweden and 10 percent in Finland. In both

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<sup>3</sup> As emphasized by Kotlikoff (2002), since the riskiness of taxes, spending and transfer payments will normally differ, the theoretically appropriate risk adjusted rates at which to discount future taxes, spending and transfers would also differ. Incomplete insurance arrangements between generations could also justify generation-specific adjustments of discount rates. He concludes that "Unfortunately, the size of these risk adjustments remain a topic for future research." (Kotlikoff, 2002).

<sup>4</sup> For a recent discussion of the home bias puzzle and issues related to international risk sharing, see Davis, Nalewaik and Willen (2000). Shiller (1999) also relates these issues to social security reforms to increase risk sharing within and between cohorts. It has been suggested that the Norwegian government should try to diversify part of its oil price risk, see Thøgersen (1994) for a discussion. An empirical analysis by Thøgersen (1997a) indicates that uninsured idiosyncratic oil price risk accounts for parts of the low consumption correlation among European OECD countries.

countries, these developments forced the governments to cut spending substantially during the 1990s despite severe unemployment problems.

A closely related problem is the so-called *Dutch Disease*, i.e. de-industrialization and other structural problems related to a booming petroleum sector and the accompanying increase in government and private spending, see Corden (1984).<sup>5</sup> In recent literature, more attention has been given to dynamic fiscal policy issues. For example, Mansoorian (1991) shows that if generations are not linked altruistically, a private resource discovery could even make future generations poorer than they would have been otherwise. This result is due to wealth effects that only benefit the generations that discovered the resource wealth, and suggests an important role for the government to redistribute welfare forward in time. However, rent-seeking and policy failures associated with huge, temporary government resource revenues appear to be widespread. Thus, empirical studies indicate that the growth performance of resource rich countries is more dismal than that of other countries, see Sachs and Warner (1995) and Gylfason, Herbertsson and Zoega (1999).

There are only a few attempts in the literature to introduce uncertainty explicitly in an optimising framework for fiscal policy strategy. From the theory of saving under uncertainty, it has been well known that strictly convex marginal utility generates a precautionary demand for saving, see Leland (1968), Sandmo (1970) and Drèze and Modigliani (1972). This idea was developed further by Caballero (1990, 1991) and others, utilizing the fact that constant absolute risk aversion (CARA utility) permits closed form expressions for optimal consumption and wealth accumulation. Thøgersen (1997b) adopted this approach in an overlapping generations framework to analyse private precautionary saving when uncertain government oil revenues generate stochastic tax rates. He showed that if a temporary tax cut is accompanied by increased oil extraction, higher precautionary saving in the private sector may offset some of the wealth effect of the tax policy for present generations.

An early contribution to the theory of fiscal planning under uncertainty was made by Leif Johansen (1980), who also adopted a CARA social welfare function to facilitate numerical analysis. Interestingly, Leif Johansen's work had an impact on Norwegian national budgeting when

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<sup>5</sup> It is interesting that in Norway, the rate of unemployment remained remarkably low despite a considerable real appreciation in the 1970s. Norway's first experience with the Dutch Disease was therefore quite different from what had been observed in Holland, U.K. and Mexico, see Steigum (1983).

oil revenues accelerated in the late 1970s. In Aslaksen et al. (1990), the CARA welfare function was used to adjust for oil price risk.<sup>6</sup> Empirically, the assumption of constant absolute risk aversion is questionable, however, because it implies that relative risk aversion increases with wealth. According to Campbell and Viceira (2001), the long-run behaviour of most market economies suggests that relative risk aversion cannot depend strongly on wealth. The empirical evidence thus limits the usefulness of the CARA utility function as the basis for normative analysis of fiscal policy and suggests that an assumption of constant relative risk aversion (CRRA) is more relevant. In our analysis, we adopt the assumption of CRRA utility. The CRRA model is also the standard paradigm of finance theory. A recent paper by Engel and Valdés (2000) derives implications of uncertain government oil revenues for optimal fiscal policy when the social welfare function is based on CRRA utility. They perform various approximations to derive closed form solutions. The present paper different in two important respects. First, our model is designed to derive implications for generational accounting. It therefore has an overlapping generation structure and captures lifecycle saving, while Engel and Valdés (2000) assume that individuals only live for one period. And secondly, we perform a quadratic approximation on the first-order conditions for a social optimum to derive a closed form solution, while Engel and Valdés (2000) give priority to simpler approximation methods in order to allow for a somewhat broader range of applications.

In the next Section we take a closer look at the fiscal policy challenges of Norway after the discovery of oil and natural gas in the North Sea. Our analytical framework developed in Section 3 is an overlapping generations model of a small open economy with a risky natural resource wealth, where the government maximizes expected social welfare exhibiting a precautionary savings motive. In Section 4 we use the model to analyse optimal fiscal policy and intergenerational risk sharing under uncertainty, with a particular focus on the precautionary savings motive. In Section 5 we adopt the constant relative risk aversion utility function and perform a quadratic approximation of the first-order condition to derive closed forms. This approximation method forms the basis for the risk adjustment of generational accounts in Section 6. The basic idea is to adjust future risky revenues such that a zero deficit in the generational accounts corresponds to the social optimum. This risk adjustment does not exclude that the government could wish to depart

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<sup>6</sup> See also Aslaksen and Bjerkholt (1987).

from full generational balance for other reasons. Up till now, there has been no systematic and transparent risk adjustment of future petroleum revenues. The proposed method shows how such a risk adjustment could be related to parameters characterizing the probability distribution of future petroleum revenues as well as to the coefficient of relative risk aversion. The method is illustrated by a numerical example calibrated to Norwegian data in Section 7. In the final Section we discuss some shortcomings and possible extensions of the analysis.

## **2. Norway's petroleum wealth and fiscal policy challenges**

In the early 1970s, the government launched an ambitious investment programme to build up its petroleum sector based on the rich discoveries of oil and natural gas in the North Sea. The rapid expansion of the petroleum sector was financed by huge capital imports; see Figure 1. The rate of investment is still substantial, and the production of oil and gas has not peaked yet. So far, the petroleum sector has turned out to be very profitable. Norway is now the third largest exporter of oil and gas in the world (after Saudi Arabia and Russia) and is running a current account surplus of about 13 percent of GDP. Norway's purchasing power corrected GDP per capita has become one of the highest in Europe. In 2000, the share of exports of petroleum in total exports was 46 percent and the share of petroleum production in total GDP was above 23 percent.

From the start, the government's involvement in the petroleum sector has been very strong. More than 80 percent of the petroleum revenues have been at the government's disposal and the share has been slowly increasing over time.

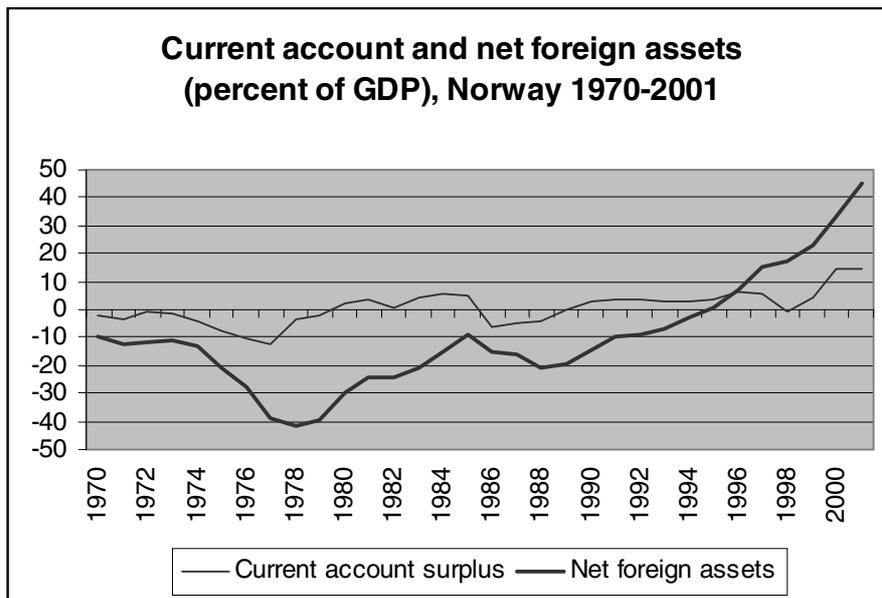


Figure 1 Current account and net foreign assets

The government petroleum wealth had an immediate impact on economic, regional and social policy. During the 1970s, monetary and fiscal policy became very expansionary, and the government built up one of the world's most generous social protection systems. Also subsidies to the agriculture and ailing industries were increased to counteract the process of structural change. The ambitious policies prevented immediate unemployment, but it also had a downside in terms of less macroeconomic stability, which also was related to a monetary policy regime involving extensive credit rationing and regulations of nominal interest rates and credit flows. Due to the non-indexed tax law and increasing inflation, after-tax real interest rates were negative to a considerable degree in the 1970s and the first half of the 1980s. The vulnerability of the Norwegian "oil-fuelled" welfare state was illustrated in 1986 when the oil price crash reduced Norway's national income by 10 per cent.<sup>7</sup> The shock reduced Norway's estimated petroleum wealth from more than 250 percent of GDP in 1985 to less than 100 per cent of GDP in 1987.

<sup>7</sup> The developments in the 1970s and 1980s suggest that Norway's real GDP is positively correlated with the real oil price. For empirical evidence based on a VAR approach, see Mork, Mysen and Olsen (1990). They also find that the corresponding correlation in most oil-importing OECD countries is negative, with the exception of U.K., where the correlation is positive, but not significantly different from zero.

This explains the decline in the fiscal surplus from 8.1 percent of GDP in 1985 to 1 percent in 1987, despite a serious fiscal policy restraint.

Nevertheless, the Norwegian economy did recover from the turbulent 1970s and 1980s with huge current account and fiscal surpluses in the second half of the 1990s.<sup>8</sup> A Government Petroleum Fund managed by the Central Bank was established in 1990. According to the rules, transfers from the Government to the fund can only take place if the Government runs budget surpluses. Moreover, deficits have to be financed by running down the fund's capital. The fund invests in foreign stock and bonds. Its capital is now about 50 percent of GDP and is expected to exceed Norway's GDP before 2010. Despite the rapid extraction of oil, the estimated present value of the remaining petroleum reserves on the continental shelf is large. Based on a 4 percent real rate of interest, the corresponding permanent income to the government is approximately 6 percent of GDP in 2002.

The idea of building up a petroleum fund to prevent excessive consumption in periods of large oil revenues goes back to the public debate in the early 1980s, but many economists argued that such a fund was not politically feasible. When the rules of the Petroleum Fund were established in 1990, it was uncertain whether the fund would ever receive transfers from the government. Most policymakers were therefore taken by surprise by the large oil revenues earned in 2000 and 2001, see Figure 1.

In the 1970s and 1980s, the optimal extraction and spending of oil revenues in an open economy received a lot of attention among Norwegian economists; see for example Aarrestad (1978, 1979), Bjerkholt, Lorentsen and Strøm (1981) and Hoel (1981).<sup>9</sup> Later, this was followed up by an impressive research effort by the Statistics Norway; see the volumes edited by Bjerkholt and Offerdal (1985) and Bjerkholt, Olsen and Vislie (1990).<sup>10</sup> In the 1990s, more attention was given to long-run fiscal policy effects and intergenerational welfare issues. Steigum and Thøgersen (1995) used a calibrated OLG-model of the Norwegian economy to assess the generational implications of various fiscal policy strategies for spending petroleum revenues in Norway. This study highlights the importance of building up a fund of foreign assets to meet the aging problem

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<sup>8</sup> As a result, the generational imbalance in fiscal policy declined substantially; see Steigum and Gjersem (1999).

<sup>9</sup> Most of the early debate among Norwegian economists and policymakers generated contributions written in Norwegian. A notable example is Eide (1974). For a more complete review of the literature; see Thøgersen (1994).

<sup>10</sup> In the late 1980s, there has also been considerable interest in adopting the option pricing approach to evaluating petroleum investment projects and other irreversible decisions, see for example Ekern (1988).

as well as preventing that only the present voters benefit from the resource wealth. The papers by Steigum (1992) and Steigum and Thøgersen (2002) look at the problem of recovering from an unexpected adverse government wealth shock (for example, triggered by a large persistent drop in the oil price), when sectoral costs of adjustment prevent a fast restructuring of the economy. Such costs involve a gradual build-up of the sector producing other tradables than petroleum. The main conclusion is that monetary policy should be sufficiently flexible to allow a sudden real depreciation and a low consumption real rate of interest during the adjustment process. Fiscal policy should be tight, but still allow budget deficits to protect the welfare of the present generations. These papers did not account for an uncertain future, however.

The remaining reserves of oil and natural gas do not appear in the National Accounts of Norway. Consequently, in years of temporarily high petroleum revenue such as in 2000 - 2002, the National Accounts give an exaggerated picture of the government's income as well as national income. In the Norwegian generational accounts, the Government's estimate of the petroleum wealth is included in the government's wealth and the corresponding petroleum income concept is therefore the permanent income from the wealth of petroleum reserves on the continental shelf. Still, the problem of the large uncertainty of the future cash flow from the petroleum sector is not handled in a satisfactory way when the government makes its petroleum wealth estimate. It has been argued that one should adjust for risk by including a risk premium in the wealth calculation, but it is not obvious on what basis such a risk adjustment should be performed. In the next Section, this problem will be addressed.

### **3. Analytical framework**

As our analytical framework, we consider an overlapping generations model of a small open economy. In each period, there are two overlapping generations, the young workers and the old retirees. There is no population growth, and the young generation's labour supply is exogenous and constant over time. There are two sectors producing internationally tradable goods, a mainland sector (ML) and a petroleum sector (PE). Mainland net output ( $Y$ ) is the *numeraire* good. There are no stochastic elements associated with the ML-sector.  $Y$  is used for private and

public consumption, investment and net exports. There is a government that levies a lump sum tax on workers, pay a pension to the old, and supplies a public consumption good. The government owns the petroleum sector. For simplicity, we assume that the PE-sector does not use labour. Moreover, petroleum is not used domestically. Due to price uncertainty, the PE-sector generates a stochastic net revenue. After a certain number of periods, however, the production will drop to zero and all uncertainty will disappear. The exogenous risk-free real rate of interest is  $r > 0$ . Both the government and the private sector have free access to risk-free international lending and borrowing.

### 3.1 Technology, preferences and budget constraints

We normalize the exogenous labour supply to one and assume for simplicity no technological progress. Exogenous and deterministic technological progress does not change the nature of the results, but makes the notation more cumbersome. The technology of the ML-sector is represented by an aggregate CRS production function:

$$(1) \quad y_t = F(k_t, 1) - \delta k_t, \quad (t = 0, 1, 2, \dots)$$

where  $y_t$  is net output,  $k_t$  is the privately owned stock of capital at the beginning of period  $t$ , and  $\delta > 0$  is the constant rate of capital decay. From the marginal productivity condition  $F_k = r + \delta$ , it follows that  $k$  and  $y$  are constant. Hence, the competitive real wage ( $w = y - rk$ ) is also constant over time.

The petroleum sector is active for  $T + 1$  periods. From period  $T + 1$  and onwards, there is no oil extraction. We express the net revenue (cash flow) from the petroleum sector as  $x_t P_t$ , where  $x_t$  is the quantity of oil produced per worker, and  $P_t$  is the relative oil price in period  $t$ ,  $t = 0, 1, 2, \dots, T$ . We follow Aslaksen et al. (1990) and Lund (1990) and consider the time profile of oil extraction as deterministic and exogenous. Future oil prices are however random variables that are generated by a stochastic process. Let  $E_t[P_{t+j}]$  be the oil price in period  $t + j$  expected in period  $t$ . The expectation operator  $E_t$  is conditional on the information known in period  $t$ , including the period  $t$  oil price (i.e.,  $E_t[P_t] = P_t$ ). The present value of future net oil revenue is the stochastic *petroleum wealth*, which we denote by  $W_t$ :

$$(2) \quad W_t = \sum_{j=t}^T R^{j-t+1} x_j P_j \quad \left( R = \frac{1}{1+r} \right)$$

Let  $\tau_t$  be the tax paid by workers,  $\pi_t$  the (after-tax) pension given to the old, and let  $g$  be the government's exogenous (and constant) consumption per worker. The government's stock of net financial assets per worker at the beginning of period  $t$  is denoted  $B_{g,t}$ . The government's budget constraint in period  $t$  is:

$$(3) \quad B_{g,t+1} - B_{g,t} = rB_{g,t} + \tau_t + x_t P_t - \pi_t - g.$$

Using (2) and (3), and excluding Ponzi game schemes, we can express the intertemporal budget constraint of the government in period 0 as:

$$(4) \quad \sum_{t=0}^{\infty} R^t g = \frac{(1+r)g}{r} = (1+r)(B_{g,0} + W_0) - \pi_0 + N_0 + \sum_{t=1}^{\infty} R^t N_t,$$

where  $N_t = \tau_t - R\pi_{t+1}$  is the present value of lifetime net taxes (taxes minus transfers) paid to the government by generation  $t$  ( $t = 0, 1, 2, \dots$ ).  $N_t$  is often called the *generational account* of generation  $t$ . Equation (4) is the basis for generational accounting. This method essentially uses (4) to calculate the last term (the tax burden placed on future generations) residually, under the assumption that the expectations of  $N_1, N_2, N_3, \dots$  are all equal, and that the present fiscal policy only applies to the generations presently alive. If we abstract from uncertainty and assume that the government wealth is known with certainty in period 0, *generational balance* is defined as a fiscal policy which involves  $N_0 = N_1 = N_2 = N_3 = \dots$ , i.e. the same net lifetime tax burden for all generations.<sup>11</sup> Then the present fiscal policy can be sustained without future fiscal restraints. We shall return to the question of generational balance below.

Turning to the behaviour of consumers, we express the lifetime utility of a member of generation  $t$  as

$$(5) \quad U_t = u(c_t) + \beta u(d_{t+1}), \quad (u' > 0, \quad u'' < 0),$$

where  $c_t$  is consumption as young,  $d_{t+1}$  is consumption as old, and  $\beta$  is the utility discount factor. Since marginal utility is decreasing ( $u'' < 0$ ), the consumers are risk adverse. It is well known that risk aversion alone does not guarantee that consumers will save more in response to more risk, i.e.

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<sup>11</sup> The method also involves an adjustment for per capita economic growth and population dynamics.

engage in precautionary saving. This question is related to the concept of *prudence*, see Gollier (2001). An agent is prudent if adding an uninsurable zero-mean risk to his future wealth raises his optimal saving. It can be shown that an agent is prudent if and only if the marginal utility of future consumption is strictly convex. Then the third-order derivative of utility is positive ( $u''' > 0$ ). It can also be shown that if absolute risk aversion is decreasing, the consumer must necessarily be prudent, see Gollier (2001, chapter 16). In what follows we will assume that the preferences exhibit positive prudence. As a benchmark, we will also consider quadratic utility, which is well known to generate zero precautionary saving.<sup>12</sup>

The young saves  $w - \tau_t - c_t$  and this accumulated private pension fund represents total private wealth ( $k + B_{p,t+1}$ ) at the start of period  $t + 1$ . ( $B_p$  is the stock of net private financial assets per worker). The old consumes

$$d_{t+1} = (1 + r)(k + B_{p,t+1}) + \pi_{t+1},$$

leaving no bequest. To simplify, we assume that the subjective discount factor  $\beta$  is equal to  $R$ . The consumers maximize (expected) utility subject to the budget constraint:

$$(6) \quad c_t + R d_{t+1} = w - N_t \quad (N_t = \tau_t - R \pi_{t+1}).$$

For generations  $t < T$ , we assume that the pension  $\pi_{t+1}$  depends on the oil price in period  $t + 1$ , permitting optimal risk sharing among generations, see below. For these generations,  $N_t$  is a random variable, and the first-order condition for optimal consumption and saving is

$$(7) \quad u'(c_t) = E_t[u'(d_{t+1})] \quad (t = 0, 1, 2, \dots, T - 1)$$

Generation  $T$  and all subsequent generations do not face uncertainty. They choose a flat consumption profile:

$$(8) \quad c_t = d_{t+1} = \left( \frac{1+r}{2+r} \right) (w - N_t), \quad (t = T, T + 1, \dots)$$

We shall assume that the generational accounts of all generations from  $t = T$  and onwards are identical. Total national wealth per worker is  $k + W_t + B_t$  at the beginning of period  $t$ .  $B_t = B_{p,t} + B_{g,t}$  is the country's net foreign assets (per worker).

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<sup>12</sup> Note that local prudence and local risk aversion are independent concepts in the sense that one could be risk-averse and prudent, risk-averse and imprudent, and even risk-lover and prudent. However, most utility functions that are used to analyse household behaviour under risk imply that consumers are both risk averse and prudent. Both risk aversion and prudence are overwhelmingly supported by data, see for example Guiso, Jappelli and Terlizzese (1996).

The intertemporal budget constraint for the entire economy is

$$(9) \quad \sum_{t=0}^{\infty} R^t (c_t + d_t) + \frac{(1+r)g}{r} = (1+r)(k + B_0 + W_0) + \frac{(1+r)w}{r}.$$

This constraint states that the present value of all private and public consumption (per worker) must be equal to the initial wealth (including the corresponding wealth income in period 0), plus the present value of wage income.

The intertemporal budget constraint of the private sector is found by deducting the government's constraint (4) from (9):

$$(10) \quad \sum_{t=0}^{\infty} R^t (c_t + d_t) = (1+r)(k + B_{p,0}) + \frac{(1+r)w}{r} + \pi_0 - N_0 - \sum_{t=1}^{\infty} R^t N_t.$$

### 3.2 Welfare optimum under certainty

Following Calvo and Obstfeld (1988), the social welfare function is written as:

$$(11) \quad U_{s,0} = \frac{U_{-1}}{\beta} + \sum_{t=0}^{\infty} \beta^t U_t = \frac{u(c_{-1})}{\beta} + \sum_{t=0}^{\infty} \beta^t (u(d_t) + u(c_t)).$$

The first term on the RHS is exogenous since  $c_{-1}$  is history. For simplicity, we don't optimise with respect to public consumption, and therefore  $g$  does not appear explicitly in (11). We assume that the social discount factor is equal to the private ( $\beta$ ) and that  $\beta = R$  as before.<sup>13</sup>

Let us first consider the case without uncertainty. The government maximizes (11) with respect to  $d_0, c_0, d_1, c_1, \dots$ , subject to the budget constraint (9). It is obvious that the optimal policy involves identical consumption across generations:  $d_0 = c_0 = d_1 = c_1 = \dots$ . Using (9), optimal consumption can be expressed as the permanent income:

$$(12) \quad c = d = \frac{1}{2} [w - g + r(k + B_0 + W_0)]$$

By using its fiscal instruments, the government can chose  $\pi_0$  and a constant generational account  $N_0 = N_1 = N_2 = N_3 = \dots$  such that the optimum is attained and the intertemporal budget constraint

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<sup>13</sup> If  $\beta > R$ , it is straightforward to show that the government wants to go into debt and the welfare of generations far into the future becomes very small. The generational accounts will always be in deficit and the generational accounts will increase over time. In this case, the assumption of a constant  $r$  appears to be too extreme. On the other hand, if  $\beta < R$ , the government wants future generations to become wealthy and it builds up an ever-increasing stock of assets. This corresponds to surpluses of the generational accounts. Now the country continuously builds up its foreign assets. We consider this case to be of minor interest.

of the government (4) is fulfilled. This policy clearly involves generational balance.<sup>14</sup> Note that under certainty, the scale of the pension system is arbitrary in the sense that a given  $N = \tau - R\pi$  can be obtained by infinitely many combinations of  $\tau$  and  $\pi$ . The scale of the pension system only affects the distribution of net financial assets between the government and the private sector, not the country's net foreign assets or consumer welfare.

The optimal policy under certainty implies that the sum of the country's total assets is constant over time. Then, since  $k$  is constant,  $B_0 + W_0 = B_1 + W_1 = \dots = B_T + W_T = B_T = B_{T+1} = \dots$ . Before period  $T + 1$ , the development in  $B_t$  must therefore be a mirror image of the development of the petroleum wealth. In periods when the petroleum wealth is increasing (which is likely when the petroleum sector is young), the country runs current account deficits. In periods of declining petroleum wealth, which must surely happen towards the end of the petroleum era, there are compensating current account surpluses. As long as the scale of the pension system is fixed, the stock of net financial assets of the private sector is constant. The government must therefore run corresponding deficits and surpluses in order to keep its total wealth constant. At the beginning of period  $T + 1$ , the petroleum wealth has been transformed into a government "petroleum fund" of foreign assets. This is clearly necessary in order to make the consumption level sustainable in the post-petroleum period.

#### 4. Oil price risk

We now explicitly take into account that oil revenue is stochastic due to oil price risk. Oil price news may change the distribution of future oil prices, for example due to positive persistence in the stochastic process. Oil prices are in fact quite persistent at the annual and smaller frequencies, but at the present level of aggregation of time periods, the persistence may be small.<sup>15</sup>

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<sup>14</sup> The generational accounting method usually takes into account steady state per capita growth by adjusting tax burdens for growth. To see what is involved, consider a CRRA utility function with the coefficient of relative risk aversion equal to  $\theta$  and a positive rate of exogenous (labour-augmenting) productivity growth ( $\lambda$ ). Now we express all variables relative to labour measured in efficiency units. It can be shown that if the social discount rate is  $(1 + r)(1 + \lambda)^{-\theta-1}$ , the optimal policy is to keep growth-adjusted taxes constant. This policy generates a modified golden rule growth path where consumption per capita is growing over time at the rate  $\lambda$ .

<sup>15</sup> See Schwartz (1997) and Pindyck (1999).

The expected petroleum wealth in period 0 (after  $P_0$  is known), can be found from (2):

$$(13) \quad E_0[W_0] = Rx_0P_0 + R^2 \sum_{j=1}^T R^{j-1} x_j E_0[P_j]$$

The change in the expected petroleum wealth from period 0 to period 1 can be expressed as:

$$(14) \quad E_1[W_1] - E_0[W_0] = -Rx_0P_0 + rE_0[W_1] + \{E_1[W_1] - E_0[W_1]\}$$

The first term on the RHS of (14) is the oil revenue extracted in period 0 discounted back to the start of period 0. It enters with a negative sign. The second term is positive, however. It reflects that the future stream of oil revenue is coming closer when one period passes. This generates a return  $rE_0[W_1]$ . The last term is the effect of new information about future oil prices on the evaluation of next period's expected petroleum wealth. This term can be positive or negative. Even if no new information arrives such that the last term is zero, expected petroleum wealth can increase over time due to the second term. Up until recently, this has been the normal case in Norway. When the last period  $T$  gets close, the expected petroleum wealth will of course approach zero such that the first term in (14) must outweigh the second.

In order to express the change in the variance of the petroleum wealth, we note that  $Var_0[W_0] = R^2 Var_0[W_1]$ . In contrast to expected wealth  $E_0[W_0]$ , the variance is independent of the current oil price  $P_0$ . The variance of  $W_1$  can be expressed as

$$(15) \quad Var_0[W_1] = R^2 \{x_1^2 Var_0[P_1] + Var_0[W_2] + x_1 Cov_0[P_1, W_2]\}$$

If there is oil price persistence, the covariance term will be positive and add to the uncertainty created by a large production of oil ( $x_1$ ) in the next period. Using (15), the change in the variance of next period's petroleum wealth is

$$(16) \quad Var_1[W_2] - Var_0[W_1] = -R^2 \{x_1^2 Var_0[P_1] + x_1 Cov_0(P_1, W_2)\} + (Var_0[W_2] - Var_0[RW_2]) + (Var_1[W_2] - Var_0[W_2])$$

This expression has an economic interpretation that corresponds to the interpretation of (14) for the change in expected petroleum wealth. The first negative term on the RHS of (16) is the reduction in variance due to the fact that the uncertainty about next period's oil price is resolved. If next period's oil production is large, this term will be substantial. This term corresponds to the first term on the RHS of (14), expect that the latter term is related to  $x_0$  not  $x_1$ . The second term on the RHS of (16) is positive and reflects that the uncertainty about  $W_2$  is coming closer when one period passes. This term corresponds to the second term on the RHS of (14). And finally, the third

term on the RHS of (16) is the effect of new information on the evaluation of the variance of  $W_2$ . This term corresponds to the third term in (14) and can have either sign. Even if the third term is zero (no new information about the variance of  $W_2$ ), we cannot exclude the possibility that the change in variance is positive in (16). This question clearly depends on the time profile of production as well as the discount factor. When  $T$  is approached, however, the variance must approach zero. In other words, sooner or later, the first term on the RHS of (16) must dominate the second.

We assume that the government cannot diversify the oil price risk through risk-sharing arrangements with other countries. This assumption is clearly more realistic than to assume full international risk sharing. We also assume that the government in each period  $t$  chooses the tax  $\tau_t$  for the young generation and the pension  $\pi_t$  to the old *after* the oil price  $P_t$  has been observed. The problem of maximizing expected social welfare in period 0 with respect to  $d_0$  and  $c_0$  can be expressed as

$$(17) \quad \max. \quad u(d_0) + u(c_0) + E_0 \left[ \sum_{t=1}^{\infty} \beta^t (u(d_t) + u(c_t)) \right],$$

subject to (9). Due to symmetry, it clearly follows that the two overlapping generations must have identical consumption, i.e.  $d_0 = c_0$ . Moreover, since the optimal strategy involves a corresponding maximization problem in each future period,  $d_t = c_t$  for all  $t$ .

After period  $T$  there are no more uncertainty due to the oil price and the optimal policy involves constant consumption over time. However, the consumption level depends on  $B_{T+1}$ , which again depends on past oil price surprises.

Before we look more closely into the case of precautionary saving, let us analyse the much simpler case of quadratic utility.

#### 4.1 Quadratic utility

It is well known that if utility is quadratic such that marginal utility is linear, zero precautionary saving is optimal. This special case is not considered to be empirically attractive because it implies increasing absolute risk aversion which is unrealistic. Still, this special case represents a useful benchmark for the analysis of prudence and precautionary saving. Following the analysis of Hall

(1978), the optimal policy can be found by taking into consideration that problem (17) yields first-order conditions:

$$(18) \quad c_0 = E_0[c_1] = E_0[c_2] = E_0[c_3] = \dots$$

This means that optimal consumption follows a random walk as long as the oil industry is active. By taking the expectation of the intertemporal budget constraint (9) and using (18), we find optimal consumption in period 0 to be:

$$(19) \quad c_0 = d_0 = \frac{1}{2} [w - g + r(k + B_0 + E_0[W_0])]$$

The only difference from the permanent income expression for optimal consumption under certainty is that the petroleum wealth is replaced by its expected value. Therefore, for the purpose of finding optimal consumption in the case of quadratic utility, the petroleum wealth should be calculated as the present value of future expected net oil revenue, using the risk-free rate of interest in the discount factor. The variance and higher order moments of the distribution of oil prices do not matter for optimal consumption. This important result has also been discussed by Aslaksen et al. (1990). Even though the social planner is risk averse and therefore does not value expected oil wealth as highly as the risk-free asset (Lund, 1990), optimal consumption is independent of the allocation of total wealth on  $B_0$  and  $E_0[W_0]$ . This also means that an increase in the variance of the oil price solely reduces the expected value of future utility, not the utility derived from present optimal consumption.<sup>16</sup> Despite the fact that  $\beta = R$ , the social planner therefore systematically allocates more utility to present consumption than to expected utility of future consumption. For the analysis of optimal investment decisions in the oil industry, a risk adjustment of the kind discussed by Lund (1990) would clearly be warranted under quadratic utility as well. It is therefore important to clarify for what purpose the risk adjustment of future oil revenue is supposed to serve.

To see the implications of the optimal plan for generational accounting, let us take the expectation of the intertemporal budget constraints of the entire economy (9) and for the private sector:

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<sup>16</sup> If future Mainland output is random as well, optimal consumption can be found by replacing  $y$  by its expected value. Neither the variance of  $y$  nor any covariance between  $y$  and the oil price would matter for optimal consumption.

$$(20) \quad \frac{(1+r)(c_0 + d_0)}{r} = c_0 + d_0 + \sum_{t=1}^{\infty} R^t (E_0 [c_t + d_t])$$

$$= (1+r)(k + B_{p,0} + B_{g,0} + E_0 [W_0]) + \frac{(1+r)(w - g)}{r}$$

$$(21) \quad c_0 + d_0 + \sum_{t=1}^{\infty} R^t (E_0 [c_t + d_t]) = (1+r)(k + B_{p,0}) + \frac{(1+r)w}{r} + \pi_0 + E_0 [N_0] + \sum_{t=1}^{\infty} R^t E_0 [N_t]$$

We express the budget constraint for entire country (20) and for the private sector (21) in these particular forms in order to facilitate the analysis of the implications of the optimal fiscal policy for the behaviour of consumers. From the fact that

$$E_0 [c_t] = \frac{1+r}{2+r} (w - E_0 [N_t]),$$

and using (18), it follows that  $E_0 [N_t] = E_0 [N_{t+1}]$  for all  $t = 1, 2, 3, \dots$ . The expected lifetime net tax burdens should therefore be equal for all generations. Moreover, in regard to the first old generation, its pension in period 0 is set such that  $d_0 = (1+r)(k + B_{p,0}) + \pi_0 = c_0$ . Expressing the LHS of (21) as

$$d_0 + \sum_{t=0}^{\infty} R^t E_0 [c_t + R d_{t+1}]$$

and using the fact that the expected budget constraints of households can be expressed as

$$E_0 [c_t + R d_{t+1}] = w - E_0 [N_t]$$

we see that this fiscal policy fulfils the expected intertemporal budget constraint of the private sector (21). Next we deduct (21) from (20) to obtain the expected intertemporal budget constraint of the government:

$$(22) \quad \frac{(1+r)g}{r} = (1+r)(B_{g,0} + E_0 [W_0]) - \pi_0 + N_0 + \frac{E_0 [N_1]}{r}.$$

Since the chosen pension in period 0 makes  $d_0 = c_0$ , and  $N_0 = E_0 [N_1] = E_0 [N_t]$  are equal, there is expected generational balance. Therefore, for the purpose of generational accounting, if utility is quadratic, the government should use a petroleum wealth concept that involves discounting of future expected oil revenues with the risk-free rate of interest.

To illustrate the optimal policy in a simple way, let us assume that  $T = 1$ , i.e. there is only one oil price shock  $P_1$  before the economy settles down in a stationary equilibrium with no uncertainty.

Now the variance of the petroleum wealth in (15) is simply  $Var_0[W_1] = R^2 x_1^2 Var_0[P_1]$ . After the oil price is known in period 1, the intertemporal budget constraint of the government can be written as

$$(23) \quad \frac{(1+r)g}{r} = (1+r)B_{g,1} + x_1 P_1 - \pi_1 + \frac{(1+r)N_1}{r}$$

Now let us write the corresponding budget constraint expected in period 0:

$$(24) \quad \frac{(1+r)g}{r} = (1+r)B_{g,1} + x_1 E_0[P_1] - E_0[\pi_1] + \frac{(1+r)E_0[N_1]}{r}.$$

The initial stock of financial assets  $B_{g,t}$  is a result of the fiscal policy and oil price in period 0. Deducting (24) from (23), and using the fact that  $r/(1+r) = 1 - R$ , yields

$$(25) \quad (\pi_1 - E_0[\pi_1]) - \frac{1}{1-R} (N_1 - E_0[N_1]) = x_1 (P_1 - E_0[P_1])$$

To find the fiscal policy rule, we note that

$$(26) \quad \pi_1 - E_0[\pi_1] = d_1 - E_0[d_1] = c_1 - E_0[c_1] = -\frac{(1+r)}{2+r} (N_1 - E_0[N_1]).$$

Substituting (26) into (25), and using the fact that  $(2+r)/(1+r) = 1 + R$ , we can find the decision rules linking fiscal policy to the oil price surprise:

$$(27a) \quad \pi_1 - E_0[\pi_1] = \frac{1-R}{2} x_1 (P_1 - E_0[P_1]),$$

$$(27b) \quad N_1 - E_0[N_1] = \frac{-(1-R^2)}{2} x_1 (P_1 - E_0[P_1]).$$

It follows that the optimal rule for fiscal policy adjustment to new information about the oil price is linear in the oil price surprise. In deriving this result, we did not need the assumption of linear marginal utility. It was crucial, however, that all future generations paid the same lifetime net tax to the government, and this is always true when  $T = 1$ . When utility is quadratic, it can be shown that this linearity is also preserved when  $T > 1$ .

The decision rule (27) is very intuitive. Since the oil revenue is  $x_1 P_1$ , the effect is proportional to oil production. Moreover, a higher interest rate (a lower  $R$ ), increases the effectiveness of a given oil price surprise in changing future net tax burdens. This fiscal policy involves a high degree of intergenerational risk sharing which is optimal due to strictly concave utility. It means that the welfare of future generations living after period  $T$  is very much dependent

on past oil price shocks. The fiscal policy therefore involves considerable government saving in response to positive oil price shocks in order to reduce the optimal expected net lifetime tax burdens of future generations.

## 4.2 Precautionary saving

Suppose that consumers are prudent, i.e. the marginal utility functions are strictly convex. We retain the assumption that  $\beta = R$ . Now consumers wish to engage in precautionary saving due to the random after-tax pension. As a precaution, they save more than in the former case of quadratic utility. Therefore  $c_0 < E_0[d_1]$ . The social planner also wishes to increase the expected consumption of future generations compared to the case of quadratic utility. She therefore increases government saving in order to reduce the expected net lifetime tax burdens of future generations living after the oil industry has been closed down.

Let us look at the expected intertemporal budget constraint of the government (4), using the fact that the optimal generational accounts are equal for generation  $T, T + 1, T + 2, \dots$ :

$$(28) \quad \frac{(1+r)g}{r} = (1+r)(B_{g,0} + E_0[W_0]) - \pi_0 + N_0 + \sum_{t=1}^{T-1} R^t E_0[N_t] + \frac{R^{T-1} E_0[N_T]}{r}.$$

We assume that the optimal values of  $\pi_0$  and  $N_0$  have been inserted into the constraint. The last term on the RHS is the present value of the expected generational accounts of all generations living after the uncertainty is resolved. In the social optimum, these generations have identical expected generational accounts. The term before the last one on the RHS is the present value of the expected generational accounts of future generations living before the oil industry has been closed down. Due to precautionary saving by the government, the optimal policy will involve different expected generational accounts for these generations. Moreover,  $E_0[N_t] > E_0[N_T]$  for all  $t < T$ . It is possible, however, that the optimal expected generational accounts are not declining monotonically for the generations living when the oil industry is active. Previously, we saw that the variance of the petroleum wealth could increase over time, see (16). Such an increase in the variance could reduce the expected generational account from one generation to the next.

The generational accounting method does not allow different expected generational accounts for future generations (generation 1, 2, 3, ...). As long as the petroleum sector is active

and the social planner faces oil price risk, the optimal policy will therefore always involve a generational account surplus. This is an argument for risk-adjusting the petroleum wealth downwards before using the generational accounting method.

Any such risk adjustment requires that we look at special functional forms of the utility function. One possibility is to use the exponential utility function, i.e. assuming constant absolute risk aversion (CARA utility). Aslaksen et al. (1990) did adopt this approach. However, they did not consider private saving and fiscal policy explicitly. Thøgersen (1997b) also applied a CARA utility function in an overlapping generations model with private precautionary saving.

In the next section we will introduce a utility function featuring constant relative risk aversion (CRRA utility) as our basis for adjusting the petroleum wealth for risk. As discussed in Section 1, this utility function is considered to be empirically more relevant than the CARA utility function, which implies increasing relative risk aversion. We shall demonstrate how the risk adjustment works in a special case of the previous model. Risk adjustment in more realistic settings would require numerical simulations, which is well beyond the scope of the present paper.

## 5. Risk adjustment with CRRA utility

We now adopt a CRRA utility function, where  $\theta = -(cu'')/u' > 0$  is the coefficient of relative risk aversion:

$$(29) \quad U_t = \frac{c_t^{1-\theta}}{1-\theta} + \beta \frac{d_{t+1}^{1-\theta}}{1-\theta}, \quad (\theta > 0, \quad \theta \neq 1).$$

Since the marginal utility function  $c^{-\theta}$  is strictly convex, it generates optimal precautionary saving. Relative prudence is defined as  $-(cu''')/u''$  and is equal to  $1 + \theta$ . Hence, relative prudence and the preference for precautionary saving are closely related to the coefficient of relative risk aversion.

To simplify further, we now assume that the generational account ( $N_t = \tau_t - R\pi_{t+1}$ ) of any generation is not subject to uncertainty in the first period of the lifecycle. In other words, the government is assumed to commit itself one period ahead to pay a certain pension  $\pi_{t+1}$  to the presently young. The pension to the first old generation ( $\pi_0$ ) is now predetermined. This means that consumers do not face uncertainty when making their saving decisions. The individual

consumption functions must therefore be the same as in (8). It is only the government's fiscal policy decision in period 0 that has to take risk into consideration. Assuming  $R = \beta$  such that  $c_t = d_{t+1}$ , and using (6), (8) and (29), as well as the fact that  $1 + R = (2 + r)/(1 + r)$ , the indirect utility function can be expressed as

$$(30) \quad U_t = \frac{(1 + R)^\theta (w - N_t)^{1-\theta}}{1 - \theta}.$$

To illustrate the risk adjustment in a simple way, we now assume that  $T = 1$ . In period 0, the next period's oil price  $P_1$  is uncertain, but after period  $T = 1$ , the oil industry has been closed down and there is no more uncertainty. The stochastic petroleum wealth can now be written as:

$$(31) \quad W_0 = Rx_0P_0 + R^2x_1P_1.$$

In period 1, the government will know the realization of  $P_1$  and chooses the tax and pension under full certainty. The optimal fiscal policy therefore involves identical generational accounts (denoted  $N_1$ ) for all future generations.

Inserting (30) into (11), expected social welfare in period 0 can be written as

$$(32) \quad E_0[U_{s,0}] = const. + \frac{(1 + R)^\theta}{1 - \theta} \left[ (w - N_0)^{1-\theta} + \frac{E_0[(w - N_1)^{1-\theta}]}{r} \right],$$

where the first term *const.* is the exogenous welfare of the first old generation.

The intertemporal budget constraint of the government (4) can now be expressed as

$$(33) \quad \frac{(1 + r)g}{r} = +(1 + r)(B_{g,0} + W_0) - \pi_0 + N_0 + \frac{N_1}{r}$$

It is convenient to use generation 0's present value of after-tax wage income,  $Z_0 = w - N_0$ , as a choice variable instead of  $N_0$ . We therefore transform the budget constraint (33) into a linear relationship between  $w - N_1$  and  $Z_0$ :

$$(34) \quad w - N_1 = -rZ_0 + V, \quad V = (1 + r)(w - g + rB_{g,0}) + r \left( -\pi_0 + x_0P_0 + \frac{x_1P_1}{1 + r} \right)$$

$V$  is a stochastic variable that represents the intertemporal constraint on the present value of private disposable income for all generations expect the first old generation. Its expectation is  $E_0[V] = \bar{V}$  and variance  $\sigma^2$ . Using the fact that  $1 - R = r/(1 + r)$ , the expectation and variance of  $V$  are related to the expectation and variance of the oil price as follows:

$$(35) \quad \begin{aligned} \bar{V} &= (1+r)(w-g+rB_{g,0})+r(-\pi_0+x_0P_0)+(1-R)x_1E_0[P_1] \\ \sigma^2 &= (1-R)^2x_1^2Var[P_1] \end{aligned}$$

Inserting (34) into (32), expected social welfare is:

$$(36) \quad E_0[U_{s,0}] = const. + \frac{(1+R)^\theta}{(1-\theta)} \left[ Z_0^{1-\theta} + \frac{E_0[(-rZ_0+V)^{1-\theta}]}{r} \right]$$

Maximizing (36) with respect to  $Z_0$  yields the first-order condition:

$$(37) \quad Z_0^{-\theta} = E_0[(-rZ_0+V)^{-\theta}] \quad (Z_0 = w - \tau_0).$$

To proceed, we use a quadratic approximation of (37). A natural benchmark policy is to set the generational account in period 0 such that  $N_0 = E_0[N_1]$ . Under quadratic preferences, this would have been the optimal fiscal policy. We know in advance, however, that the optimal  $N_0$  must be larger than  $E_0[N_1]$  due to precautionary government saving. Therefore,  $Z_0 < E_0[w - N_1]$ . We use the benchmark policy as a point of departure for a quadratic Taylor approximation of (37). For details, see the Appendix.

Let  $a$  be the value of  $Z_0$  which corresponds to the benchmark policy:  $N_0 = E[N_1]$ . From (34) it follows that

$$(38) \quad a = \frac{\bar{V}}{1+r}$$

The quadratic approximation of the first-order condition yields the following solution:

$$(39a) \quad Z_0 = \frac{[1+(1+\theta)(1-r)-H]a}{(1+\theta)(1-r)}, \quad (r \neq 1), \quad H = \sqrt{1 + \frac{(1+\theta)^2(1-r)\sigma^2}{1+r a^2}}$$

$$(39b) \quad Z_0 = \left[ 1 - \frac{1+\theta}{4} \left( \frac{\sigma}{a} \right)^2 \right] a \quad (r = 1).$$

In the Appendix we derive sufficient conditions that makes the approximate solution economic meaningful. Note that when there are only two overlapping generations, one time period corresponds to 25-30 years, and the rate of interest could easily be larger than 1. We see from (39) the important role played by relative prudence  $(1 + \theta)$ , which exerts a negative influence on  $Z_0$ . Not surprisingly,  $Z_0$  also declines if the variance of the future oil price increases.

In the case of quadratic utility, the optimal  $Z_0$  is equal to  $a$ , which is a linear function of  $x_1$  (see (35) and (38)):

$$(40) \quad a = A + a_1 x_1, \quad \left( \begin{array}{l} A = w - g + rB_{g,0} + rR(-\pi_0 + x_0 P_0) > 0, \\ a_1 = R(1-R)E_0[P_1] > 0 \end{array} \right).$$

The optimal marginal propensity to consume next period's expected oil revenue is therefore  $a_1$  when utility is quadratic. To find the corresponding effect when marginal utility is strictly convex, we differentiate  $Z_0$  in (39) with respect to  $x_1$ , using the fact that  $\sigma$  is linear in  $x_1$  (see (35)). In the special case  $r = 1$ , it is immediately seen from the expression (39b) that the derivative of  $Z_0$  with respect to  $x_1$  must be smaller than  $a_1$ . In the general case  $r \neq 1$ , we first note that the derivative of  $H$  with respect to  $x_1$  ( $H'$ ) is positive if  $r < 1$  and negative if  $r > 1$ . Differentiating (39a), the derivative can be expressed as:

$$(41) \quad \frac{dZ_0}{dx_1} = \frac{Z_0}{a} a_1 - \frac{aH'}{(1+\theta)(1-r)} < a_1.$$

The last term of this expression is always negative. Moreover,  $Z_0 < a$ . The marginal propensity to consume  $x_1$  in advance is therefore always lower than  $a_1$ . The intuition is straightforward. An increase in  $x_1$  will – in addition to increasing the expected petroleum wealth – also increase the variance. The increase in the risk makes the optimal fiscal policy more prudent.

It is important to note that the effect on the optimal policy in period 0 of an increase in  $x_0$  is quite different from the effect of an increase in  $x_1$  because in the former case, the variance of the petroleum wealth does not change. Let  $a_0 = rRP_0$  be the effect of a one-unit increase in  $x_0$  on  $Z_0$  in the case of quadratic utility, see (40). Differentiating (39a) with respect to  $x_0$  (assuming that  $x_1$  does not change) yields:

$$(42) \quad \frac{dZ_0}{dx_0} = \left[ \frac{Z_0}{a} + \frac{(1+\theta)\sigma^2}{(1+r)Ha^3} \right] a_0.$$

Although the first term in the square brackets in (42) is smaller than one, the second term is positive and could make the total effect stronger than  $a_0$ . In other words, we cannot exclude that the policy maker would reduce the generational account  $N_0$  by more than in the case of quadratic utility. The intuition is that when the economic policy is optimally adjusted to future risk at the outset, more safe income in period 0 will not necessarily trigger more precautionary saving at the margin. For a more general analysis, see Gollier (2001).

Let us now look at the difference between the optimal generational account  $N_0$  and the expected optimal  $N_1$ . This difference is denoted  $\Delta = N_0 - E_0[N_1]$ . From (34), we see that  $\Delta$  can be expressed as

$$(43) \quad \Delta = (1+r)(a - Z_0) > 0.$$

From (40) and (41), we see immediately that  $\Delta$  must be an increasing function of  $x_1$ . Using (39), it can be shown that this difference can be expressed as:

$$(44a) \quad \Delta = \frac{(H-1)(1+r)a}{(1+\theta)(1-r)}, \quad (r \neq 1)$$

$$(44b) \quad \Delta = \frac{(1+\theta)\sigma^2}{2a}, \quad (r = 1).$$

From the definition of  $H$  it follows that  $(H - 1)$  always has the same sign as  $(1 - r)$ . Therefore, the expression in (44a) is always positive.

## 6. Risk adjustment of generational accounts

We now show how the generational accounts can be adjusted for the oil price risk. The idea is to adjust the petroleum wealth downwards such that the optimal policy corresponds to adjusted generational balance. Taking the expectation of the government's intertemporal budget constraint (34):

$$(45) \quad \frac{(1+r)g}{r} = (1+r)B_{g,0} + x_0P_0 + \frac{x_1E_0[P_1]}{1+r} + N_0 + \frac{E_0[N_1]}{r}$$

If no uncertainty or quadratic utility,  $\Delta = 0$ . Then there is generational balance in an expected sense. In optimum  $\Delta > 0$ , however, and we have shown that the expected generational accounts must show a surplus under the optimal fiscal policy, i.e.  $N_0 > E_0[N_1]$ . We therefore add  $\Delta$  to  $E_0[N_1]$  on the RHS of (45) and reduce the petroleum wealth correspondingly:

$$(46) \quad \frac{(1+r)g}{r} = (1+r)B_{g,0} + x_0P_0 + \frac{x_1E_0[P_1] - x_1\mu_0}{1+r} + N_0 + \frac{E_0[N_1] + \Delta}{r}.$$

The risk adjustment of the expected oil price ( $\mu_0$ ) follows from

$$(47) \quad x_1 \mu_0 = \frac{(1+r)\Delta}{r} = \frac{(1+r)(H-1)\bar{V}}{r(1-r)(1+\theta)}.$$

Now all future generations pay a risk adjusted generational account ( $E_0[N_1] + \Delta$ ), which is equal to  $N_0$ , i.e. the generational accounts are in a risk-adjusted balance under the optimal policy.

To see more clearly what is involved in the risk adjustment term ( $\mu_0$ ) for the oil price, let us consider the special case  $r = 1$ , see (39b) and (44b). If the period length is defined as 25 years, this corresponds to an annual real rate of interest of 2,81 percent. Inserting (44b) into (47), we obtain:

$$(48) \quad \mu_0 = \frac{(1+\theta)Var_0[P_1]x_1}{4A + E_0[P_1]x_1}, \quad \left( A = w - g + rB_{g,0} + \frac{(-\pi_0 + x_0 P_0)}{2} \right).$$

The positive constant  $A$  has previously been defined by (40). We first note that the risk adjustment term is now proportional to  $(1 + \theta)Var_0[P_1]$ . The economic intuition is straightforward. The constant term  $A$  in (48) is the part of the private permanent income (excluding the income of the first old generation) that is not related to future oil revenues. An increase in  $A$  warrants a reduction in the risk adjustment term. This is a reflection of constant relative prudence ( $1 + \theta$ ): It is easier to bear a given future income risk when the economy gets wealthier. For the same reason, an increase in the expected oil price should also *reduce* the risk adjustment term.

We also observe that the risk adjustment term in (48) depends positively on  $x_1$ . Higher future oil production would therefore call for an increased risk adjustment of the expected oil price. In general the sign of this effect is ambiguous if  $r$  is different from 1. If  $x_1$  is increased, the variance of the petroleum wealth increases for a given expectation and variance of the future oil price. The isolated effect of the increased variance is to increase the risk adjustment of the oil price. However, increased future oil production also increases the expected government petroleum wealth, and since relative prudence is constant, the isolated effect is to reduce the risk adjustment factor. The sign of the total effect on the risk adjustment factor of the expected oil price is therefore ambiguous. This relationship highlights that the risk adjustment of the oil price also depends positively on the share of future expected oil revenue in the private permanent income. To see this we imagine that  $A$  is reduced and  $x_1$  increased such that the denominator in (48) is

constant. Since this increases the variance of the stochastic petroleum wealth, the risk adjustment term must be raised.

## 7. A numerical example

We illustrate the risk adjustment method by a numerical example designed to reflect the stylised facts about the Norwegian economy. We assume that the annual real rate of interest is 4 percent and that the annual rate of capital decay is 3 percent. One period is defined as 25 years. This implies that  $r = 1.6658$  and  $\delta = 1.36325$ . The ML-sector is represented by a Cobb-Douglas production function with a capital income share 36 per cent of GDP. This corresponds to  $k = 0.035865$ ,  $y = 0.30177$ , and  $w = 0.193135$ . Government consumption is 25 per cent of Mainland GDP, i.e.  $g = 0.07544$ . Government wealth is 70 per cent of the Mainland capital stock, which implies  $E0[V] = 0.425238$ . Assume that 50 per cent of total government wealth is expected discounted cash flow from period 1.

Table 1 shows the future expected tax in per cent of the present tax under various assumptions about the petroleum risk and the elasticity of marginal utility ( $\theta$ ). Table 2 shows the corresponding risk premiums. The results indicate that the optimal generational policy is fairly sensitive to the petroleum risk.

**Table 1 Present after -tax wage in percent of expected future after-tax wage (net taxes)**

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	$\frac{\sigma}{E_0[V]}$		
θ	0.02	0.05	0.1
2	99.6	97.4	89.6
3	99.4	96.5	86.0

**Table 2 Optimal risk premiums (annual, per cent)**

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	$\frac{\sigma_v}{EV}$		
θ	0.02	0.05	0.1
2	0.05	0.32	1.52
3	0.07	0.42	2.31

## 8. Conclusions

Thirty years have passed since Norway entered into the petroleum age, but the oil price risk will still be substantial for many years to come. In fact, the peak of the petroleum exhaustion still lies in the future, most likely in the near future. The issue of risk adjustment in the context of fiscal planning will therefore be important for a long time.

The main conclusion of this paper is that what matters for the appropriate risk adjustment of the petroleum wealth is the issue of precautionary saving, i.e. the existence of prudence. Risk aversion in itself does not warrant risk adjustment of the petroleum wealth. In other words, what is important for the question of risk adjustment of the petroleum wealth is the convexity of the marginal utility function. If the social welfare function exhibits linear marginal utility, no risk adjustment of expected oil prices is warranted when using the general accounting method or other fiscal planning tools, even if risk aversion is present. Still, for optimal investment decisions in the petroleum sector, risk aversion would clearly matter even if marginal utility is linear, assuming also that the oil price risk cannot be diversified internationally; see Lund (1990). It is therefore important to separate the question of risk adjustment in the context of petroleum investment decisions from the question of adjusting for risk when making estimates of the petroleum wealth for the purpose of generational policy.

Another conclusion we emphasise is that when the government bear the brunt of the oil price risk, it is welfare improving to index pensions to the oil price for the purpose of intergenerational risk sharing. The present pension system is indexed to the real wage, which again is mainly influenced by the growth prospects of the Mainland economy. A more complete analysis of the issue of intergenerational risk sharing is however outside the scope of the present paper.

Finally, the paper suggests a method for adjusting the petroleum wealth for oil price risk when the social welfare function exhibits constant relative risk aversion. This is shown in a special case. The idea is to adjust the petroleum wealth downwards such that the optimal fiscal policy corresponds to adjusted generational balance in the structural model. This method of quantifying the risk adjustment term can be applied even if the actual fiscal policy does not imply generational balance in a real world application. The adoption of a stylised structural model with a welfare-maximizing government is solely for the purpose of measurement of the petroleum wealth

correction term. The structural model itself is however too stylised to replace the generational accounting method.

The derived risk adjustment factor for the future oil price has a natural economic interpretation. It increases with the variance of the petroleum wealth and the relative prudence. In general it is not linear in future oil production. If future oil production estimates are increased, the variance of the petroleum wealth increases for a given expectation and variance of the future oil price. The isolated effect of the increased variance is to increase the risk adjustment of the oil price. However, increased future oil production also increases the expected government petroleum wealth, and since relative risk aversion is constant, the isolated effect is to reduce the risk adjustment factor. The sign of the total effect on the risk adjustment factor of the expected oil price is therefore ambiguous.

The method was illustrated using stylised facts about the Norwegian economy. More information about the stochastic properties of the petroleum wealth is however needed before more reliable assessments of the quantitative importance of the risk adjustment can be made.

Constraining the model to only one source of risk can be justified as a first step towards more general models encompassing several sources of uncertainty. One natural generalization would be to introduce a stochastic return from the State Petroleum Fund and to relate the optimal portfolio choice of the Petroleum Fund to the oil price risk exposure from the petroleum wealth. An even more ambitious generalization would be to permit stochastic wage rates with repercussions into the tax-transfer system. These extensions represent natural topics for future research.

## Appendix: The quadratic approximation of the first-order condition

The quadratic approximation  $q(X)$  of the function  $X^\theta$  around  $a$  is:

$$q(X) = a^{-\theta} \left[ (1+\theta) \left( 1 + \frac{\theta}{2} \right) - \theta(2+\theta) \frac{X}{a} + \frac{\theta(1+\theta)}{2} \left( \frac{X}{a} \right)^2 \right].$$

Adopting this approximation on each side of (37) yields the following equation in  $Z_0 = X$ :

$$(A1) \quad \frac{(1+\theta)(1-r^2)}{2} \left( \frac{Z_0}{a} \right)^2 - \left[ (2+\theta)(1+r) - (1+\theta)r \left( \frac{\bar{V}}{a} \right) \right] \frac{Z_0}{a} + (2+\theta) \frac{\bar{V}}{a} - \frac{(1+\theta)}{2} \left[ \frac{\sigma^2}{a^2} + \frac{\bar{V}^2}{a^2} \right] = 0.$$

We first look at the solution of (A1) in the special case  $r = 1$ . Then the coefficient in front of the quadratic term is zero. Using that  $(1+r)a = \bar{V}$  yields the following solution for  $Z_0$ :

$$(A2) \quad Z_0 = \left[ 1 - \frac{1+\theta}{4} \left( \frac{\sigma}{a} \right)^2 \right] a, \quad (r=1).$$

For this solution to be meaningful,  $Z_0$  must be positive, i.e. the risk must not be too large. The following condition must therefore be fulfilled:

$$(A3) \quad \frac{\sigma^2}{a^2} < \frac{4}{1+\theta}.$$

Let us now consider the general case involving  $r$  different from 1.

Assuming  $r \neq 1$ , the solution of (A1) becomes:

$$(A4) \quad Z_0 = \frac{[1 + (1+\theta)(1-r) - H]a}{(1+\theta)(1-r)}, \quad H = \sqrt{1 + \frac{(1+\theta)^2(1-r)\sigma^2}{1+r a^2}}.$$

Let us first consider the case  $r < 1$ , and then  $r > 1$ .

If  $r < 1$ , we must be sure that the numerator in (A4) is positive. This requires that

$$(A5) \quad H < 1 + (1+\theta)(1-r).$$

Squaring on both sides of (A5) and using the definition of  $H$ , we obtain the following condition

$$(A6) \quad \frac{\sigma^2}{a^2} < \frac{(1+r)(2+(1+\theta)(1-r))}{1+\theta}.$$

The RHS of (A6) declines when  $r$  approaches 1. In the limit, condition (A6) is identical to (A3). If  $\sigma^2/a^2$  is greater than the RHS of (A6) such that the condition is violated, the Taylor approximation does not work.

If  $r > 1$ , we must be sure that  $H$  is a real number, i.e.

$$(A7) \quad \frac{\sigma^2}{a^2} < \frac{1+r}{(1+\theta)^2(r-1)}.$$

Condition (A7) places an upper bound on the risk, depending on the coefficient of relative risk aversion ( $\theta$ ) and the interest rate. If the risk is greater than this, the quadratic approximation is not meaningful because the exact optimal solution is too far away from  $Z_0 = a$ . We assume that (A7) always holds if  $r > 1$ . In the calculations underlying Table 1 and 2,  $r > 1$  and condition (A7) is satisfied.

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