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Implementing Stochastic Bio-Economic Model For North-East Artic Cod Fishery

by

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Implementing Stochastic Bio-Economic Model for North-East Arctic Cod Fishery

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Abstract

This paper studies how a stochastic model can be used to determine optimal levels of exploitation of the Norwegian cod fish species. A noncritical depensation growth model is developed for this species in order to examine both deterministic and stochastic results. Estimation of the biological and the noise term parameters in the stochastic biomass dynamics involved simulation and the empirical North East Arctic Cod (NEAC) data sets for the years 1985 – 2001. Kolmogorov Smirnov criterion based estimation method is used to estimate both drift and diffusion parameters simultaneously. The estimates are quite reasonable and the model is able to capture the salient features of the NEAC. The model is used to derive numerically optimal level of exploitations under different diffusion functions in the stochastic case and various discount rates in the deterministic case. Optimal catches are compared to the historical catch records. A striking feature of our modeling approach is that these records fit surprisingly well the infinite discounting tracks, i.e., the bliss solution. Our general results indicate that over fishing resulted from lack of long term planning as well as non adequate response to uncertainty.

Key words: North East Atlantic Cod, Stochastic bio-economics model, Optimal control, Parameter estimation

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1 Introduction

Biomass uncertainty is prevalent for renewable resources and it can play an important role in applied fisheries management. The presence of stochastic fluctuations in resource stocks implies uncertainty regarding not only present and future stock abundance but also the effects of exploitation on resource productivity. Yet simple rules that can be used to account for biological uncertainty and the possibility of resource extinction are still not available to resource managers.

The literature in this area has grown dramatically during the last few years. The fruitful discussions by Clark [5] contain important practical implications although his work was mostly focused on deterministic models. He points out that many of the stock-growth and stock-recruitment relationships are poorly understood and difficult to be estimated by using the existing noise corrupted data. This leads to the issue of uncertainty and its effects on fishery management.

A fundamental intention of most marine management regimes is to ensure conservation of the fishery resource into the future and hence the sustainability of fishery exploitation. Because of seemingly random fluctuations it is very difficult to reach either of these objectives in reality. Management procedures are therefore imposed upon the industry in order to control the output, catches or landings, of the fishery. Feedback rules for adaptive management of fishery resources can be introduced to find the suitable yield levels where the stock dynamics are either deterministic or stochastic. A deterministic feedback rule is used by Grafton, Sandal and Steinshamn [7] and a deterministic and stochastic model by McDonald, Sandal and Steinshamn [12] to verify Canada's Northern Cod fishery and Southern Bluefin Tuna fishery respectively.

The purpose of this paper is to study the implications of uncertainty on the optimal management of the North East Arctic Cod (NEAC) fishery using an aggregated bio-economic model. We introduce the issue of process uncertainty in the form of a stochastic model of NEAC biomass dynamics and leave other sources of uncertainty for future investigation. The NEAC is the most important species along the cost of Norway and Northern Russia. This fishery has played an important economic role within the coastal communities for the past hundreds of years. It has large variations in the annual harvesting quantities. The stock size fell from its highest level in 1946 to the lowest level in 1981. However, the stock seems to be recovering from the depleted state in the 1990s due to improved management strategies.

A bio-economic model was developed in Arnason et al. [4] for NEAC in a deterministic context in order to examine the economic efficiency of the Norwegian , Danish and Icelandic cod fisheries. In the present work, a stochastic continuous time bio-economic model is developed for NEAC. The random component in the resource dynamics becomes an Itô variable and the population dynamics is described by a stochastic differential equation. Managing fishery in this context suggests formulating the management problem as a stochastic optimal control problem where the objective is to find a harvesting rule that in our case will be suitable for the North east Arctic Cod stock.

The model presented here is an aggregate bio-economic model, that uses lump variables and provides rules of thumb for quota management of the stock. This helps to avoid over parameterization of the model and lack of over view and causality in the dynamics. For this reason the parameter estimations should not be judged as an econometric work but rather as an attempt to keep the number of parameters low in order to make a simple and representative aggregated dynamic model.

Estimating the parameters for both drift (biological) and diffusion terms of this non-linear model has also been done by using the NEAC stock data. This paper employs the technique of Kolmogorov Smirnov criterion based estimation method (McDonald and Sandal [11]) in which all the available information is used in the model of the NEAC stock. Our model which is based on parameter estimation in a stochastic differential equation is novel for this particular fishery and has obvious advantages compared to traditional approaches.

Parameter estimation in both drift and diffusion terms in stochastic differential equations is a complicated undertaking as the estimation in both terms has to take place simultaneously and one can not expect stationarity. Our method can be used to estimate both drift and diffusion parameters of the stochastic differential equation simultaneously. The way we estimate the parameters can be used even when the number of observations in each replicate is quite few (less than twenty).

In order to study optimal harvest levels for a marine resource that is subject to both stochastic and deterministic dynamics we have to resort to numerical methods. We compare our derived optimal exploitations with the actual exploitation of the NEAC. The results reported in this work provide a benchmark for the real management policy.

It is hard to construct a detailed empirical model for any fishery. In the NEAC fishery we have acsess to a fair amount of data although the qualiy may be variable. The method and results proposed here do not provide the full story but it is an important step towards a comprehensive bio-economic model and provides the approximate key levels of optimal harvesting policies. It is an efficient tool for investigating the exogenous changes in the optimal yield paths (policies) and allows us to compare harvesting policies in different environments.

This article is organized as follows: First a stochastic bio-economical model is formulated and a solution method for the stochastic optimal control problem is discussed. Then a condensed study of the parameter estimation is presented and followed by a brief history of the NEAC fishery. Finally, optimal harvest levels are presented and discussed for deterministic as well as stochastic cases.

2 An optimal management strategy

Bio-economic models have two principle components, a *biological part* which defines the natural constraints and an *economic part* which characterizes the policy of the fishery management. Our attention here will basically focus on a non-linear mathematical model of a renewable marine resource with stochastic fluctuation.

In recent years an increasing number of stochastic optimization models have appeared in the resource economics literature. The real world resource and environmental economics are always uncertain. An important characteristic of stocks of renewable resources, such as a fish population, is volatility and fluctuations. The variation in a fish stock appears to be related to migrations and spawning patterns and changes in the marine ecological system, among others. A stochastic and fluctuating environment make the stock a random variable. A convenient way of modeling the random stock dynamics is to use a *Stochastic Differential Equation* (SDE) of the form

$$ds_t \;=\; \left[g\left(s_t\right) \;-\; u_t\right] dt \;+\; \sigma\left(s_t\right) dB_t\,. \tag{1}$$

The biomass s_t is a random variable on $[0\,,\,\infty)$, $g\,(.\,)$ is the growth of the biomass, $\sigma\,(.\,)$ is the diffusion term. The functions $g\,(.\,)$ and $\sigma\,(.\,)$ are twice continuously differentiable. The term dt is a time increment and dB_t is an increment of a standard Brownian motion which is independent and identically distributed.

We suppose that $g\,(0) = 0$ and that there exists $K > 0$ such that $g\,(.\,)$ is strictly positive on $(0, K)$ with $g\,(K) = 0$ and $g\,(s) < 0$ for $s > K$. K is called the *environmental carrying capacity* of the resource. In addition we assume that $\sigma\,(.\,)$ is strictly positive on $(0, \infty)$ and $s_t = 0$ is an absorbing boundary, (i.e.) , if s_t ever becomes zero, the resource becomes extinct. This is a direct extension of Clark's [5] bio - economic framework to include stock uncertainty. The specification assumes that the size of the resource stock in the current period is known without error and that the change in stock size is composed of a deterministic part $[g(s_t) \;-\; u_t]\; dt$ and a random part $\sigma(s_t)\; dB_t$. This stochastic process describes the dynamics of the aggregated stock biomass.

A well defined resource management problem needs a clear objective. In order to determine an optimal level of exploitation for the fishery an economic component to model is also required. A reasonable approach will be to use the net revenue, i.e., the total revenue from fish harvest less total operating costs, as given by

$$\Pi \;=\; \Pi\left(s_t, u_t\right) \;=\; p\left(u_t\right) u_t - c\left(s_t, u_t\right), \tag{2}$$

where Π is *net revenue* at time t from having a resource stock of size s_t and harvest u_t. Here $p\,(.\,)$ and $c\,(.\,)$ have their usual meanings. The inverse demand function for the landings of cod is assumed to be linear and decrasing, the cost function a decreasing function of biomass and an increasing function of harvest.

When the growth of the resource is stochastic, the objective of the management is to maximize the expected present value of the return from a harvest schedule, u, over an infinite horizon. This is given by

$$\mathbf{J}(s_t, u_t) \;=\; \mathbb{E}\left[\int_0^\infty \; e^{-\,\delta\,\tau}\;\Pi(s_\tau, u_\tau)\;d\tau\right], \tag{3}$$

were $\delta > 0$ is the rate of discounting and $\mathbb{E}$ denotes the expectation operator. The dynamic optimization problem can be written as

$$\mathbf{V}(s) \;=\; \max_{u \geq 0} \quad \mathbf{J}(s_t, u_t); \quad s(t=0) \;=\; s_0 \;=\; s, \tag{4}$$

subject to the dynamics of the fish biomass constraint given by Eq. (1) and the appropriate boundary conditions. The optimal control (harvest) can be found by solving the associated *Hamilton Jacobi Bellman* (HJB) equation:

$$\delta\mathbf{V} \;+\; g(s)\mathbf{V}_s \;+\; \frac{1}{2}\,\sigma^2(s)\mathbf{V}_{ss} \;+\; \max_{u \geq o}\;[\Pi(s,u) \;-\; u\mathbf{V}_s] \;=\; 0, \tag{5}$$

where the subscripts denote derivatives. From Eq. (5) it follows that the policy relation is given by

$$u^* \;=\; \begin{cases} 0 & \text{if } \;\Pi_u(s,0) - \mathbf{V}_s < 0 \\ \text{u} \geq \;0 & \text{if } \;\Pi_u(s,u) - \mathbf{V}_s \geq 0 \end{cases}. \tag{6}$$

Solving the HJB equation together with appropriate boundary conditions is a difficult task. This problem was adressed in Sandal and Steinshamn [14]. They produced closed form approximations by using perturbation techniques. We solve this problem using numerical methods. Numerical algorithms for optimal stochastic control problems can be found in, e.g., Kushner and Dupuis [10]. Their approach is based on on probability theory and is referred to the *Markov chain approximation method*, which we adopt in this work.

3 Criterion function based parameter estimation

Over the last few years various new tools have emerged in marine resource modeling and financial markets leading to a demand for adaptable estimation methods for relevant model parameters. In the present work we have used a method based on Non - Parametric techniques which estimates simultaneously drift and diffusion coefficients of the stochastic population dynamics. It is a *Kolmogorov Smirnov criterion based estimation* which to our knowledge was first utilized in this context by Sandal and McDonald [11].

The method relies on the existence of observed (real) replicated time series data and replicated simulations uses the specified stochastic process. The parameter estimation is then feasible by optimizing the fit between the real and simulated data over a reasonable interval for the SDE parameters (Eq. (1)).

The Kolmogorov Smirnov two sample goodness of fit test is used to compare the empirical distribution functions of two multiple time series samples. Following Gibbons [6], the Kolmogorov Smirnov two sample statistic

$$D_{m,n} = \max_{x} |S_n(x) \; - \; S_m(x)| \, ,$$

where $D_{m,n}$ is the maximum absolute difference between the two empirical distributions $S_m(x)$ and $S_n(x)$, has an asymptotic distribution

$$\lim_{m,n \to \infty} \left\{ \mathbb{P} \left[\sqrt{\frac{mn}{m+n}} D_{m,n} \; \leq \; D \right] \right\} \; = \; L(D) \tag{7}$$

with the Kolmogorov Smirnov likelihood

$$L(D) \; = \; 1 \; - \; 2 \sum_{i=1}^{\infty} (-1)^{i-1} e^{-2\, i^2 \, D^2}. \tag{8}$$

These empirical distribution functions were derived from the ordered statistics and correspond to the two random samples of sizes m and n from the observed and the simulated time series respectively. The empirical distribution function $S_\nu(x)$ for a random sample of size ν is defined as

$$S_\nu(x) = \begin{cases} 0 & \text{if} \quad x < X_{(1)}, \\ \frac{k}{\nu} & \text{if} \quad X_{(k)} \leq x \leq X_{(k+1)}, \qquad \text{for } k = 1, 2, ..., \nu - 1, \\ 1 & \text{if} \quad x \geq X_{(\nu)}. \end{cases}$$

where $X_{(1)}, X_{(2)}, ..., X_{(\nu)}$ denote the order statistics of a random sample and $\nu \; \equiv \; [m, n]$. The Kolmogorov Smirnov statistic can be used to test the hypothesis that the population distributions have been drawn from the same population.

In this paper, one of these samples is the real data and the other is generated from the specified stochastic differential equation. Here we consider replicated time series data from both real and simulated data sets. This provides the opportunity to evaluate the empirical distributions and to determine the Kolmogorov Smirnov statistic, D_t, for each time period t. Using the asymptotic null distrbution, we take the criterion function, Φ, to be the product of the Kolmogorov Smirnov likelihoods, $L(D_t)$, computed at each time step:

$$\Phi \; = \; \prod_{t=1}^{T} L(D_t) \tag{9}$$

where T is the length of the time series and

$$L(D) \; = \; 1 \; - \; 2 \sum_{i=1}^{N} (-1)^{i-1} e^{-2\, i^2 \, D^2} \tag{10}$$

is the truncated likelihood function from (8).

In order to implement this method a scheme for the numerical integration of equation (1) is required. The strong order Milstein scheme (Kloeden and Platen [9]) is used in this application. To apply a numerical method to Eq. (1) over a finite interval $[0, T]$, we first discretize the interval. Let $\Delta t = \frac{T}{l}$ for some $l \in \mathbb{N}$ and $\tau_j = j\Delta t$. Our numerical approximation to $s(\tau_j) = S_j$ is given by

$$S_{j+1} = S_j + [g(S_j) - u_j]\,\Delta t + \sigma(S_j)\Delta B_j + \frac{1}{2}\sigma(S_j)\,\sigma'(S_j)\left[\Delta B_j^2 - \Delta t\right] \tag{11}$$

where $\Delta B_j = B_{j+1} - B_j$ and $\sigma'(S_j) = \frac{\partial \sigma_j}{\partial S_j}$.

4 North-East Arctic Cod Fishery

Cod (*Gadus morrhua*) is the main basis of the Norwegian commercial white fish industry. There are basically two types, the *coastal cod* and the *migratory cod* (North - East Arctic Cod, NEAC, also called Barents Sea cod) which is one of Norway's key fish stocks. The NEAC has its main distribution from the west cost of Norway to the Spitsbergen and Novaja Semlja islands. It is found in an area about 3116000 km^2 in north to approximately 81^0N, which includes the Barents Sea (area I) and the Norwegian Sea (area II) west to 11^0W and south to $63^0 - 64^0$N (see Fig.1 for details [1]).

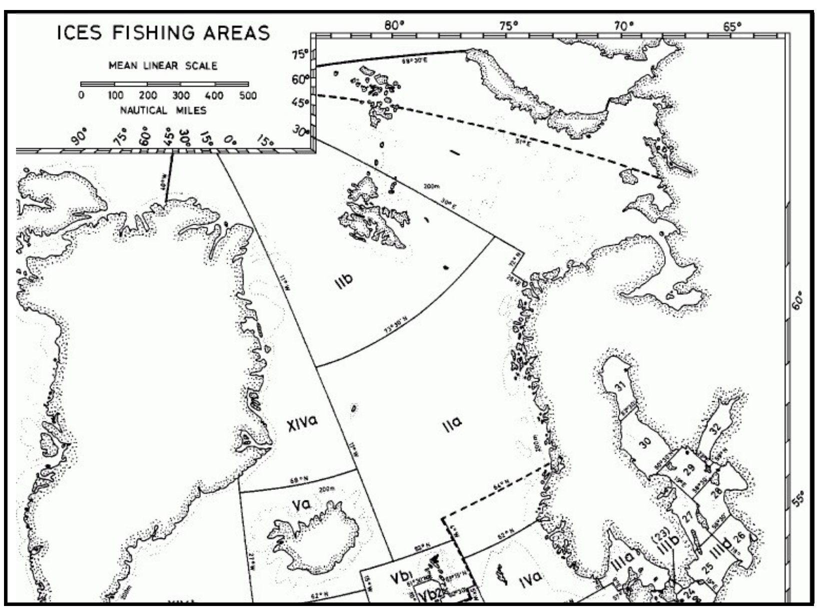

Figure 1: ICES areas I (*Barents Sea*), IIa and IIb (*Norwegian Sea*)

Topographic structures within the Nordic Seas divide the area into three sub areas I, IIa and IIb (Figure.1) and they are referred as ICES (International Council for the Exploration of the Sea) fishing areas. The large shallow ocean area, the blending of the water masses, cold and warm streams meeting, the ice melting and retreating are all factors forming the basis for the production that makes the Barents Sea one of the richest ocean areas. The Barents Sea is capable of maintaining large fish populations including Cod, Capelin, herring and approximately 150 other species of fish.

The NEAC spawns along the Norwegian coast from 62^0N and northwards with the main spawning at Lofoten - Vesterålen. Eggs and larvae drift into the Barents sea and the juveniles feed there until they mature at an age of 6 - 7 years. Maturing cod migrate to the Norwegian coast to spawn and back to the Barents Sea after spawning. Recruitment to the fishery starts at age 3.

Barents Sea cod stock is potentially the largest cod stock in the world (Jakobsson [8]) with capelin and herring being important as prey. As mentioned, cod is brought into commercial exploitation at an age 3 - 5 years. It is sexually mature at an age of 7. It takes 4 years for a good species to reach a weight of one kilo. The average harvest of cod in the period 1950 - 2002 was 665 thousand tonnes. Norway and Russia regulate the cod fishery co-operatively and give quotas to third countries such as EU, Faeroes and Iceland. In addition, Norway is responsible for a number of local stocks in fjords along the Norwegian coast.

Various management strategies are implemented and enforced including total allowable catch (TAC) and minimum catch size, etc. by the Advisory Committee on Fishery Management (ACFM) in the ICES. TAC policies have been reviewed by Odd Nakken, Steinshamn and Per Sandberg in their article [13] recently. In November 2002, Norway and Russia agreed on a long - term harvesting strategy for cod that will enter into effects from year 2004. As regards the 2003 quota for NEAC, both countries have agreed upon TAC of 395 thousand tonnes.

In recent years, the harvesting level of cod stock proceeds with the same level of fishing mortality (see Arctic Fisheries Working Group report [3]). The Ministry of Fisheries, Norway believes that if this similarity continues then this NEAC stock may decline. This would cause the volume of future catches to sink and greatly diminish the yield for those who make their living within the fisheries sector. Based on the development in the NEAC fishery we are trying to present a stochastic model which we believe to be more realistic than previously available models for this fishery by using time series for biomasses and landings from 1985 to 2001.

5 Application and Discussion

The growth of biological populations is affected by natural regulatory factors such as environmental variations, random changes in rates of survival or reproduction etc. Factors that can influence populations in relation to the size of populations are referred to as density dependent factors. The concept of density dependence is the application of surplus population modeling in fisheries management. In the classical work of Clark [5], the population growth models are categorized as compensation and depensation models.

According to the theory of compensation populations will grow when population density is low and will decline when density is high. In this way, population size remains relatively stable. The growth function $g(.)$ is defined over the interval $s \geq 0$ is strictly concave. For such growth functions, the proportional growth rate, $\frac{g(s)}{s}$, falls as stock grows, for all stock sizes. On the other hand, depensation can occur when the birth rates or survival rates decrease at the low densities. Depensation tends to destabilize the population. For this case the proportional growth rate is an increasing function of s for certain values of s and the growth function is initially convex then concave.

Different functional forms of the stock - growth relationship represent different hypotheses about the response of surplus growth to changes in the density of the stock. In this regard, the following generalized logistic function

$$g(s) = rs^2\left(1 - \frac{s}{K}\right), \tag{12}$$

where r is an *intrinsic growth rate* per unit time and per unit stock and K is the *environmental carrying capacity*, is used as a population growth function. With this growth function we have depensation in $0 < s < \frac{1}{3}K$ and MSY at $s = \frac{2}{3}K$.

Parameter estimation represents the estimation of the both drift and diffusion parameters in the stock dynamic constraint. For this purpose, it is worthwhile considering some families of SDE to Eq. (1):

$$ds = \left[rs^2\left(1 - \frac{s}{K}\right) - u\right] dt + \sigma(s)\, dB, \tag{13}$$

where the functional forms for the diffusion term were specified as $\sigma_0 s$, $\sigma_0 s^{\beta}$ and $\sigma_0 s e^{-\beta s}$ with positive diffusion coefficients. To make use of this specification we need a replicated biomass time series data for NEAC stock.

The time series data on biomass and catch are based on ICES Arctic Fisheries Working Group (AFWG) reports. We have used the latest working group reports available to us, namely ICES report for the year 2002 [3]. The AFWG gives a time series of biomass for ages $3-13^{+}$ calculated by Virtual Population Analysis (VPA), as well as catch in numbers by age, and time series for weight in catch by age for the years 1948 – 2001 from areas I, IIa and IIb (Figure 2).

When estimating biological and stochastic parameters by using the Kolmogorov and Smirnov procedure we need multiple biomass time series, i.e., several independent stock survey results for abundance biomass, for this cod stock. It is possible to compute this multiple annual data sets from the recent AFWG report for a short period of time, namely, from 1985 to 2001.

Initially, we considered a reference biomass time series which is based on the VPA calculation and an yield time series which includes the landings from the North East Arctic regions I and II. Using the data from ICES [3], the cod biomass for each year from 1985 to 2001 has been calculated as the sum of the biomasses for age $3-10^{+}$. This biomass

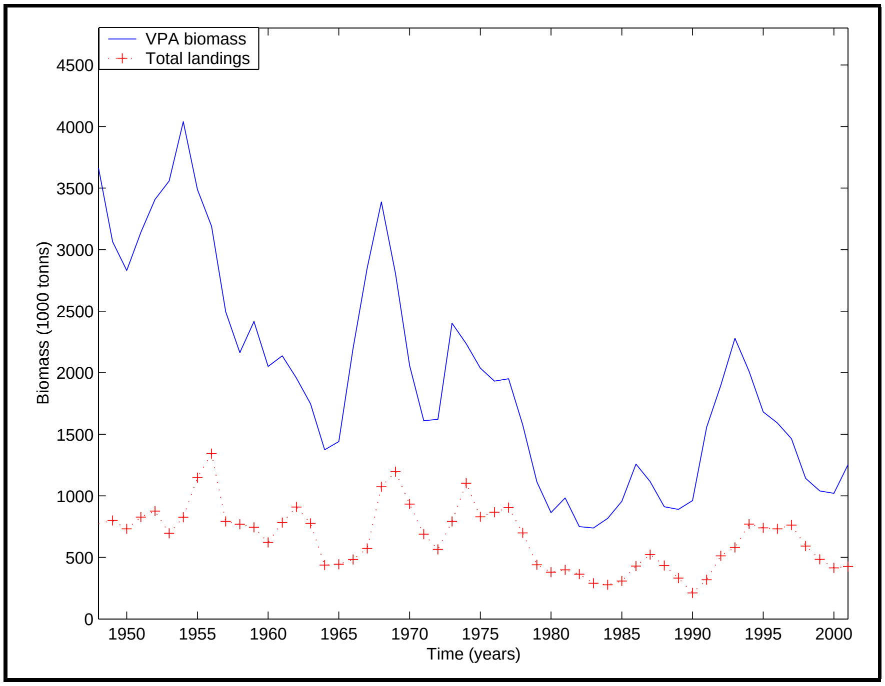

Figure 2: VPA biomass time series with Total Landings for year 1948 – 2001

time series is called the *reference* biomass of the NEAC stock. Catches for each age for the years 1985 – 2001 have been calculated as the product of catch in numbers by age and weight in catch for that age, whereupon the catches for ages $3 - 10^{+}$ were summed to give total landings for that year.

Apart from that, the AFWG report [3] produces several independent and individual survey results for abundance stock numbers which depends on both age from $1 - 10^{+}$ and length from the Barents and the Norwegian Seas. These acoustic, autumn and bottom trawl surveys were made by Russian and Norwegian vessels not only in various seasons but also in different sub areas. The biomasses of cod from age $3 - 10^{+}$ for the years 1985 – 2001 have been calculated by using stock weight at age for each survey (Figure 3). This calculation produces multiple time series data for 7 observed data points including reference biomass, with 17 replicates for the NEAC stock.

The biomass time series, which is represented as a biomass matrix, is obtained from the surveys and the VPA calculation have not been calculated from the same fishing region of NEAC. That is, the abandon stock numbers were counted from different regions for each survey and the VPA biomass represented all fishing regions of NEAC. Thus the time series are not in similar pattern. Therefore, it is problematic when we use this biomass matrix directly to our estimation process.

In the mean time, we need a simulated multiple time series from the proposed SDE (13) with appropriate initial guess for unknown parameters to proceed in our estimation method. This leads us to consider the necessity of the catch data for each individual survey. It is not possible to obtain this information from the AFWG report. There is

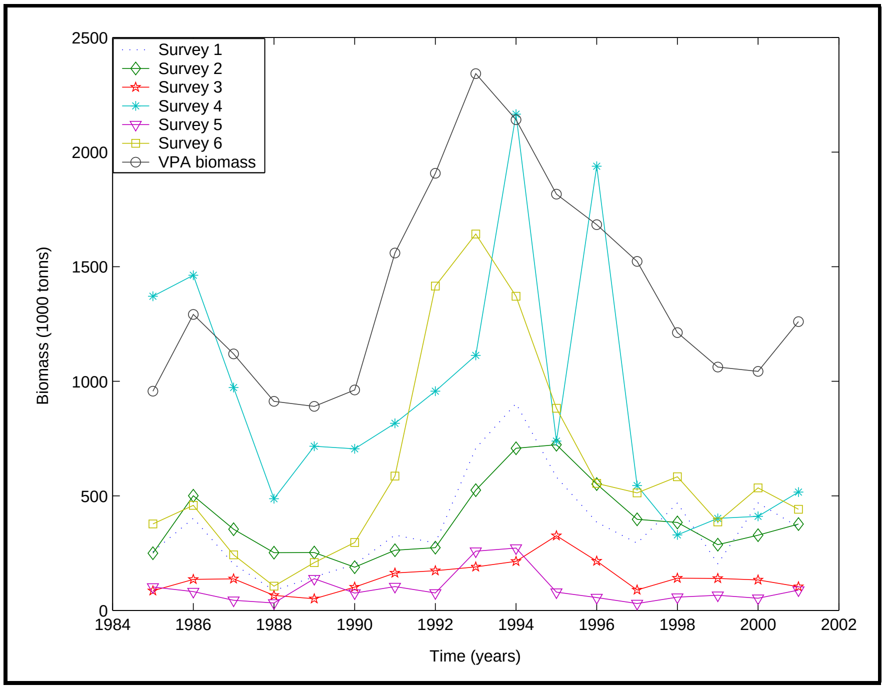

Figure 3: Biomass time series from individual acoustic, autumn and bottom trawl surveys for year 1985 – 2001. Russian vessels made the Surveys 4 and 5 and Norwegian vessels did the rest

only one time series available for landings which covers the whole NEAC area. Here we also have problem to produce a simulation biomass matrix which we can compare with the biomass matrix using the Kolmogorov Smirnov statistic.

Because of these inconveniences, we shall simplify the problem by regressing each biomass time series from the survey results with the VPA biomass. We used a method called weighted robust polynomial regression to regress the available data in the survey results. This produce a new multiple biomass time series and, thus, this new matrix is referred as a biomass matrix of this cod stock.

Estimation was obtained using the criterion function (8). The global minimization procedure with respect to the drift, r and K, and the diffusion parameters produced the estimated values. The observed biomass matrix has seven realizations ($n = 7$) with time series length seventeen ($T = 17$). The simulated biomass matrix was produced from the proposed stock dynamics (13) using the initial stock $s(0) = \overline{s}$ and the initial guess for parameters. The initial stock was calculated as the average stock biomass in year 1985. The dimension of the simulated matrix was also identical to the observed matrix.

Relying on some physical information from biologists, the suitable ranges for the parameters are chosen. The estimation was executed 1000 times and the mean estimated values were registered with their standard deviations (in parenthesis) of parameter estimates in Table 1.

It is clear from table 1 that the estimates are statistically acceptable. There is no

Parameters	r	K	σ	β
Linear diffusion	6.31818e-004	2648.4	0.49204	–
	[2.9014e-005]	[265.94]	[0.11949]	–
Convex diffusion	6.51533e-004	2640.3	0.20687	1.21595
	[3.48546e-005]	[273.42]	[0.07476]	[0.11597]
Concave diffusion	6.67267e-004	2651.7	0.51756	7.93e-005
	[3.40781e-005]	[282.01]	[0.10469]	[1.347e-006]

Table 1: Estimated values of the drift and the diffusion coefficients of the bio - dynamic constraint. The carrying capacity K is measured in 1000 metric tonnes. Results in parenthesis denote standard errors of the estimated parameters

obvious way of choosing among these models. They are all produced by fitting an a priori given dynamical structure to the same data. As such, we are dealing with phenomenological models both with respect to the drift and diffusion. It is not our task to make a detailed description of the biomass dynamics and its volatility. We try to produce a submodel that can be implemented in a management or economic setting. The level of details in the economic submodel is rather limited. We do not want to clutter the analysis with partial models containing more details than other parts. The stochastic optimization procedure (SOC) creates policies that are on an aggregated level. We belive it to be a bad idea trying to make very sophisticated management policies dependent on a lot of more or less unknown details. Practicality is a key attribute to most successful optimal management strategies. Differences in the biological production functions need not imply significant differences in the final policy output. We investigate their impact by comparing the optimal policies that stem from these models with policy in place as it can be viewed from the historical landings.

Biological species grow by the gift of nature. The structure of the growth is quite complicated and it requires sophisticated mathematical functions to adequately model them. Fortunately, there are simpler models that reasonably and approximately represent the growth models. We already used a depensation model to represent the growth function of this particular fishery. Now we use a pure compensation growth model in our setup for the comparison.

The management of a renewable resource has generally been based on the concept of maximum sustainable yield (MSY). This is the level where the surplus production equals the sustainable yield and usually occurs where the growth function has its maximum. As such MSY is a measure of the models statment about the productivity of the stock. Comparing MSY from various models is a commen way of comparing productivity statments from different models.

We consider the following stock dynamic constraint:

$$ds = [g_l(s) - u] \, dt + \sigma(s) \, dB, \tag{14}$$

where $g_l(s) = r_l s(1 - \frac{s}{K_l})$ represents the Schaefer logistic growth function, $\sigma(s) = \sigma_0 s$ is a linear diffusion term and r_l, K_l have their usual meanings which we defined previously. Using the same procedure, the Kolmogorov- Smirnov method, we estimate the parameters in the equation (14). The Table below shows some quantities of practical interest pertaining to the NEAC. Here we considered the results from the depensation growth with a linear diffusion model for the purpose of comparison.

Parameters	Depensation	Logistic
r	6.31818e-004	0.4649
K	2648.4	5735.2
MSY	656.53	665.7

Table 2: Estimated biological parameter values for the logistic and the depensation growth functions with sustainable values. The carrying capacity K and MSY are measured in 1000 metric tonnes.

Estimates of MSY quantities are shown in row 3 of Table 2. Both models produce almost the same estimate of MSY even though they give different biological parameter values. The MSY is around the values of TAC in the late 90's. Thus, our comparative studies about MSY implies that the estimated parameter values are quite appealing and acceptable for the NEAC fishery.

A possible interpretation of Table (2) is that the depensation model suggestes that the stock is relatively small and productive whilst the logistic model indicates the opposite.

The next step is to investigate the implications of these models (specifications) on the resource exploitation policy. Thus we introduce the economic component to the problem. The demand and the cost functions are

$$p(u) = p_0 - p_1 u \tag{15}$$

$$c(s,u) = c_0 \frac{u}{s} \quad , \quad c_0 = 8864. \tag{16}$$

The linear inverse demand function Eq. (15) was estimated by using annual catch and price data [2] in the period 1985 to 2000. A robust linear regression method with bi-square weighting function was used and it turned out that for the best statistical fit we obtained $R^2 = 0.43$. The cost function (16) was adapted from an article by Arnason , Sandal , Steinshamn and Vestergaard [4]. The net revenue function is

$$\Pi(s,u) = \left(p_0 - \frac{8864}{s}\right) u - p_1 u^2, \tag{17}$$

with the estimated values, $p_0 = 10.527$ (1.0056) and $p_1 = 0.005973$ (0.000307), where standard errors appears in parenthesis. The biomass and the harvest are measured in 1000 metric tonnes for all parameter estimations.

We now consider the biological and the economical functional forms required to evaluate the implications of harvesting quotas of the NEAC fishery. We focus on the catch that is produced by the optimization scheme. We solve numerically the HJB by using the *Markov chain approximation methode*, Kushner and Dupuis [10] as previously outlined. The optimal landing levels are then compared with real harvest.

For the comparison purpose two main features, deterministic with different discount rates and stochastic with non-identical diffusion terms, are considered. Using already derived functions for $\Pi(s, u)$, $g(s)$ and $\sigma(s)$ with Eqs. (5) and (6), the optimal yield path as a function of stock are calculated. The *bliss harvest rate*, which can be interpreted as the *upper limit* for the optimal control, is also presented along with optimal yield paths. The bliss can be evaluated by maximizing the net revenue $\Pi(s, u)$ with respect to the control variable u:

$$\mathrm{u}_{\mathrm{Bliss}} = \frac{p_0 - \frac{c_0}{s}}{2p_1},$$

where p_0, p_1 and c_0 are arbitrary constants and this is a measure where the instantaneous profit is maximized without taking the future effects on the stock into account. To see that this we can reason in the following way:

1) If *Bliss* is achiveable, it maximises the intgrand, that is the net reveneu, at each point in time. It is clear that this is the best you can hope for.

2) In the limit where the discount rate goes to ∞ future does not matter. This means that optimum is achived as above. Hence *Bliss* can be interpreted as the optimal policy when future is unimportant. This means that if the future does not count there is nothing to gain by reducing present harvest.

When considering the deterministic case the diffusion term, $\sigma(s)$, in the HJB equation is approached by a very small value (i.e. $\sigma(s) \approx 0$) and it is interpreted as a negligible diffusion term. Figure. 4 is included to display optimal harvest paths for various discount rates with the bliss control path and the historical yields. Displaying historical harvest data with optimal yield paths allow easy comparison between the proposed results from Eq. (6) and the real harvesting over the period from 1985 - 2001.

A noticeable aspect of these curves is that the bliss follows the historical harvest points quite closely. This curve looks like a regression line of these actual harvest points. We have earlier pointed out that bliss is an upper limit on the feasible harvest path that the optimal policy approaches in the limit of an infinite discount rate, i.e., when the future has no intrinsic value in the fishery. This suggests that the stock has been managed without a long term perspective and emphasize has been on the immediate return. Economically overfishing has taken place. When the discount rate or time preference is increasing, the optimal harvest path shifts upwards towards medium to large stock levels and the harvest moratorium is invoked at radically lower stock levels which put the stock at risk for medium to small stock levels.

Inclusion of the stochastic term to the stock dynamics implies additional effect on the optimal harvest paths. In Figure. 5 we again show the optimal paths for both deterministic

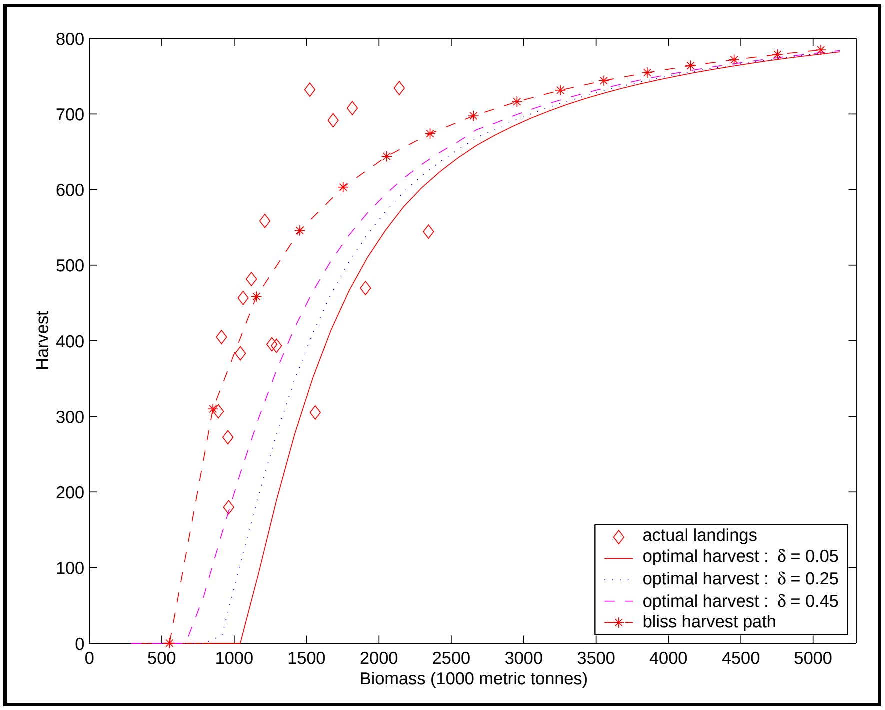

Figure 4: The optimal landings from deterministic case for different discount rates along with actual harvest and the bliss control path.

and stochastic cases. It displays the quotas suggested by the stochastic rule for three different diffusion functions along with an optimal path implied by the deterministic rule. Optimal paths are calculated using discount rates of 5 % for both the stochastic and the deterministic cases.

Figure. 5 shows a very interesting feature, that is, the addition of a stochastic element to the stock dynamics has similar effect to increasing the discount rate in the deterministic case. When compared to the deterministic case, the stochastic optimal harvest quotas were approximately similar at moderately large stock levels and higher at small stock levels. As the degree of the uncertainty rises, harvest moratoria are implemented at progressively smaller stock levels, a result which is consistent with increased discount rates in the deterministic case. This implies that the higher degree of uncertainty, among the other things, may also response to an important cause for fishing down.

In order to understand the dynamic nature of this harvesting strategy, Figure. 6 presents the time series of optimal and actual harvests. Optimal harvests are derived from both the stochastic case for a 5 % discount rate with various diffusion terms and the deterministic case for various discount rates. The paths belong to deterministic for 25 % discount rate and stochastic with linear diffusion term have similar pattern, but they have noticeble depature from the actual harvest path in most of the time. Although the trend of the time series belong to optimal and actual harvestings looks like similar, the results shown in Figure. 6 clearly exhibit that the actual harvest exceeded the optimal harvest most of the time for the period from 1985 - 2001 and it indicates that the economic over

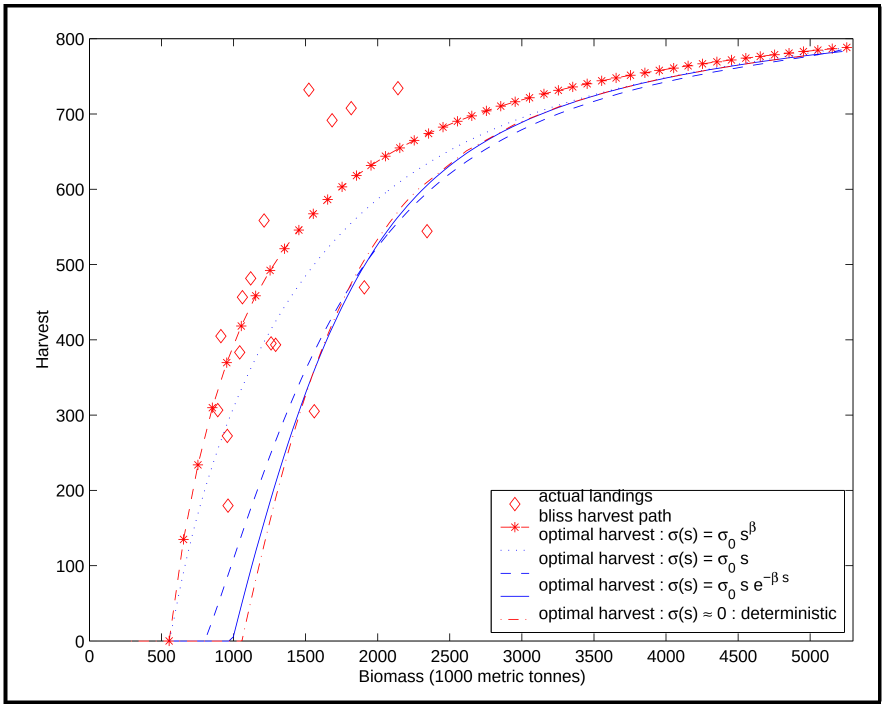

Figure 5: The optimal landing paths, which are derived using 5 % of discount rate, from stochastic case for different diffusion terms are plotted. A deterministic optimal path, which is derived using same discount rate, is also included.

fishing has taken place.

Summary

The North East Arctic Cod is an important species that urges to be regulated in order to prevent its stock from being moderately depleted. Owing its migratory nature and natural fluctuations it is useful to study the implications of a stochastic bio-economic model for this fishery. Focusing on the fluctuating environment, we introduced the issue of process uncertainty in the form of stochastic model of NEAC biomass dynamics and left other sources of uncertainty for future investigation. Parameter estimation in both economical and biological functions have been considered to derive optimal harvesting paths using industry data of NEAC fishery. Optimal harvests obtained have then been compared to the historical yields.

An interesting feature of these results is that the bliss harvest path, which can be interpreted as the optimal harvest for infinite discount rate or the limiting curve for the optimal harvest rule, tracks the historical harvests quite closely. There is a clear conclusion from all of our models. The NEAC fishery has been poorly managed in the sense that historical harvest data can only be explained if one assumes a very high discount rate or and the lack of accounts for the significant uncertainties which clearly are present. Both these views indicate that the management of the sock has been short sighted and narrow minded.

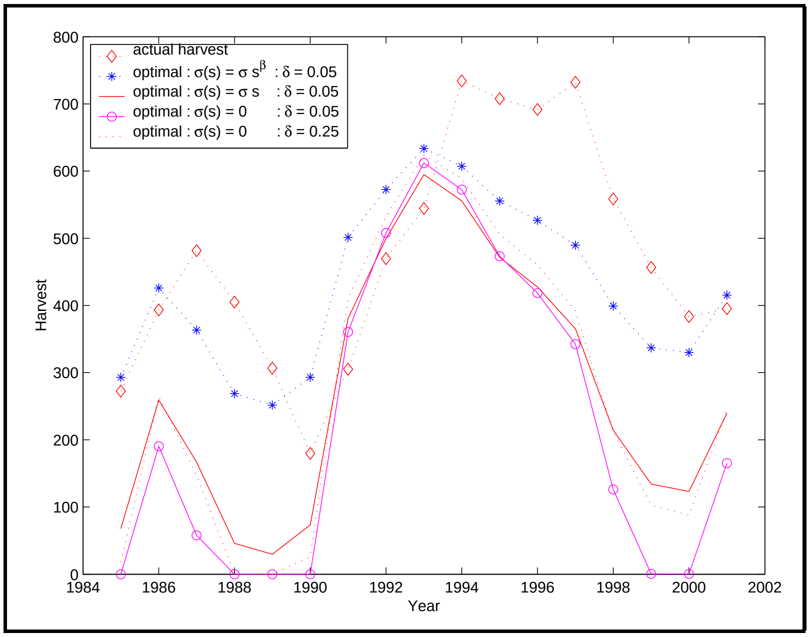

Figure 6: Time series of harvest for actual data and optimal harvests from deterministic ($\delta = 25\%$) and stocastic ($\delta = 5\%$) cases.

In the deterministic case, optimal paths closely moved toward the actual catches at the very high discount rates. A similar pattern was observed in the case of stochastic dynamics at the lower discount rates with the higher diffusion terms which represent one of the many sources of uncertainty. For the comparison with the deterministic case, the stochastic case produced similar harvesting patterns at moderate to large stock levels, but it increased optimal harvests at small stocks, thus reducing the stock level at which moratorium is suggested. This result is quite surprising, but it is consistent with deterministic optimal harvests for higher discounting rates.

We also noticed that there is quite large difference in the moratorium levels when considering the same discount rates for both cases. There is, however, a pitch fall here. It is dangerous to think about the deterministic limit as merely putting the diffusion term to zero. Surely this creates the deterministic case, but not the corresponding one as one must keep in mind that the Kolmogorov-Smirnov approach calculates all parameters simultaneously , i.e., the parameter values in the drift and diffusion terms influence each other. One must therefore expect that the deterministic parameter values become different when we start out with no diffusion term. This is why parameter estimation in general is a though undertaking in continuous time stochastic processes described by an Ito differential equation. Therefor the deterministic rule may perform quite well at the small stock levels and can be used at lower discount rates, if we estimate a new set of parameters for the deterministic dynamics setup. We here used a negligible coefficient for the diffusion term in our stochastic procedure to derive a optimal harvest rule for the deterministic case. However, it is important to use new estimated parameter values for deterministic procedure. Because our estimation process describes that the change in both biological and stochastical parameters occurs simultaneously and thus, the inclusion of a negligible value (or zero value) for the diffusion coefficient in the stock dynamics leads to the production of new estimated values for the other parameters.

Although the harvest moratorium in the stochastic case appears much lower level than the level suggested by the deterministic rule, the optimal stochastic yields along with actual harvests clearly exhibit the economic over fishing for much of the time period in past 17 years. It is important to look carefully into these results, because they have relation with particular form of adopted uncertainty. It is always believed that overfishing has been due to lack of concern for stock recovery and future generations. Besides, our study reveals that the major cause of over fishing and fish depletion is a high degree of uncertainty in the models used.

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