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Capacity Charges: A Price Adjustment Process for Managing Congestion

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Abstract

In this paper we suggest a procedure based on capacity charges for managing transmission constraints. The system operator states nodal capacity charges for transmission prior to market clearing. Market clearing brings forth a single market price for energy. For optimal capacity charges the market equilibrium coincides with that of nodal pricing. Capacity charges are based on technical distribution factors and estimates of the shadow prices of network constraints. Estimates can be based on market information from similar congestion-situations, or they can be adjusted near the optimal values through an iterative process.

1 Introduction

The goal of deregulating the electricity market has been to achieve efficiency through competition between supply and demand. A special feature of electricity is the reliance on a common network for transmission, where network constraints have special implications for optimal economic dispatch, due to the externalities created by the loop flow. Highly different market designs have been chosen for handling network constraints, with different implications for efficiency. Our objective is to combine several of the suggested approaches, and see whether it is possible to find a good approximation to optimal nodal prices by using system operator announced capacity charges.

In the capacity charge approach capacity constraints are handled by issuing nodal capacity charges. Market clearing brings forth a single market price for energy common to the entire pool. The net nodal price thus equals the common market price less the nodal capacity charges. For positive (negative) capacity charges, the net nodal price is lower (higher) than the market price. Optimally set capacity charges allow the market to reach optimal dispatch as net nodal prices equal optimal nodal prices. Capacity charges are issued by the system operator, and are based partly on technical load factors, and partly on the shadow prices of congested lines. Implementation of the capacity charges approach can be on an ex post or an ex ante basis.

If capacity charges are announced ex post, i.e. *after* bidding and market clearing, the approach is merely a different representation of the nodal pricing method, where the aggregate effect of capacity constraints in the grid is priced explicitly for each node. With the ex post announcement of capacity charges, the system operator has full information of shadow prices, and the calculation of capacity charges is straightforward.

On an ex ante basis capacity charges are issued *prior to* bidding and market clearing, and market participants respond to the issued capacity charges in submitting their supply and demand curves. Without full information, as with the ex ante announcement, however, the shadow prices of congested lines have to be estimated. With the ex ante announcement of capacity charges, the implementation of the approach further depends upon the overall design of the electricity market. Organization of competitive electricity markets varies with respect to how real-time balancing is organized, ranging from solutions with electricity spot markets which in effect are real-time markets, to solutions with a day-ahead scheduling market, and where real-time balancing is handled in a separate real-time balancing market.

Within the market framework of a separate day-ahead scheduling market and a real-time balancing market, we assume that the function of the day-ahead market is to reach an efficient schedule which is feasible with respect to expected capacity constraints. The market participants state their bids after the announcement of capacity charges. Thus, the market clearing brings forth a market equilibrium consistent with the estimated shadow prices of expected capacity constraints. The efficiency of the market equilibrium can be improved through an iterative adjustment process to reach an optimal and feasible market solution.

Within the framework of a real-time spot market, the ex ante announcement of capacity charges may affect demand and supply bids. These estimated capacity charges will however not be able to clear the market alone, and there is no room for a direct iterative process. This requires using e.g. ex post nodal prices together with the pre-announced capacity charges.

The capacity charge approach offers several advantages. A main issue is the role of capacity charges as important market signals for demand and supply, signaling geographical differences in the nodal cost of aggregate network constraints. Compared to approaches such as zonal pricing, the method also incorporates the advantages of nodal pricing. Further, as capacity charges apply to all contracts of physical delivery, the method enables spot and bilateral contracts to coexist. Also, as capacity charges are issued by the system operator, and are based on technical information and estimates of shadow prices, this enables a clearer distinction between the role of the exchange and the system operator, and might facilitate coordination in areas where there for some reason exist multiple exchanges. As for implementing ex ante announced capacity charges in a market based on a real-time spot market only, the combination of ex post nodal prices with pre-announced capacity charges may be a source of enhancing market efficiency. In a market which is cleared real-time, we note that the producer or consumer has to be able to respond automatically to prices in order to submit price-elastic bids. If not, only price-inelastic bids can be submitted. The pre-announcement of capacity charges enables this group to respond on the signals conveyed by the capacity charges, reducing or increasing planned demand according to the signals of expected congestion cost conveyed by the capacity charges.

The rest of the paper is organized as follows: Section 2 discusses the approach in relation to other methods for handling congestion. Section 3 presents the foundation for the capacity charge approach, and shows its relation to optimal dispatch by nodal prices, using a model with a "DC"-approximated network. Section 4 exemplifies the approach of optimal capacity charges in a six-node model. Section 5 discusses iterative approaches for implementing the capacity charge method, illustrating with a standard gradient method. Section 6 discusses a heuristic approach in obtaining feasible flows. Section 7 concludes the paper, and future research is discussed.

2 Literature Review

The concept of nodal prices is discussed by Schweppe et al. (1988). Optimal nodal prices are produced by the solution of the welfare maximization problem as the dual prices of the power flow equations, and are interpreted as the value of power in each node (cf. Wu et al. (1996)). A mechanism enforcing optimal nodal prices, where generators and consumers adapt to the local (nodal) market price when deciding on output, ensures social optimum in the short run. Wu et al.

(1996) point to several counter-intuitive and possibly troublesome characteristics of implementing the nodal pricing approach. For instance, for the system operator to calculate the optimal economic dispatch and implement it, suppliers and consumers must truthfully reveal cost and demand functions, and they may not be willing to give away such strategic information.

On the other hand, the price system suggested by Chao and Peck (1996) represents a system for "explicit congestion pricing", where, instead of providing locational energy prices as nodal prices do, the use of scarce transmission resources is priced. This is accomplished through the design of a trading rule, based on load factors or distribution factors, specifying the transmission capacity rights that traders must acquire in order to complete an electricity transaction. In optimum, Chao-Peck prices are consistent with optimal nodal prices, and in accordance with the shadow prices of the transmission constraints of the optimal dispatch problem. A slight modification of Chao-Peck prices is suggested by Stoft (1998), where a "hub" price is determined by allowing energy bids at any given node (or "hub") in the network. Both mechanisms rest upon a market bringing forward the prices of transmission rights on the links, and the number of prices these systems have to derive is usually far less than the number of nodes in the network. However, although Chao and Peck (1996) and Stoft (1998) give some indications, the specific design of a market mechanism to determine the Chao-Peck prices is still regarded an unresolved problem.

The coordinated multilateral trade model suggested by Wu and Varaiya (1995) is intended to attain optimal dispatch without requiring the system operator to collect private information, i.e. supply and demand curve bids. Instead, brokers carry out profitable multilateral trades under feasibility constraints. More specifically, central coordination is achieved through an iterative process, where, in the case of transmission constraints, loading vectors are announced by the system operator, based on which brokers must evaluate the feasibility of the trades in question. Consequently, the decision mechanisms regarding economics and the reliability of system operation are separated. Economic decisions are carried out by private multilateral trades among generators and consumers, while the function of reliability is coordinated through the system operator who provides publicly accessible data, based upon which generators and consumers can determine profitable trades that meet the secure transmission loading limits.

In relation to the optimal dispatch problem, the coordination models can be interpreted as different relaxation schemes, with competitive players in generation and consumption and the system operator solving different sub-problems, and information is exchanged back and forth. The decompositions corresponding to nodal pricing and Chao-Peck pricing are price-driven. In the case of nodal prices, the system operator hands out the optimal nodal prices of energy obtained after solving the optimal dispatch problem, and optimal dispatch is achieved as producers and consumers adapt to their local prices. For Chao-Peck prices, a market is supposed to bring forth the competitive prices of transmission rights, while the system operator provides information on how trades affect every single link. When traders adapt to the transmission charges of the links imposed by the prices of the transmission rights, the overall problem is solved. The coordinated multilateral trade model can be interpreted as a Benders decomposition, where the market players maximize net profit, and quantities are communicated to the system operator, which checks feasibility and generates constraints. The new constraints must be taken into consideration when additional trades are placed and the process continues. Due to the complexity of electric networks, each method has its limitations in practical use.

In this paper, we combine several of the above approaches. Our objective is to find good approximations of the optimal nodal prices based on an uncongested system price and the loading vectors of congested lines. This approach may be interpreted as a Chao-Peck pricing approach including a "hub", as suggested by Stoft (1998), where we estimate/guess the shadow prices of congested lines. Our approach is also similar to the coordinated multilateral trade model of Wu and Varaiya (1995) in that we need not rely on the disclosure of private information. Compared to Wu and Varaiya (1995), instead of announcing the constraints through the publication of the loading vector, the grid operator announces a set of nodal capacity charges that is based on an estimate/guess of the shadow price of the constraint in question, and the loading vector. The approach is also similar to that of Glavitch and Alvarado (1997) who use market information to estimate cost parameters.

3 The Capacity Charge Approach

The optimal market equilibrium is the market solution which gives the maximum social surplus attainable within the constraints of the system, i.e. the equilibrium which replicates the solution to the optimal economic dispatch problem. In this section we compare the market equilibrium of the capacity charge approach with the optimal economic dispatch.

Consider an electricity market where supply and demand are located in n nodes which are interconnected by a constrained transmission grid. Demand for power in node i is described by the non-increasing demand function $p_i^d(q)$, and supply of power in node i, by the non-decreasing supply function $p_i^s(q)$. Social surplus, Π_{ss}, is defined as total willingness to pay, less total cost of production, as shown in (3.1),

$$\Pi_{ss} \equiv \sum_{i=1}^{n}\left[\int_0^{q_i^d} p_i^d(q)dq - \int_0^{q_i^s} p_i^s(q)dq\right] \tag{3.1}$$

where q_i^d and q_i^s are the quantities demanded and supplied in node i. Social surplus may be decomposed into consumer surplus, supplier surplus and grid revenue, the latter due to congestion. These are shown respectively as the three terms of (3.1').

$$\begin{aligned}\Pi_{ss} \equiv &\sum_{i=1}^{n}\left[\int_0^{q_i^d}\left(p_i^d(q) - p_i^d\left(q_i^d\right)\right)dq\right] + \sum_{i=1}^{n}\left[\int_0^{q_i^s}\left(p_i^s\left(q_i^s\right) - p_i^s(q)\right)dq\right] \\ &+ \sum_{i=1}^{n}\left[p_i^d\left(q_i^d\right)q_i^d - p_i^s\left(q_i^s\right)q_i^s\right]\end{aligned} \tag{3.1'}$$

The transmission grid consists of several lines connecting the nodes. To illustrate the nodal capacity charge approach, we consider real power using the lossless linear "DC" approximation of power flow equations, with reactances equal to 1 on every link[1]. Let q_{ij} be the flow along line ij in the direction from node i to node j. Under the lossless "DC" approximation we have $q_{ij} = -q_{ji}$. Also, let the net injection in node i be defined by:

[1] The "DC" approximation is the customary approximation used in literature when dealing with the management of transmission constraints. Under these assumptions, and with well-behaving cost and benefit functions, the optimal dispatch problem is convex. For the specifics of the "DC" approximation, see for instance Wu and Varaiya (1995), Chao and Peck (1996) or Wu et al. (1996). In the "DC" approximation both losses and reactive power are left out.

$$q_i \equiv q_i^s - q_i^d \tag{3.2}$$

The power flow on each line is determined by Kirchhoff's junction rule, Kirchoff's loop rule, and the Law of conservation of energy.

$$q_i = \sum_{i \neq j} q_{ij} \qquad i = 1, \ldots, n-1 \tag{3.3}$$

$$\sum_{ij \in L_\ell} q_{ij} = 0 \qquad \ell = 1, \ldots, m-n+1 \tag{3.4}$$

$$\sum_i q_i = 0 \tag{3.5}$$

Kirchhoff's junction rule (3.3) states that the current flowing into any node is equal to the current flowing out of it. There are n nodes, and there are n-1 independent equations. Equation (3.4) follows from Kirchhoff's loop rule that states that the algebraic sum of the potential differences across all components around any loop is zero. The number of independent loops is given by m-n+1, where m is the number of lines in the grid. $(\overline{\mathrm{L}}) = (\mathrm{L}_1, \ldots, \mathrm{L}_{\mathrm{m-n+1}})$ is the set of independent loops[2] and L_ℓ is the set of directed arcs ij in a path going through loop ℓ. The law of conservation of energy (3.5), states, in the absence of losses, that total generation is equal total consumption.

In general, for a given network and load, the power flows may be summarized by a matrix of load factors. Each load factor β_{ij}^{lm} shows the fraction of an injection in node l with withdrawal in node m that flows along line ij. Note that $\beta_{ji}^{lm} = -\beta_{ij}^{lm}$ and $\beta_{ij}^{ml} = -\beta_{ij}^{lm}$. Under the "DC" approximation the load factors are constants, i.e. independent of load[3]. By introducing a reference point r for withdrawals, the load factors may be represented by a loading vector

[2] See Dolan & Aldous (1993).

[3] In general AC systems the load factors depend on the distribution of loads over the network. Our method applies also for general AC systems, however, requiring recalculations of the load factors according to the load.

$\overline{\beta}_{ij}(r) \equiv (\beta_{ij}^{1r} \quad \beta_{ij}^{2r} \quad \ldots \quad \beta_{ij}^{nr}) \equiv (\beta_{ij}^{1} \quad \beta_{ij}^{2} \quad \ldots \quad \beta_{ij}^{n})$ for each link *ij*. Element *k* of loading vector $\overline{\beta}_{ij}(r)$ shows the flow along line *ij* if 1 MW is injected into node *k* and withdrawn in the reference point *r*. A general trade between node *l* and *m*, may be viewed as a combined trade between node *l* and *r* and between *r* and *m*. Thus, we have $\beta_{ij}^{lm} = \beta_{ij}^{lr} + \beta_{ij}^{rm} = \beta_{ij}^{lr} - \beta_{ij}^{mr} = \beta_{ij}^{l} - \beta_{ij}^{m}$. With net injections given, the line flow along line *ij* is consequently:

$$q_{ij} = \sum_i \beta_{ij}^i q_i \tag{3.6}$$

Capacity constraints $CAP_{ij} \geq 0$ and $CAP_{ji} \geq 0$ on line *ij* require that $q_{ij} \leq CAP_{ij}$ and $q_{ji} \leq CAP_{ji}$. The capacity constraints may thus be stated as[4]:

$$\sum_i \beta_{kl}^i q_i \leq CAP_{kl} \qquad k = 1,\ldots,n, \quad l = 1,\ldots,n, \quad k \neq l \tag{3.7}$$

Under the "DC" approximation, optimal economic dispatch is then given by the following convex optimization problem:

$$\begin{aligned}
&\textit{maximize} && \Pi_{ss} \equiv \sum_{i=1}^{n} \left[\int_0^{q_i^d} p_i^d(q)dq - \int_0^{q_i^s} p_i^s(q)dq \right] && \\
&\textit{subject to} && q_i = q_i^s - q_i^d && i = 1,\ldots,n \\
& && q_i = \sum_{i \neq j} q_{ij} && i = 1,\ldots,n-1 \\
& && \sum_{ij \in L_\ell} q_{ij} = 0 && \ell = 1,\ldots,m-n+1 \\
& && \sum_i q_i = 0 && \\
& && \sum_i \beta_{kl}^i q_i \leq CAP_{kl} && k = 1,\ldots,n, \quad l = 1,\ldots,n, \quad k \neq l
\end{aligned} \tag{3.8}$$

[4] Note, that for a non-existing line *ij*, we have $\beta_{ij}^{lm} = \beta_{ji}^{lm} = 0$, and $CAP_{ij} = CAP_{ji} = 0$.

In the unconstrained case, where neither of the capacity constraints of (3.7) are binding, there will be a uniform price in the market. For the capacity constrained case, where at least one capacity constraint is binding, nodal prices will differ and may be different for all nodes[5]. If the constraint $q_{kl} \leq CAP_{kl}$ is binding, we have $q_{kl} \geq CAP_{kl}$ and thus $q_{kl} \geq 0$. As $q_{lk} = -q_{kl}$, the corresponding constraint $q_{lk} \leq CAP_{lk}$ is not binding. Define the shadow prices of (3.7) as $\mu_{kl} \geq 0$. Thus, if $\mu_{kl} > 0$, we have $\mu_{lk} = 0$.

Under the capacity charge approach, the system operator first provides nodal capacity charges, cc_i. On receiving this information, the participants determine supply and demand bids. Market clearing, results in an equilibrium energy price, p, which is common to the entire pool. The capacity charges may be positive or negative. A positive capacity charge, $cc_i > 0$, is defined as the amount cc_i the suppliers in the node pay per unit supplied, or equivalently the amount cc_i the consumers receive per unit consumed. For a negative charge, consumers pay, while producers are compensated. The net nodal price thus equals $p_i = p - cc_i$.

Proposition: *The market equilibrium of the capacity charge approach is in accordance with optimal economic dispatch when capacity charges are optimally defined.*

Proof. If we relax the capacity constraints in (3.8), we obtain the Lagrangian function:

$$L(\overline{\mu}) = \sum_i \left[\int_0^{q_i^d} p_i^d(q)dq - \int_0^{q_i^s} p_i^s(q)dq \right] + \sum_k \sum_l \mu_{kl} \left[CAP_{kl} - \sum_i \beta_{kl}^i \left(q_i^s - q_i^d \right) \right] \tag{3.9}$$

For a given vector $\overline{\mu}$ consisting of shadow prices for all lines of the network, the relaxed problem $h(\overline{\mu}) = \{\max L(\overline{\mu}) \text{ s.t. } (3.2)\text{ - }(3.5)\}$ provides an upper bound on the objective function value of (3.8). This follows from weak duality. Because of strong duality, solving the dual problem $\min_{\overline{\mu}} h(\overline{\mu})$ also provides the solution to our original problem (3.8). Considering the objective function of the dual problem and rearranging terms, we get:

[5] Refer to Wu et al. (1996) for the characteristics of optimal nodal prices.

$$L(\bar{\mu}) = \sum_i \left[\int_0^{q_i^d} p_i^d(q)dq + \sum_k \sum_l \mu_{kl} \beta_{kl}^i q_i^d \right] - \sum_i \left[\int_0^{q_i^s} p_i^s(q)dq + \sum_k \sum_l \mu_{kl} \beta_{kl}^i q_i^s \right]$$
$$+ \sum_k \sum_l \mu_{kl} CAP_{kl} \qquad (3.10)$$
$$= \sum_i \int_0^{q_i^d} \hat{p}_i^d(q)dq - \sum_i \int_0^{q_i^s} \hat{p}_i^s(q)dq + \sum_k \sum_l \mu_{kl} CAP_{kl}$$

The rearranged Lagrangian function is quite similar to the original (3.1), however, with two alterations. First, the original supply and demand functions are perturbed, and have been shifted by the term $\sum_k \sum_l \mu_{kl} \beta_{kl}^i$, as shown in (3.11).

$$\begin{aligned} \hat{p}_i^d(q) &\equiv p_i^d(q) + \sum_k \sum_l \mu_{kl} \beta_{kl}^i \equiv p_i^d(q) + cc_i \\ \hat{p}_i^s(q) &\equiv p_i^s(q) + \sum_k \sum_l \mu_{kl} \beta_{kl}^i \equiv p_i^s(q) + cc_i \end{aligned} \qquad (3.11)$$

This perturbation is equivalent to the shift in supply and demand curves resulting from the capacity charge approach, where suppliers and consumers in node *i* face a capacity charge

$$cc_i = \sum_k \sum_l \mu_{kl} \beta_{kl}^i \qquad (3.12)$$

Secondly, we have the addition of the last term $\sum_k \sum_l \mu_{kl} CAP_{kl}$. For a given shadow price vector μ , this term is a constant. For optimal shadow prices, μ_{kl}^*, the term is equal to the Merchandising surplus (cf. Wu et al. (1996)), which is equivalent to our definition of grid revenue in (3.1'). Thus, social optimum is achieved by the system operator issuing optimal capacity charges $cc_i^* = \sum_k \sum_l \mu_{kl}^* \beta_{kl}^i$, and subsequently solving the unconstrained optimal dispatch problem by clearing the market according to the perturbed supply and demand functions of (3.11). ■

In our approach capacity constraints are managed by means of nodal capacity charges, which cause shifts in the supply and demand curves. Thus, constraints are implicitly taken care of, and market equilibrium results from clearing the market on a unique price, *p*, the system price of energy. In optimum, the net prices of each node, $p_i = p - cc_i$, are equivalent to the optimal nodal prices of the nodal pricing approach. Note, however, that although optimal capacity charges cc_i are given by (3.12), they are not uniquely defined, but are associated with the load factors. When using load factors associated to a reference point, both the level of the capacity charges and the system price are affected by the chosen reference point. Using optimal shadow prices will however always ensure the same optimal net nodal prices, regardless of which reference point is chosen. As market participants optimally adapt to the net price, i.e. the market energy price corrected by the capacity charge, market equilibrium is not affected by the choice of reference point.

4 Capacity Charges: An Example

To illustrate the nodal capacity charge approach, we construct an electricity market of six nodes, each with production and consumption. As a reference point we show the outcome of optimal unconstrained and constrained dispatch, the latter using nodal prices. With optimally defined capacity charges, we show that the outcome of the capacity charge approach is identical to that of nodal pricing.

Model and Parameters

Generators are assumed to have quadratic cost functions, with a profit function π^s of the general form $\pi^s = (p - cc_i)q_i^s - \frac{1}{2}c_i {q_i^s}^2$, which gives us linear supply curves. We also assume linear demand functions. Supply and demand curves, including capacity charges, are shown as:

$$p = c_i q_i^s + cc_i \qquad (4.1)$$

$$p = a_i + cc_i - b_i q_i^d \qquad (4.2)$$

where $a_i > 0$, $b_i > 0$ and $c_i > 0$ are positive parameters. The parameters of our example are shown in table 4.1.

Table 4.1 Parameters

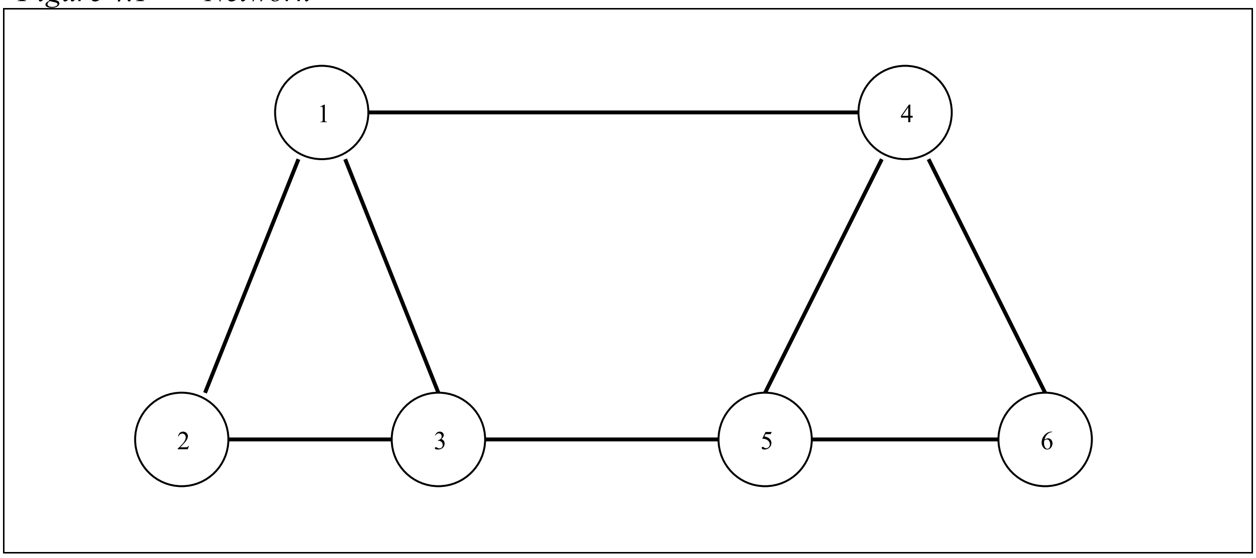
	a_i	b_i	c_i
Node 1	20	0.05	0.2
Node 2	20	0.05	0.1
Node 3	30	0.10	0.7
Node 4	20	0.05	0.2
Node 5	30	0.10	0.7
Node 6	30	0.10	0.1

Social surplus, decomposed into the surpluses of suppliers and consumers, and grid revenue due to congestion, is given by (4.3).

$$\Pi_{ss} \equiv \sum_{i=1}^{6} \tfrac{1}{2}(a_i - p + cc_i)q_i^d + \sum_{i=1}^{6} \tfrac{1}{2}(p - cc_i)q_i^s + \sum_{i=1}^{6} cc_i\left(q_i^s - q_i^d\right) \tag{4.3}$$

The network connecting the six nodes is shown in figure 4.1.

Figure 4.1 Network



We apply the lossless linear "DC" approximation of the power flow equations, with reactances equal to 1 on every link. For given net injections, power flows are determined according to (3.3), (3.4), and (3.5), here stated as follows:

$$\begin{aligned} q_1 &= q_{12} + q_{13} + q_{14} \\ q_2 &= -q_{12} + q_{23} \\ q_3 &= -q_{23} - q_{13} + q_{35} \\ q_4 &= -q_{14} + q_{45} + q_{46} \\ q_5 &= -q_{35} - q_{45} + q_{56} \end{aligned} \tag{3.3'}$$

$$\begin{aligned} q_{13} &= q_{12} + q_{23} \\ q_{13} &= q_{14} + q_{45} - q_{35} \\ q_{13} &= q_{14} + q_{46} - q_{56} - q_{35} \end{aligned} \tag{3.4'}$$

$$q_1 + q_2 + q_3 + q_4 + q_5 + q_6 = 0 \tag{3.5'}$$

The load factors β_{ij}^{lm} show the power flow on line *ij* following from an injection of one unit in node *l* and withdrawing it in node *m*. By solving (3.3'), (3.4') and (3.5') with $q_l = 1$ and $q_m = -1$, we can represent the power flow of network by a matrix of load factors. The matrix of load factors for our example network is shown in table 4.2.

Table 4.2 Load factors

β_{ij}^{lm}		**TRADES (lm)**														
		12	**13**	**14**	**15**	**16**	**23**	**24**	**25**	**26**	**34**	**35**	**36**	**45**	**46**	**56**
	12	19/30	4/15	1/10	1/6	2/15	- 11/30	- 8/15	- 7/15	- 1/2	- 1/6	- 1/10	- 2/15	1/15	1/30	- 1/30
L	**13**	4/15	8/15	1/5	1/3	4/15	4/15	- 1/15	1/15	0	- 1/3	- 1/5	- 4/15	2/15	1/15	- 1/15
I	**14**	1/10	1/5	7/10	1/2	3/5	1/10	3/5	2/5	1/2	1/2	3/10	2/5	- 1/5	- 1/10	1/10
N	**23**	- 11/30	4/15	1/10	1/6	2/15	19/30	7/15	8/15	1/2	- 1/6	- 1/10	- 2/15	1/15	1/30	- 1/30
E	**35**	- 1/10	- 1/5	3/10	1/2	2/5	- 1/10	2/5	3/5	1/2	1/2	7/10	3/5	1/5	1/10	- 1/10
S	**45**	1/15	2/15	- 1/5	1/3	1/15	1/15	- 4/15	4/15	0	- 1/3	1/5	- 1/15	8/15	4/15	- 4/15
(i-j)	**46**	1/30	1/15	- 1/10	1/6	8/15	1/30	- 2/15	2/15	1/2	- 1/6	1/10	7/15	4/15	19/30	11/30
	56	- 1/30	- 1/15	1/10	- 1/6	7/15	- 1/30	2/15	- 2/15	1/2	1/6	- 1/10	8/15	- 4/15	11/30	19/30

For example, a trade consisting of injecting 1 MW in node 1 and withdrawing it in node 3, gives a flow over line 12 equal to $\frac{4}{15}$ in direction from 1 to 2. Likewise, injecting 1 MW in node 2 and withdrawing it in node 3, results in a flow over line 12 of $-\frac{11}{30}$, i.e. a flow of $\frac{11}{30}$ from 2 to 1. If combined, the flows over line 12 from the two trades partly cancel, resulting in a net flow over line 12 equal to $q_{12} = \frac{4}{15} + \left(-\frac{11}{30}\right) = \frac{1}{10}$.

Alternatively, by introducing a reference point *r* for withdrawals, the load factors may be represented by the loading vectors $\overline{\beta}_{\mathrm{ij}}(r)$ for each link *ij*. The load factors using reference point 3 are for example given by β_{ij}^{l3} derived from the five columns for trades with node 3 in table 4.2, (columns 13, 23, 34, 35 and 36), and noting that $\beta_{ij}^{ml} = -\beta_{ij}^{lm}$ and that $\beta_{ij}^{33} = 0$ by definition. This loading vector shows line flows for trades from any injection point to the reference point only. All information of table 4.2 is, however, contained in these five columns. A general trade between e.g. node 1 and 6, may be viewed as a combined trade between node 1 and 3 and between 3 and 6. For example, the flow on line 12 resulting from this trade is $\beta_{12}^{16} = \beta_{12}^{13} + \beta_{12}^{36} = \frac{4}{15} + (-\frac{2}{15}) = \frac{2}{15}$.

Unconstrained optimal dispatch

Assuming no congestion in the network, optimal dispatch and maximum social surplus results from aggregating supply and demand curves, and clearing the market so that the prices of all regions are the same, i.e. $p_1 = p_2 = p_3 = p_4 = p_5 = p_6$. Due to the absence of constraints, all resulting flows are feasible, and capacity charges and grid revenue due to congestion are thus equal to 0. In our example, the market price of energy in the scenario of zero capacity charges is 17.09. Table 4.3 shows the optimal unconstrained dispatch of our example. The table also displays total social surplus, 7552.33, and its allocation to production, consumption and the grid.

Table 4.3 Unconstrained dispatch

	Price	Supply	Demand	Net Injection	Supply Surplus	Demand Surplus	Grid Revenue	Total Surplus	Line	Flow
									12	-34.40
Node 1	17.09	85.47	58.14	27.33	730.43	84.51	0.00	814.93	**13**	43.99
Node 2	17.09	170.93	58.14	112.79	1460.86	84.51	0.00	1545.36	**14**	17.73
Node 3	17.09	24.42	129.07	-104.65	208.69	832.95	0.00	1041.64	**23**	78.39
Node 4	17.09	85.47	58.14	27.33	730.43	84.51	0.00	814.93	**35**	17.73
Node 5	17.09	24.42	129.07	-104.65	208.69	832.95	0.00	1041.64	**45**	43.99
Node 6	17.09	170.93	129.07	41.86	1460.86	832.95	0.00	2293.81	**46**	1.07
Total		561.63	561.63	0.00	4799.96	2752.37	0.00	7552.33	**56**	-42.93

Nodal prices

Now, assume that the capacity of the lines are as shown in table 4.4. In the example we assume that $CAP_{ij} = CAP_{ji}$ [6]. These capacity constraints make the unconstrained optimal dispatch infeasible, as the thermal constraints of lines 23, 35 and 45 would be violated at this solution.

Table 4.4 Line Capacities

Line	Capacity
12	60
13	60
14	60
23	60
35	10
45	30
46	8
56	60

With the nodal pricing approach optimal nodal prices result in optimal dispatch. Table 4.5 displays optimal dispatch, nodal prices, and the allocation of social surplus. In optimal dispatch, we find that the capacity of lines 23, 45 and 46 are binding. Line 35, although expected, is not constrained, while the flow direction of line 46 has changed and the flow limit is binding in optimal dispatch. The table also shows optimal shadow prices for each line. Since $\mu_{ji} = 0$ if $\mu_{ij} > 0$, we have displayed only one shadow price per line. If positive, it indicates that the constraint CAP_{ij} is binding. If negative, it indicates that the constraint CAP_{ji} is binding, where the absolute value of μ_{ji} is the shadow price of the constraint CAP_{ji}.

[6] In reality, this may not be so, as the grids may be operated with different capacities depending on the direction of the flow over the interconnection.

Table 4.5 Optimal dispatch - nodal prices

	Price	Supply	Demand	Net Injection	Supply Surplus	Demand Surplus	Grid Revenue	Total Surplus	Line	Flow	Shadow price
									12	-24.42	0.00
Node 1	17.05	85.23	59.08	26.15	726.42	87.26	-445.78	813.68	**13**	35.58	0.00
Node 2	16.15	161.47	77.05	84.42	1303.69	148.43	-1363.17	1452.11	**14**	14.99	0.00
Node 3	18.71	26.73	112.89	-86.17	250.06	637.26	1612.20	887.32	**23**	60.00	3.46
Node 4	16.28	81.40	74.39	7.01	662.62	138.36	-114.08	800.98	**35**	9.41	0.00
Node 5	19.48	27.82	105.24	-77.41	270.95	553.74	1507.73	824.69	**45**	30.00	6.14
Node 6	17.30	173.00	127.00	46.00	1496.45	806.45	-795.80	2302.90	**46**	-8.00	-1.16
Total		555.66	555.66	0.00	4710.18	2371.50	401.11	7482.79	**56**	-38.00	0.00

This nodal pricing approach follows the concept of nodal prices, as discussed by Schweppe et al. (1988) and Hogan (1992). In order to implement such a system of nodal prices, it is required that the system operator calculates optimal nodal prices on the basis of information of the network, supply and demand.

Optimal Capacity Charges

Under the capacity charge approach a positive or negative capacity charge cc_i is issued to each node, while the market is cleared at a single equilibrium *p*. If capacity charges are announced prior to market clearing, consumers and producers will take the announced capacity charge into account when deciding supply and demand bids. Market response to optimally defined capacity charges, will result in a feasible and optimal market equilibrium. Table 4.6 shows the optimal capacity charges, when the system price is defined as the unconstrained energy price. Note that the net nodal prices, $p_i = p - cc_i$, equal the optimal nodal prices, and that the resulting market equilibrium and social surplus of the two methods coincide. Likewise, if capacity charges are announced *after* bidding and market clearing, we see that it is straightforward to represent nodal prices by a common market price and nodal capacity charges, so that $p_i = p - cc_i$.

Table 4.6 Capacity charges

	Price	Capacity Charge	Supply	Demand	Net Injection	Supply Surplus	Demand Surplus	Grid Revenue	Total Surplus	Line	Flow
										12	-24.42
Node 1	17.09	0.05	85.23	59.08	26.15	726.42	87.26	1.23	814.91	**13**	35.58
Node 2	17.09	0.95	161.47	77.05	84.42	1303.69	148.43	79.83	1531.95	**14**	14.99
Node 3	17.09	-1.62	26.73	112.89	-86.17	250.06	637.26	139.37	1026.69	**23**	60.00
Node 4	17.09	0.81	81.40	74.39	7.01	662.62	138.36	5.69	806.68	**35**	9.41
Node 5	17.09	-2.38	27.82	105.24	-77.41	270.95	553.74	184.50	1009.19	**45**	30.00
Node 6	17.09	-0.21	173.00	127.00	46.00	1496.45	806.45	-9.52	2293.38	**46**	-8.00
Total			555.66	555.66	0.00	4710.18	2371.50	401.11	7482.79	**56**	-38.00

Optimal capacity charges are defined by (3.12), using the optimal shadow prices from table 4.5, and load factors defined by the physical characteristics of the grid from table 4.2. Load factors are defined relatively to the chosen reference point. The level of both the system price and the capacity charges depend on this chosen point of reference. The net nodal price, $p_i = p - cc_i$, as well as the nodal differences between both net prices and between capacity charges, however, are the same, regardless of the chosen point of reference. Table 4.7 shows examples of optimal sets of energy price and capacity charges, depending on the chosen reference point.

Table 4.7 Optimal capacity charges and energy prices

		Basis for determining capacity charges						
	Node	Unconstrained energy price	Reference Point 1	Reference point 2	Reference point 3	Reference point 4	Reference point 5	Reference point 6
Energy Price		17.09	17.05	16.15	18.71	16.28	19.48	17.30
Capacity Charge	**1**	0.05	0.00	-0.90	1.66	-0.77	2.43	0.25
	2	0.95	0.90	0.00	2.56	0.13	3.33	1.15
	3	-1.62	-1.66	-2.56	0.00	-2.43	0.77	-1.41
	4	0.81	0.77	-0.13	2.43	0.00	3.20	1.02
	5	-2.38	-2.43	-3.33	-0.77	-3.20	0.00	-2.18
	6	-0.21	-0.25	-1.15	1.41	-1.02	2.18	0.00

As market participants face identical net nodal prices in all cases, their resulting supply and demand will be the same as in optimal dispatch, i.e. as shown in table 4.5. We will find the same production, consumption, line flows, social surplus and allocation of surplus, including identical grid revenues. Moreover, the grid revenue is equal to the merchandizing surplus under optimal nodal prices. However, although all market participants are equally well off in all cases, we may find that their perception of the situations may differ. For the individual market participant, it may be difficult to see the relation between the market price and the capacity charges. A market participant facing a capacity charge would thus be likely to think of his burden due to the constraint as the capacity charge cc_i he faces, with a total burden of $cc_i q_i^s$ for suppliers and $-cc_i q_i^d$ for consumers. Table 4.8 displays the *perceived* burdens of the consumers and producers due to the constraints.

Table 4.8 Perceived burdens of the constraint

	Node	Basis for determining capacity charges						
		Unconstrained energy price	Reference point 1	Reference point 2	Reference point 3	Reference point 4	Reference point 5	Reference point 6
Producer	**1**	4.00	0.00	-76.60	141.86	-65.27	207.13	21.64
Consumer		-2,77	0.00	53.09	-98.33	45.24	-143.58	-15.00
Producer	**2**	152.70	145.12	0.00	413.88	21.46	537.54	186.12
Consumer		-72,87	-69.25	0.00	-197.50	-10.24	-256.50	-88.81
Producer	**3**	-43.23	-44.49	-68.51	0.00	-64.96	20.47	-37.70
Consumer		182,61	187.91	289.37	0.00	274.36	-86.45	159.24
Producer	**4**	66.16	62.34	-10.82	197.83	0.00	260.16	83.01
Consumer		-60,46	-56.97	9.89	-180.80	0.00	-237.77	-75.86
Producer	**5**	-66.31	-67.62	-92.62	-21.31	-88.92	0.00	-60.55
Consumer		250,81	255.75	350.33	80.59	336.34	0.00	229.03
Producer	**6**	-35.81	-43.93	-199.41	244.02	-176.41	376.50	0.00
Consumer		26,29	32.25	146.39	-179.14	129.50	-276.39	0.00
Total		401.11	401.11	401.11	401.11	401.11	401.11	401.11

For instance, considering the producer in node 6, the coice of node 5 as the reference node leads to a total payment of 376.50, whereas the choice of node 2 as the reference point, induces a total compensation of 199.41. However, since the net prices are identical in all situations, the surpluses of each participant will be the same as shown in table 4.5. For example, the producer in node 6 has a supply surplus of 1496.45 in all cases.

5 An Iterative Adjustment Process

We found that optimally stated capacity charges lead to optimal dispatch. From equation (3.12) we see that the informational requirements for issuing optimal capacity charges are the loading vectors and the shadow prices of congested lines. Loading vectors are technical information, which we assume are readily available. Shadow prices are in principle found by solving the optimal dispatch problem, thus requiring that the system operator has information on cost and benefit functions. When capacity charges are issued prior to bidding and market clearing, shadow prices have to be estimated. Estimated shadow prices can be improved through an iterative process, making use of market responses to obtain good estimates of the shadow prices. Such an iterative approach is similar to that of Wu and Varaiya (1995), however, while they use feasibility and clever market agents (brokers) to arrange multilateral trades, we will use prices and market response to coordinate the nodal markets.

The problem of the system operator in this case is to state capacity charges, based on estimates of the shadow prices of the congested lines, and improved through an iterative process. The iterative approach may be interpreted and implemented in a direct or indirect manner. The direct approach involves a series of actual iterations in clearing the market, where participants after each market clearing receive adjusted capacity charges to which they respond with adjusted supply and demand curve bids. The final market price and capacity charges will be those of the last iteration. While this direct approach would contribute to ensuring "correct" prices for each point in time, the transaction costs of several iterations for each market clearing could be quite large. Alternatively, the iterative approach may be implemented indirectly. On observing a pattern of congestion similar to earlier periods, the iteration comes about when the system operator uses information on earlier market responses to improve estimates. This may be a more cost efficient method, and justifiable if congestion situations last for a period of time. In this case, market responses to earlier capacity charges may be used to obtain better estimates. It is also possible to start with an estimate based on information obtained from earlier estimates and market observations, and improve capacity charges through a few iterations.

We will illustrate the iterative process by using a simple updating procedure in our example, assuming the direct interpretation, or alternatively, identical market conditions in consecutive time slots. In the example we start with capacity charges set to zero, implying that no lines are congested. This results in the unconstrained dispatch solution, which is not feasible. Alternatively, starting points may be based on forecasts of congested lines and shadow prices.

In each iteration shadow prices, and consequently capacity charges, are updated, to relieve the congested lines. The objective here is to illustrate the approach, rather than finding the most efficient rule in this case. There is a vast literature on algorithms for updating, see for example Minoux (1986). For illustration, we have employed a standard gradient method, where the shadow prices are updated on the general form:

$$\mu_{ij}^{t+1} = \mu_{ij}^{t} + \lambda_{ij}^{t} \frac{\gamma_{ij}^{t}}{\left\| \gamma_{ij}^{t} \right\|} \tag{5.1}$$

where μ_{ij}^t is the estimated shadow price of line *ij* at iteration *t*, γ_{ij}^t is a gradient of the objective function valued at iteration *t* of our objective function, $\left\|\gamma_{ij}^t\right\|$ is a normalization of the gradient, and λ_{ij}^t is the step chosen at time *t*. In the example, we have defined the terms as follows:

$$\gamma_{ij}^t = \frac{\partial L^t}{\partial \mu_{ij}} = CAP_{ij} - \sum_k \beta_{ij}^k q_k^t = CAP_{ij} - q_{ij}^t \tag{5.2}$$

$$\left\|\gamma_{ij}^t\right\| = CAP_{ij} \tag{5.3}$$

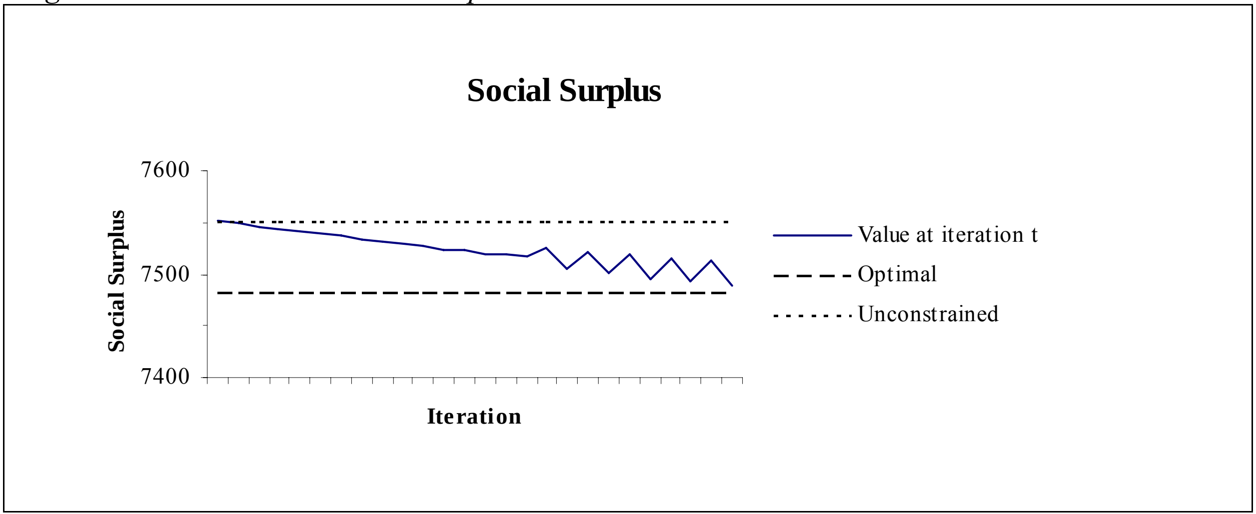
where we see that γ_{ij}^t is the under- or over-utilization of the line, and by normalizing by CAP_{ij}, we have the relative under- or over-utilization of the line.

If $CAP_{ij} - q_{ij} < 0$, the line is congested, requiring the shadow price estimate of line *ij* to be raised. If $CAP_{ij} - q_{ij} > 0$, the line is not congested and any shadow price estimate for the line has to be driven towards 0. Thus the step λ_{ij}^t is negative. The size of the step λ_{ij}^t determines the speed of change. In our example relatively small steps induce a slow convergence towards the optimal value, while larger, but still moderate steps give a faster convergence. However, steps which are large relative to the congestion of the line, cause an oscillation of capacity utilization and shadow price around the optimal values. A mixed definition related to the degree of capacity utilization may give a faster convergence when over-utilization is high, while reducing oscillation around the optimal value, for example as shown in (5.4).

$$\lambda_{ij}^t = \begin{cases} -1 & \text{for} \quad \dfrac{\left|CAP_{ij} - q_{ij}^t\right|}{CAP_{ij}} > 0.1 \\ -0.1 & \text{for} \quad \dfrac{\left|CAP_{ij} - q_{ij}^t\right|}{CAP_{ij}} \leq 0.1 \end{cases} \tag{5.4}$$

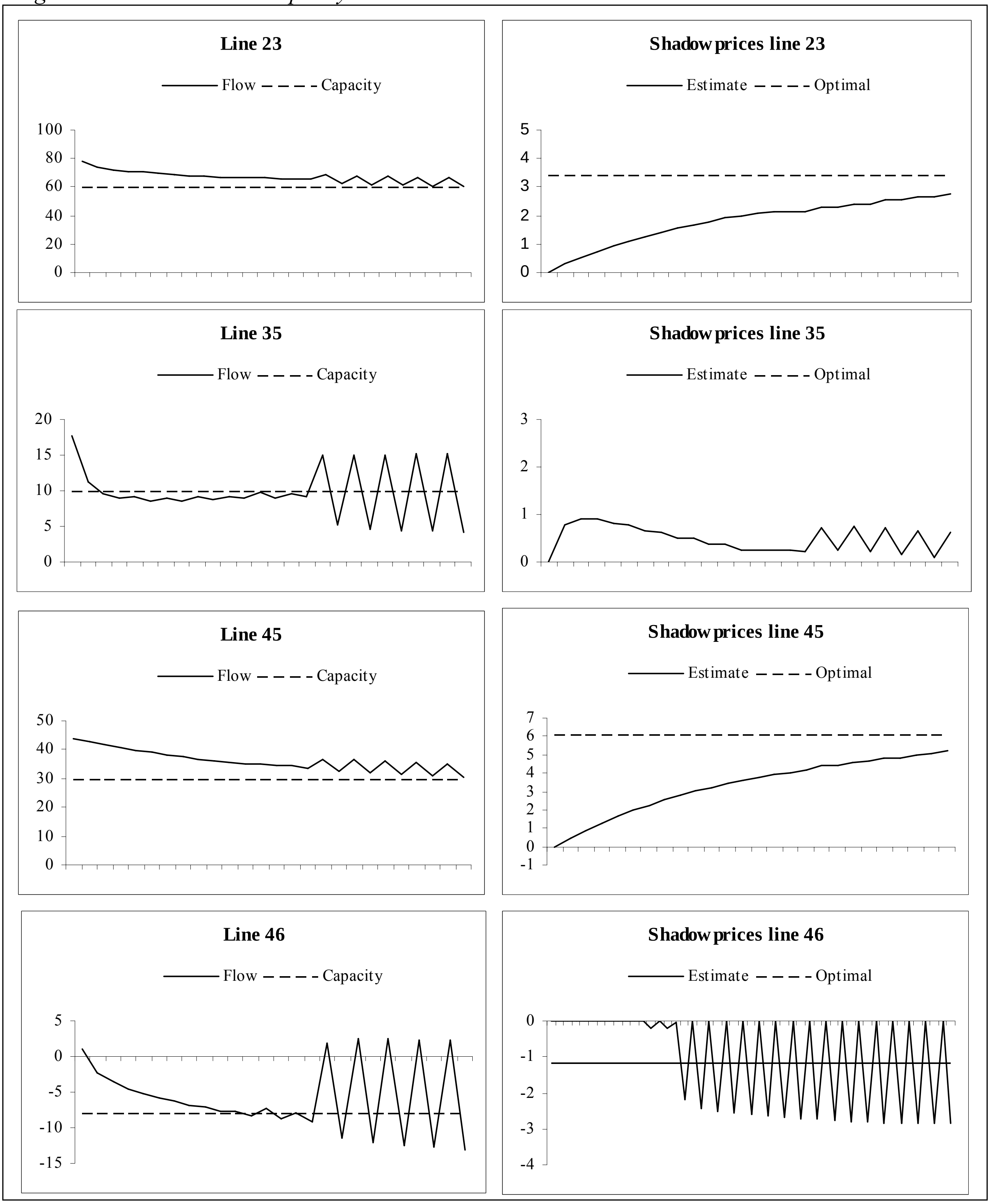
Figures 5.1 shows the resulting development of social surplus in 25 iterations defined by (5.1)-(5.4) and non-negativity constraints on the shadow prices. Starting from the value connected to the infeasible unconstrained case, social surplus evolves towards the level of the optimal case, however oscillating due to our rather crude definition of the iteration process.

Figure 5.1 Iterations: Social surplus



The corresponding development of capacity utilization and shadow prices is displayed in figure 5.2. The left hand side displays the capacity and capacity utilization of the lines that are constrained starting with zero capacity charges, and/or that are constrained in optimal dispatch. As in the above tables, capacities and flows of the lines *ij* are displayed as a positive number when the flow is in the direction *i* to *j*, and as a negative number when the flow is in the direction *j* to *i*. The right hand side shows the estimated shadow prices, where we equivalently have determined $\mu_{ij} > 0$ as the shadow price for the capacity in the direction of *i* to *j*, while $|\mu_{ij}|$ is the shadow price of μ_{ji} if $\mu_{ij} < 0$.

Figure 5.2 Iterations: Capacity Utilization and Shadow Prices



In the unconstrained market solution, we find that the flows over lines *23*, *35* and *45* exceed capacity. This indicates a positive shadow price for each of these lines, and the adjustment rule induces a rise in the shadow price estimate. Nodal capacity charges are issued based on these estimates, and the result in the second market clearing is a reduction of the actual flow over these lines. Iterations gradually increase the shadow price estimates of the lines, further reducing the flow over the capacitated lines towards a feasible solution.

It should be noted that the flow over all lines changes as a result of the capacity charges and the corresponding market equilibrium. Line *35* is initially infeasible in the unconstrained solution with no capacity charges, and the shadow price of this line is estimated to be positive. However, though initially infeasible, changes in the flow of other lines due to capacity charges, actually induce a lower flow over line *35*, making the resulting flow feasible in optimum. Lines *12*, *13*, *14*, *46* and *56* are initially feasible. New market solutions as a result of the stated capacity charges causes changes in lines *12*, *13*, *14*, and *56* with flows still within the line capacity. Line *46* starts with an initial feasible flow of 1.07 in the direction from node *4* to node *6*. Changes elsewhere in the network gives a reduced flow, and subsequently a change in the flow direction, from node *6* to node *4*, as illustrated in the figure by negative numbers. Further changes in the network implicates a required flow from *6* to *4* beyond the capacity of the line, thus inducing a positive shadow price estimate.

The above iteration procedure illustrates the use of the gradient as indicating the direction for updating the shadow price estimate. In our case example, we find that with a constant small step, e.g. 0.1, the line flow is driven asymptotically towards the capacity, albeit necessitating a large number of iterations. A higher step size would speed the process, when the shadow price is far from the optimal value, but result in an oscillation around the optimal value when near, as shown in the example. The engineering of more efficient algorithms may reduce the number of iterations called for. However, taking into account the costs of iterations, in order to obtain feasible flows within a small number iterations, the adjustment procedure has to be combined with some other mechanism as e.g. curtailment or counter purchases.

6 A Heuristic Procedure

Above we have illustrated how iterations based on a simple standard gradient method can bring the market solution towards optimal dispatch, and lower the line flows of constrained lines towards capacity limits. We see that this procedure may require a rather large number of iterations to reach the optimal solution. Comparing the cost of further iterations with the gain in social surplus, it may be optimal to terminate the iterative procedure before reaching the optimal solution. However, note that the illustrated procedure is an upper bounding procedure, where the line flows of constrained lines are driven to the capacity limit from above. By prior termination of the iteration procedure, the resulting flow would not be feasible. To find a feasible flow, an alternative is curtailment, however, a problem is to curtail such that the resulting quantities constitute a market equilibrium. An infeasible flow can also be corrected through a secondary market, for instance a market organizing counter-purchases. An alternative or supplement to such cut-off mechanisms is to “force” the iteration itself to reach a feasible solution. This section discusses a heuristic approach for finding a feasible flow that is also a market equilibrium, and as we will see, also brings us near the optimal solution.

The proposed heuristic procedure is based on Everett (1963) and reviewed below. Let us first summarize the problem. Focusing on the capacity constraints, the optimal dispatch problem of (3.8) may be formulated as follows:

$$\begin{aligned} &\textit{maximize} && \Pi_{ss}(\overline{q}) \\ &\textit{subject to} && q_{kl}(\overline{q}) \le CAP_{kl} \qquad \forall\, kl \end{aligned} \tag{3.8'}$$

where $\overline{q} = (q_1^s, \ldots, q_n^s, q_1^d, \ldots, q_n^d)$ is the vector of production and consumption in each node, and $q_{kl}(\overline{q})$ is the flow on line *kl*. The shadow price vector $\overline{\mu}$ gives us the shadow prices for all lines in both directions. The Lagrangian function is:

$$L(\overline{\mu}) = \Pi_{ss}(\overline{q}) + \sum_k \sum_l \mu_{kl} \left[CAP_{kl} - q_{kl}(\overline{q}) \right] \tag{3.9'}$$

In this setting Everett’s theorem can be stated as follows:

1. Choose an arbitrary vector $\overline{\mu}$ of non-negative shadow prices for all lines.
2. Find a solution $\overline{q}^*$ which maximizes the unconstrained Lagrangian function $L(\overline{\mu})$.
3. Then, $\overline{q}^*$ is the solution to the constrained maximization problem with the same objective function as (3.8'), but with modified capacity limits $CAP_{kl}^{'}$, given by $CAP_{kl}^{'} = q_{kl}(\overline{q}^*)$.

The ex ante announcement of capacity charges is easily interpreted within this theorem. In order to set capacity charges according to (3.12), i.e. $cc_i(\overline{\mu})$, the system operator has to choose an arbitrary set of shadow prices $\overline{\mu}$. Market participants bid on the basis of the capacity charges. By clearing the market on a single spot price, we thus actually find the unconstrained maximum of the new Lagrangian function. The solution $\overline{q}^*$ is a true market equilibrium, and is both feasible and optimal with respect to the *modified* constraints. Our problem is to reach a feasible and optimal solution within the *real* constraints of the network, calling for an adjustment of shadow prices through an iterative process. Different sets of $\overline{\mu}$ lead to different flows. However, if the chosen $\overline{\mu}$ gives a feasible solution, and the flow actually equals the real constraint, the Everett theorem states that this solution also is the optimal solution to the original constrained problem.

Based on this insight, we have slightly modified the updating procedure. The main issue is to reach shadow prices (and thus the capacity charges) that will give rise to a flow that matches the constraints. As a heuristic we have tried to speed movement towards a feasible solution by simply exaggerating the over-utilization by redefining the capacity size used in calculating both the gradient and the step. Starting with a market equilibrium that causes an infeasible flow over line *ij*, adjustment, the updating procedure is still given by (5.1) - (5.4), with one alteration. We have substituted the real capacity CAP_{ij} with $CAP_{ij}^{'} = \alpha_{ij} CAP_{ij}$, where $\left|\alpha_{ij}\right| < 1$. The relative over-utilization of the line will then be exaggerated, the normalized gradient will be larger, and with the specified step of (5.4), the step also might be higher. This causes a faster adjustment of the shadow prices towards a choice that gives a feasible flow. Note that not all lines become feasible within the same iteration. Thus, even though a given line becomes feasible within a given market equilibrium, further iterations may be called for to obtain feasible flows over other lines.

Subsequent iterations however slightly alter equilibrium supply and demand, causing changes in line flow over all lines. There will thus still occur changes in the line flow over lines that have become feasible. The moment the flow of a given line becomes feasible with respect to the real capacity, any further adjustments in shadow prices have been made using the real capacity in (5.1) - (5.4).

To obtain a market equilibrium, supply and demand must balance according to the net prices of the nodes, and at the same time obtain a flow that is feasible. There exists many market equilibria, though they do not represent the optimal dispatch, cf. Wu et al. (1996). Market clearing ensures that supply and demand balances. By Everett's theorem we find that if the line flows defined by the market solution are feasible and equal the capacity limit for binding constraints, the resulting market solution is indeed the optimal dispatch.

Figures 6.1 and 6.2 illustrate a simple version of the heuristic, where $\alpha_{ij} = 0.7$ is constant and equal for all lines. In figure 6.1 we see that social surplus more quickly advances the optimal level of social surplus than is the case without the heuristic.

Figure 6.1 Iterations: Social surplus

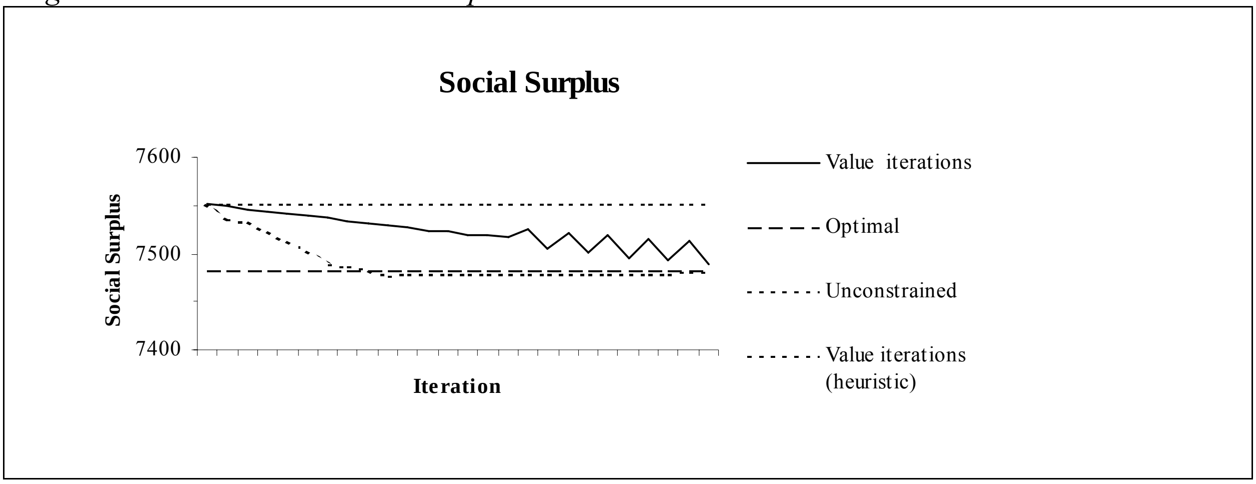
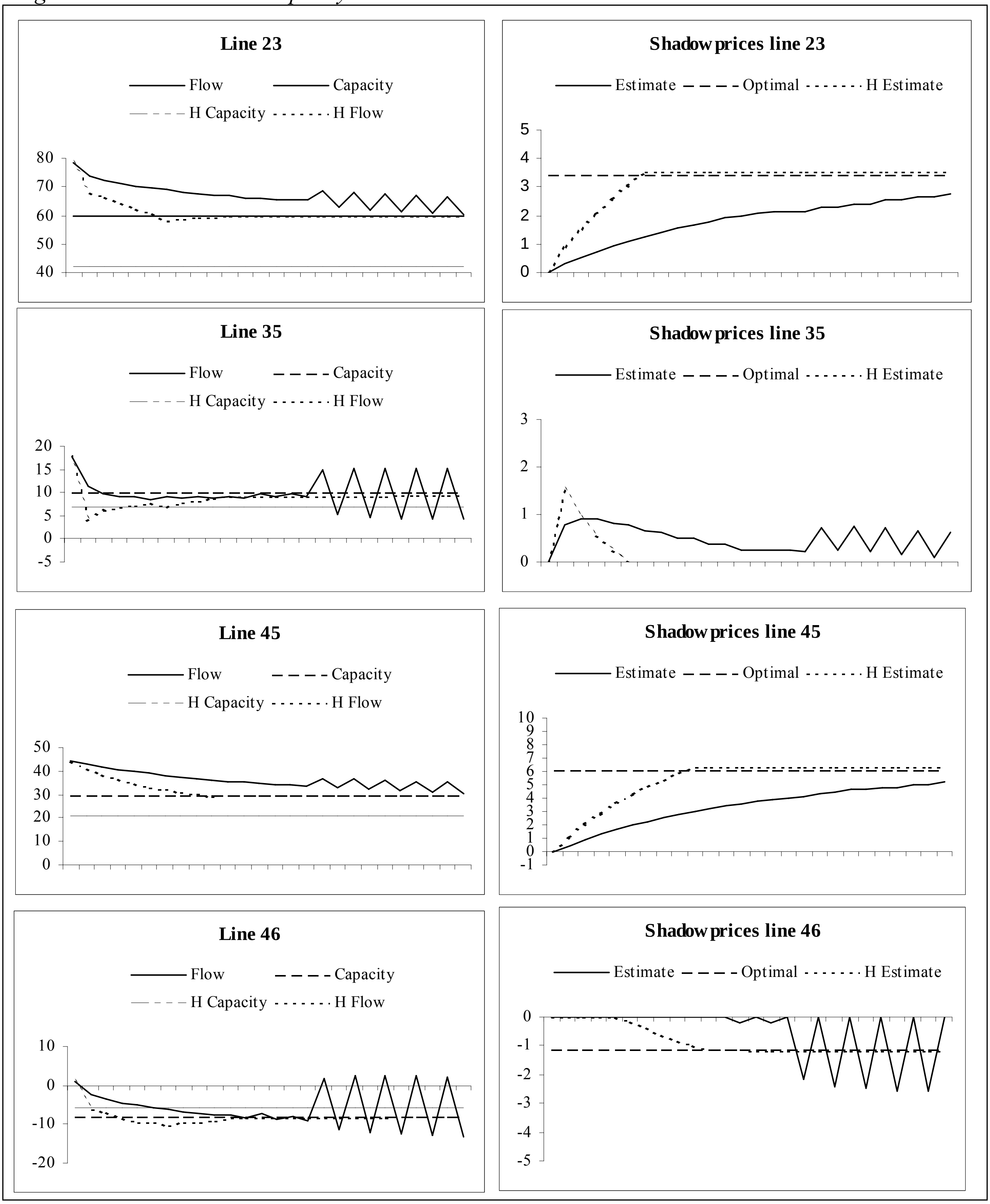


Figure 6.2 shows how the heuristic effects line flow and shadow price estimates. Even with this rather crude definition of the heuristic, using the same α_{ij} for all lines, we see that the line flows of the constrained lines become feasible in less iterations. Also note that the estimated shadow prices come close to the optimal shadow prices.

Figure 6.2 Iterations: Capacity Utilization and Shadow Prices



7 Summary and Topics for Future Research

Electricity markets have been reorganized, introducing competition in supply and demand to achieve greater efficiency. Capacity constraints in electricity networks represent rather complex constraints on an efficient market solution, due to the externalities created by the loop flow. Several different market designs have been chosen for handling network constraints. Optimal nodal prices, as the solution of the welfare maximization problem, represent an efficient market equilibrium, where optimal nodal prices reflect both supply and demand conditions, together with the constraints of the network. In the capacity charge approach we combine several of the suggested approaches, while at the same time trying to withhold the optimal characteristics of nodal prices. An objective is to find good approximations of the optimal nodal prices based on a competitive market price and nodal capacity charges issued by the system operator.

A main issue of the approach is the role of capacity charges as important market signals for demand and supply. The capacity charge approach is not a flowgate based approach, but signals the nodal cost of aggregate congestion in the network. Compared to a approach such as zonal pricing, the method incorporates the advantages of nodal pricing. Capacity charges apply to all contracts of physical delivery, facilitating the co-existence of exchange traded and bilaterally traded contracts. The calculation of capacity charges is based on technical information, together with estimates of shadow prices. The capacity charges are issued by the system operator. This recognizes the coordination task performed by the system operator, as well as the system operator's access to important information on physical dispatch. At the same time, the approach enables a clearer distinction between the role of the exchange and the system operator, and might facilitate coordination in areas with separate exchanges. The efficiency of the approach is however contingent on the quality of shadow price estimates. We show that estimated shadow prices can be improved through a (direct or indirect) iterative process, making use of market responses to obtain good estimates of the shadow prices. A potential source of inefficiency is related to the system operator's incentives for stating capacity charges that boost their revenue. This is an issue for further investigation, and should be seen in connection with the regulation of the grid-company.

In principle, capacity charges may be announced before or after market bidding. If capacity charges are announced after market clearing, the approach is equal to the nodal pricing approach. When announced prior to bidding and market clearing, capacity charges give signals to the market as to expected congestion. Within the market framework of a separate day-ahead scheduling market, and a separate real-time balancing market, as the Norwegian system, market participants act upon this information in stating their supply and demand bids. The efficiency of the market equilibrium can be improved through an iterative adjustment process to reach an optimal and feasible solution. Within the framework of a real-time spot market, the ex ante announcement of capacity charges signals expected congestion prices, and may induce a shift in supply and demand which otherwise would be inelastic with respect to the real-time spot price.

We think the capacity charge approach may be a realistic procedure for managing transmission constraints in a competitive electricity market, for example in the Norwegian scheduled day-ahead power market. The capacity charge approach bears a close relation to the system used in Norway today. The Norwegian system operates with a general price referred to the "system price of energy" which is the price of unconstrained dispatch. Estimated congestions at the time of market clearing are mainly managed through zonal pricing. With the capacity charge approach, however, capacity charges are announced prior to market clearing, and for each node instead of only for a few zones. Also, the use of nodal capacity charges is similar to what is already done for marginal losses, in that loss factors are published for every connection point in the central high voltage grid. In trading there is now an approximate 70/30 split on bilateral trading vs. trading on the pool. We also believe that the suggested procedure will easily facilitate bilateral trading to go alongside with the pool. As for the iterative process, we note that many congestion situations in the Norwegian system seem to last for a period of time, and that the zonal division only comes into practice when the congestion is expected to last for several days. In this case, market responses of initial capacity charge estimates might be used to obtain better estimates. To ensure real-time balance of the system, this is handled by counter-purchases through the regulating market, a market that is already present.

Further development of the capacity charge approach, however, still leaves a number of questions to be investigated. First, we would like to do numerical tests on larger networks, and the exact procedure used for adjusting shadow price estimates is of special interest. In this, we can rely on optimization theory, for instance to find an adjustment scheme producing near-optimal dual variables within a few iterations. In this setting, we should also consider how close to optimality we need to be in order for the system to perform satisfactorily. In this case the procedure has to be combined with some other mechanism to obtain feasible flows.

Moreover, it would be interesting to perform simulations, where market data are slightly perturbed in each step. This would simulate how grid revenue develops in an adjustment process where we use market information from similar congestion situations to obtain initial guesses on the shadow prices. Since the suggested procedure also involves a mechanism like for instance curtailment or counter purchases to obtain feasible flows, the specific design of the mechanism is of special interest, also taking into account how it affects the performance of e.g. separate real-time balancing power markets. Furthermore, gaming possibilities and more generally, regulatory issues should be examined.

Employing the "DC" approximation of the power flow equations, we do not focus on transmission losses, although a complete system for transmission pricing should address losses as well. We believe that marginal losses can be readily taken care of in our approach by issuing nodal loss factors similar to what is already done in the present Norwegian system, but possibly with a "hub" as a reference point when computing loss factors. If the pool is located at the "hub", the clearing price of the pool could be used in pricing marginal losses. One of the other simplifications of the "DC" model is that load factors are constants. The non-linear nature of the AC power flows implies that loss factors depend on loads, however our approach is still valid if the load factors we use are marginal load factors (Stoft (1998)). So, if we are to use the full AC power flow model, in principle, we have to recalculate the load factors whenever the load changes, in addition to solving non-linear systems.

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