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Modeling the Norwegian Sea 'pelagic complex' An application of the Ensemble Kalman Filter

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by

## Nils-Arne Ekerhovd Sturla F. Kvamsdal

**SNF Project No. 5172:** Bioeconomic Multispecies Analysis of Marine Ecosystems

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# Modeling the Norwegian Sea 'pelagic complex'. An application of the Ensemble Kalman Filter

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#### Abstract

We have estimated the parameters of a modified logistic ecosystem model of the pelagic fish stocks in the Norwegian Sea with the Ensemble Kalman Filter. Our model only contains four parameters. The model appear to capture much of the dynamics in the system as well as the interactions between the different species. The interactions are competitive, mutually destructive interactions, where Norwegian Spring Spawning herring, Northeast Atlantic blue whiting and Northeast Atlantic mackerel prey upon the same food source(s), thus, limited by a common 'carrying capacity'. Increase in one species' biomass leads to reduced growth for all three species. While the main, dynamic features seems to be picked up, and most observations are within the forecast range, the forecasts are often too small, and a deterministic forecast has less or no downward bias.

**Keywords:** Ecosystem Management, Pelagic Fisheries, Norwegian Sea, Ensemble Kalman Filter, Bioeconomics

## Introduction

The rationale for developing a bioeconomic multispecies model for mackerel, herring and blue whiting is to describe the behavior of the harvestable populations for use in theoretical analysis. The bioeconomic modeling involves identifying the patterns of changes in the biomass of fish over time as a function of additions to the stock due to natural growth and deductions from the stock due to natural mortality and harvesting. In addition, multi-species models must take into account inter-specific effects between the fish populations. During the period 2006-2009 there has been a strong build up of biomass of planktivorious fish (herring, mackerel and blue whiting<sup>1</sup>) in the Norwegian Sea. The negative relationships between length at age and stock biomass, the pronounced reduction in zooplankton abundance witnessed in the Norwegian Sea in recent years, and the expansion in spatial distribution of fish indicate that the biomass of planktivours fish in the area has been above the carrying capacity (Huse et al. 2012). All stocks showed sign of density-dependent length growth, whereas for herring and blue whiting there were also significant effects of interspecific competition. Huse et al.'s results support the hypothesis that the planktivorous fish populations feeding in the Norwegian Sea have interactions that negatively affect individual growth, mediated through depletion of their common zooplankton resource. It will be important to include these findings in the future ecosystem based management of the Norwegian Sea.

The Northeast Atlantic sustains a number of pelagic fish stocks, the most important of which are Norwegian Spring Spawning (NSS) herring, Northeast Atlantic blue whiting and Northeast Atlantic mackerel (Skjoldal et al. 2004). All these stocks are classified as straddling stocks in the sense that they not only cross boundaries between the EEZs of coastal states, but also traverse the high seas areas between those boundaries (Bjørndal and Munro 2003). NSS herring mainly inhabit Norwegian waters throughout the life cycle, but can migrate into Russian waters during the juvenile phase, and into Faroese, Icelandic

<sup>&</sup>lt;sup>1</sup>These zooplankton feeding stocks have substantial spatial and dietary overlap, and are often collectively referred to as the 'pelagic complex' in the Norwegian Sea.

and international waters as adults during the feeding period in the summer (Holst et al. 2004). The feeding migration pattern, especially for large herring, has changed several times over the last 60 years (Holst et al. 2002; Utne et al. 2012), varying with the size of spawning stock biomass and possibly ocean conditions as well. Mackerel spend most of the year in EU waters, but a large part of the stock migrates into the eastern part of the Norwegian Sea and the North Sea from June to October (Belikov et al. 1998; Iversen 2004). In recent years Icelandic waters have also been inhabited by mackerel (Nøttestad and Jacobsen 2009) possibly due to changing water temperatures. Blue whiting is mainly found in the Norwegian Sea throughout the year, but spawns west of the British Isles in February-May (Bailey 1982). The stock is located in Norwegian, Icelandic, Faroese and EU waters, but the large scale distribution pattern varies and is related to total stock size and water temperature (Utne et al. 2012).

The migratory patterns of these stocks have undoubtedly made it more difficult to attain and to uphold international agreements on catch quotas. While agreements on the less migratory demersal stocks (cod and haddock, for example) between Russia and Norway have remained unchanged since the early 1980s, the agreements on the pelagic stocks have sometimes broken down or taken a long time to establish. After lifting the moratorium on North Sea herring in 1981 it took several years to establish a lasting sharing agreement between Norway and the EU. An agreement on the NSS herring was established in 1996, several years after its recovery, but it broke down in the period 2003-2006 because of disagreement over allocation of national quotas. This shows that the agreement lacked time consistency which is a fundamental condition for a cooperative agreement to be stable over time. An agreement on blue whiting was reached in 2005, after many years of intensive exploitation where total catches in some years were four fold the recommended ICES quota (Bjørndal 2009).

The relevance of our research is clearly emphasized by the recent mackerel dispute between Norway and EU on one side and Iceland and the Faroe Islands on the other, the so-called mackerel war (Hannesson 2012). There has for several years been an unsolved dispute between these nations about the size of their respective quotas. Norway and EU had originally an agreement with 10-years duration about the size and distribution of mackerel quotas. Then the mackerel started to change its migration pattern such that a larger share of the stock entered Iceland's and the Faroe Islands' economic zones. This caused these two countries to multiply their previous harvest of mackerel. Norway has responded by refusing landings of mackerel from Iceland and the Faroe Islands in Norway, and EU has recently warned that they may do the same. The present threat is that if this dispute is not solved fairly soon and sustainable harvesting is resumed, the increased harvest pressure on the mackerel may cause the whole stock to collapse implying severe problems both for fishermen and the pelagic fishing industry in all countries involved for a long period. The scientific advice for total harvest in 2011 was 650 000 tonnes whereas actual harvest was about one million tonnes. So far no agreement has been reached. Even if an agreement is reached, it may be interesting to compare it with an optimal agreement based on bioeconomic modeling under various scenarios, and therefore our line of research is of interest no matter what actually happens. The so-called mackerel war is a classic example of the commons problem for which the relevance and importance of bioeconomic modeling and analysis is well established.

## Method

In order to model the pelagic fisheries in the Norwegian Seas, simplifications are necessary. We resort to a aggregated biomass model which permits analysis of optimal agreements and quota decisions, but still maintain species interactions and stock dynamics. To make the model as representative as possible, we apply the ensemble Kalman filter, a data assimiliation method for chaotic, nonlinear models, to fit the model to aggregated data. The method allows for adaptive parameters which make it possible for relatively simple models to capture the complex dynamics observed in the data. The method also extends to forecasting which can be useful in the analysis. An important part of the method is the quantification of model uncertainty which could be important for management and in negotiations.

We briefly spell out the central parts of the ensemble Kalman filter methodology; see Evensen (2003) for a comprehensive treatment. The continuous time state-space model is written

$$\mathrm{d}x = f(x)\mathrm{d}t + \sigma\mathrm{d}B \tag{1}$$

$$d = M(x) + v \tag{2}$$

The state equation, equation (1), describes the time evolution of the state vector x. The sum of the drift term f(x)dt and the stochastic diffusion term  $\sigma dB$  amounts to an incremental change dx in the state vector. When x is an aggregated biomass vector, f(x) is the multi-dimensional growth function. The stochastic, Brownian increments dB are independent, identical, and normal distributed with mean zero and variance dt. The measurement equation, equation (2), relates the state vector to the observations d via the measurement functional M(x). When the state vector is directly observed, the measurement functional is the identity operator. v is a normal distributed error term with mean zero and covariance R.

The ensemble Kalman filter is a generalization of the classical Kalman filter, and the overall structure of the procedures are similar. The state vector is integrated forward in time according to the state equation until measurements become available. In the classical Kalman filter, the model is linear, and integration is straight forward; in the extended Kalman filter, the model is locally linearized; in the ensemble Kalman filter, a Markov Chain Monte Carlo approach is invoked and the full, nonlinear model is used. Measurements are used to update the state vector via the Kalman gain matrix  $K_e$ . Integration then continues. Parameter estimation is facilitated by increasing the state-space to include parameter dimensions. Thus, when the state vector is updated with measurements, the state equation is also updated if the drift or diffusion terms contain

unknown parameters. Parameters are treated as unobserved, but constant model states; they have zero drift and diffusion terms.

The forward integration of the state vector is tantamount to solving the Fokker-Planck equation for the time evolution of the probability density. To solve the Fokker-Planck equation is inconvenient in practical settings. The ensemble Kalman filter approximates the solution with a Markov Chain Monte Carlo method. An ensemble of states, a cloud of points in the state-space, represents the probability density function and each, individual member is integrated in time according to the state equation. Errors are simulated. The integrated ensemble represents a forecast of the probability density. The only approximation is the limited number of ensemble members (Evensen 2009).

The state vector is updated with measurements in a linear weighting between the forecasted (integrated) state vector and the measurements. We write  $x^f$  for the forecast state vector and  $X^a$  for the updated state vector (in the technical literature, the update operation is called the analysis). The weighting is given by the ensemble-based Kalman gain  $K_e$ . In order to maintain the proper covariance structure in the state ensemble, it is necessary to perturb the observations to account for observation uncertainty (Burgers et al. 1998). In practice, the observations are represented by an ensemble D which has as many members as the state ensemble X. The observation ensemble has the observation d as its mean, and its covariance, written  $R_e$ , represents the observation uncertainty. For ensemble member i, the update is written

$$X^{a}(i) = X^{f}(i) + K_{e} \left( D(i) - M X^{f}(i) \right)$$
(3)

The ensemble Kalman gain is given by

$$K_e = C_e^f M' (M C_e^f M' + R_e)^{-1}$$
(4)

where  $C_f$  is the covariance of the forecast ensemble  $X^f$ . Apostrophe denotes the transpose. The ensemble is assumed to be of sufficient size, such that inverted matrices

are nonsingular. See Evensen (2003) for derivations and further details.

The ensemble represents the probability density function of the state vector. At any given time, the estimate of the state is the mean of the state ensemble, with the ensemble covariance representing uncertainty in the estimate. The initial ensemble should reflect belief about the initial state of the system. An advantage with the approach outlined above is that state and parameter variables are estimated simultaneously, taking model error into account (Evensen, 2009, pp. 95-97). The filter produces estimates conditional upon observations up until and including a given time. When estimates conditional upon the full information set is relevant, the estimates should be smoothed with the ensemble Kalman smoother. The ensemble Kalman smoother can be formulated sequentially in terms of the filtered estimates; see Evensen (2003, p. 360) for details.

The ensemble should be large enough to sufficiently represent the probability density function in the state space. Computational limitations may restrict viable ensemble sizes. In the meteorological and oceanographic sciences, where the ensemble Kalman filter is applied to high-dimensional, chaotic systems, it is generally held that relatively small ensembles can ensure statistical convergence (Evensen 2009). When the ensemble is small, the covariance will be underestimated, ultimately leading to so-called filter divergence where observations receive very small weights in the update (the state vector will then not depend on the observations and diverges). An *ad hoc* remedy is to artificially increase the ensemble covariance; the method is known as inflation (Anderson and Anderson 1999).

As an aid to compare goodness of fit of models and between models, we consider different measures. The technical literature (see Evensen, 2003, and references therein) often considers root mean squared errors and root mean squared innovations. (The term innovations are used for the ensemble updates, given by the second right-hand term in equation (3).) Errors in the parameter ensembles decline over time by construction, but should stabilize before the end of the time series in the ideal case with a long enough time series and an appropriate model. Innovations does not decline by construction, but are also expected to stabilize in the ideal case. Weak or nonexistent signs of stabilization of either errors or innovations means either that the model has not converged (inappropriate initialization or simply too few observations) or that the model is not a good model. While useful, errors and innovations have limited ability to inform about model choice between alternative models. To compare between models, we consider the Akaike (AIC) and the Bayesian (or Schwarz) Information Criteria (BIC). With the ensemble Kalman filter, the estimated density is represented by a discrete cloud of points in the state-space. While it is possible to make distributional assumptions and carry out calculations of the criteria, we apply a more rudimentary approach where distributional assumptions are avoided. First of all, to make distributional assumptions with an involved covariance structure in a highdimensional space can be cumbersome. Second, distributional assumptions lead to heavy calculations as the entire distribution has to be considered. In our rudimentary approach, which simply consider a local density relative to the observation, any kind of covariance is accommodated and the calculations are comparatively simple. The approach considers a given neighborhood in the state-space around each observation where the density is given by the relative weight of the neighborhood compared to the remainder of the state-space. Weights are decided by the distribution of ensemble members within and outside the neighborhood. The neighborhood should be as small as possible without being empty. When comparing models, the neighborhood size which in related methodology would be referred to as bandwidth should be kept constant. How to control for neighborhood size is currently not clear to us. Given that the exact distribution of the ensemble members vary for different runs of the filter, criteria calculations from more than one run should be compared. When comparing different models, the basis for comparison grows exponentially with the number of runs, such that relatively few runs (less than ten) could form solid ground for comparisons. (The exact distribution depends on the Markov Chain Monte Carlo mechanism, which has a strong, random element. In the limit. where the ensemble size goes to infinity, the random element is cancelled out and the criteria calculations are unique. In practice, one has to consider a number of runs, where for example the mean difference in criteria are considered. Each run is equally

representative for the criteria.)

## Data and modeling

The international Council for the Exploration of the Seas (ICES) publish stock estimates and landings of NSS herring, Atlantic mackerel and Atlantic blue whiting. For blue whiting reporting on total stock and landings started in 1977. As juvenile individuals the NSS herring spend their time in the coastal waters of northern Norway or in the Barents Sea, and only appear in the Norwegian Sea along with the mature part of the stock at the age of 3 - 4 years old. Therefore, and since the overlap and interactions with the two other stocks mainly takes place in the Norwegian Sea, we use the spawning stock biomass as the state variable for herring. For mackerel and blue whiting the choice of state variable is not so clear cut. Both juvenile and adult blue whiting spend time in the Norwegian Sea, while the mature individuals migrate west of the British Isles to spawn, (some of) the juveniles remains in the Norwegian Seas. This is for a large part the case with mackerel; a large part of the stock, both young and adult individuals, spend time in the Norwegian Sea. For this reason we use the total stock biomasses for mackerel and blue whiting as state variables in our model.

The biomass of the three stocks are the state variables; herring is denoted  $x_1$ , mackerel is denoted  $x_2$ , and blue whiting is denoted  $x_3$ . The harvest rates are denoted  $h_1$ ,  $h_2$ , and  $h_3$  for herring, mackerel and blue whiting. The parameters are denoted  $c_k$ . The dynamic model for the system is written:

$$dx_1 = \left(c_1 x_1^{m_1} \left[1 - \frac{x_1 + x_2 + x_3}{c_4}\right] - h_1\right) dt + \sigma_1(x) dB_1$$
(5)

$$dx_2 = \left(c_2 x_2^{m_2} \left[1 - \frac{x_1 + x_2 + x_3}{c_4}\right] - h_2\right) dt + \sigma_2(x) dB_2$$
(6)

$$dx_3 = \left(c_3 x_3^{m_3} \left[1 - \frac{x_1 + x_2 + x_3}{c_4}\right] - h_3\right) dt + \sigma_3(x) dB_3$$
(7)

The stochastic increments  $dB_i$  are independent, with mean zero and variance dt. Correlations in the noise processes are reflected in the scaling term  $\sigma_i(x)$ . The scaling term is geometric,  $\sigma_i(x) = \Sigma \cdot x$ , where the upper triangular matrix  $\Sigma$  reflects covariation.

The first terms  $(c_1 - c_3)$  in each model equation is equivalent to the modified  $(m_i)$  logistic growth function, and the parameters  $c_1$ ,  $c_2$ ,  $c_3$ , and  $c_4$  are interpreted accordingly. The growth equations (5 - 7) are modified by the following modification terms:  $m_1 = 1.773$ ,  $m_2 = 1.856$  and  $m_3 = 1.807$  for an initial  $c_4$  parameter equal to 30 million tonnes, and  $m_1 = 1.801$ ,  $m_2 = 1.779$  and  $m_3 = 1.798$  for an initial  $c_4$  parameter equal to 20 million tonnes. (The idea of carrying capacity; the standard interpretation of the  $c_4$  parameter in the modified logistic, becomes unclear in an ecosystem setting. The capacity of the ecosystem to harbor any one specie depends on the state of the entire system. Hence, intrinsic, single species notions such as the carrying capacity must be treated with caution in all multi species approaches. Moreover, here we assume that there is a common carrying capacity for all three species.)

All parameters are log-normal distributed, and are thus always positive. The signs of the interactions are negative, for example, the herring stock is negatively affected by the size of its stock as well as the size of the mackerel and blue whiting stocks.

The initial ensemble is drawn from a multivariate normal distribution. For the three state variables, we use the first observations as the mean of the initial ensemble and 30% of the first observation as standard deviation. As parameters enter the model equations as  $c_k = \exp(\alpha_k)$ , the parameter variable ensembles are defined in terms of the  $\alpha_k$ 's, which may be called shadow parameters. Means and variances for the shadow parameter variable ensembles are listed in Table 1. The table also lists the implied parameter mean  $\exp(\bar{\alpha}_k)$ . Since it is intuitively much easier to relate to the actual parameters  $c_k$  rather than the shadow parameters  $\alpha_k$ , we refer to the actual parameters in the discussion that follows.

For mackerel and blue whiting the single species intrinsic growth rates are estimated to lie between 0.3 to 0.4 (Hannesson 2013; Ekerhovd 2003), and for herring it is assumed

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| Parameter                                     | Implied Mean $(\exp \bar{\alpha}_k)$ | Ensemble Mean $(\bar{\alpha}_k)$                   | Ensemble St. Dev. |
|---|--------------------------------------|--|-------------------|
| <i>c</i> <sub>1</sub> - <i>c</i> <sub>3</sub> | 1/1000                               | -6.69078   | 1.0               |
| $c_4$   | 30000ª<br>20000                      | $\begin{array}{c} 10.30895 \\ 9.90349 \end{array}$ | 0.2               |

Table 1: Initial ensemble parameters and standard deviations

<sup>a</sup> Thousand tonnes

to about the same. However, here the growth equations are modified logistic functions, and the intrinsic growth rates must be scaled accordingly. Hence, the initial means for the growth parameters for herring, mackerel and blue whiting,  $c_1 - c_3$ , were set to 1/1000.

Utne et al. (2012) calculated the consumption of zooplankton by herring, mackerel and blue whiting in 1997, which was estimated to be 82 million tonnes. This gives a consumption/biomass ratio in the range 5.2 - 6.3. The total biomass of the pelagic fish stocks was estimated to be between 13 and 16 million tonnes. However, the pelagic fish stock is subject to substantial commercial fisheries and the question remains what would the pelagic fish biomass be if there was no fisheries? Is the total 'carrying capacity' biomass of the pelagic fish stocks substantially larger than the biomass we observe in the current situation? Moreover, all three fish stocks spend a substantial amount of time in waters where they do not interact with each other. This indicates that the  $c_4$  parameter could be substantially larger than the observed biomass. Thus, the initial  $c_4$  parameter value was set to 30 million tonnes and 20 million tonnes to test what fitted the data best.

## Results

Figure 1 shows the estimated stock levels with the  $c_4$  parameter initialized at 30 million tonnes (solid lines) for all three species herring (top panel: State 1), mackerel (middle panel: State 2), and blue whiting (bottom panel: State 3). The figure also shows the observed stock levels (x-marks) which, except for three observations of herring (in 1988, 1989 and 1990), lie within two standard errors from the estimate (the shaded areas show two standard errors in each direction from the estimate). It is also worth noticing that



Figure 1: Estimated stock levels (solid lines), initial  $c_4 = 30$  million tonnes, with two standard deviation intervals (shaded areas), stock observations (x-marks), and catch data (circles).

for blue whiting several of the observations is close to the upper limit of the two standard deviation band. The general impression is that the model captures much of the dynamics of the ecosystem.

Figure 2 shows the estimated stock levels with the  $c_4$  parameter initialized at 20 million tonnes (solid lines) for all three species herring (top panel: State 1), mackerel (middle panel: State 2), and blue whiting (bottom panel: State 3). The figure also shows the observed stock levels (x-marks) and the two standard deviation intervals (shaded areas). The general impression is that the model does not captures as much of the dynamics of the ecosystem as Figure 1 with the  $c_4$  parameter initialized at 30 million tonnes. For example, several of the blue whiting observations lie outside the two standard deviation band.

Figure 3 shows the parameter estimates when the  $c_4$  parameter is initialized at 30 million tonnes, with two standard deviations intervals for the model in equations 5 -



Figure 2: Estimated stock levels (solid lines), initial  $c_4 = 20$  million tonnes, with two standard deviation intervals (shaded areas), stock observations (x-marks), and catch data (circles).



Figure 3: Estimated parameter values (solid lines), initial  $c_4 = 30$  million tonnes, with two standard deviation intervals (shaded areas).

7. All parameters are fairly stable over most of the time series, with some tendency of decreasing spread over time. The exception here is the carrying capacity parameter (parameter 4) which is also fairly stable, but with a wide spread (initially approximately 17 million tonnes reduced to about 12.5 million tonnes at the end).

Figure 4 shows the parameter estimates when the  $c_4$  parameter is initialized at 20 million tonnes, with two standard deviations intervals for the model in equations 5 - 7. All growth parameters  $(c_1 - c_3)$  behave similar to the ones in Figure 3, i.e., when the  $c_4$  parameter was initialized at 30 million tonnes. However, the parameters are not as stable and the two standard deviations areas do not continue to decrease after the initial drop. In fact, for parameter 3 it increases slightly towards the end. The development of the carrying capacity parameter (parameter 4) is not as stable as compared to Figure 3, but width of the spread is decreasing over time from about 12 million tonnes at the beginning to about 8.5 million tonnes at the end.



Figure 4: Estimated parameter values (solid lines), initial  $c_4 = 20$  million tonnes, with two standard deviation intervals (shaded areas).

Figure 5 shows the root mean squared innovations and root mean squared errors of the  $\alpha_i$  ensembles when the  $c_4$  parameter is initialized at 30 million tonnes. Only parameter 1 have errors well below half of the maximum at the end of the time series. However, the innovations for all parameters show decreasing trends towards the end of the time series.

Figure 6 shows the root mean squared innovations and root mean squared errors of the  $\alpha_i$  ensembles when the  $c_4$  parameter is initialized at 20 million tonnes. The growth rate parameters (parameters 1, 2, and 3) all have errors close to or below half of the maximum at the end of the time series, and the innovations show decreasing trends towards the end of the time series. For the carrying capacity parameter, the ensemble convergence is less pronounced but the innovations appear stable with no increasing trend.

Table 2 shows the last estimated parameter values  $c_1 - c_4$  for both cases of initializations of parameter 4 at 30 and 20 million tonnes, respectively. We see that the growth parameters  $c_1$ ,  $c_2$  and  $c_3$  when  $c_4$  is initialized at 20 million tonnes are higher than



Figure 5: Root mean squared innovations and root mean squared errors, initial  $c_4 = 30$  million tonnes, for all parameters.



Figure 6: Root mean squared innovations and root mean squared errors, initial  $c_4 = 20$  million tonnes, for all parameters.

| Parameter | Initial $c_4 = 30$ million tonnes | Initial $c_4 = 20$ million tonnes |
|-----------|-----------------------------------|-----------------------------------|
| $c_1$     | 0.000811                          | 0.001211                          |
| $c_2$     | 0.000270                          | 0.000837                          |
| $c_3$     | 0.000294                          | 0.000439                          |
| $c_4$     | $26106^{\mathrm{a}}$              | $18498^{\mathrm{a}}$              |
| AIC       | 49.8280                           | 67.3558                           |
| BIC       | 55.9334                           | 73.4613                           |

Table 2: Estimated parameters values and information criteria statistics. Average over 5 run with a state-space neighborhood equal to 2000, and 1500 ensemble members

<sup>a</sup> Thousand tonnes

when  $c_4$  is initialized at 30 million tonnes. When it comes to the estimate of the carrying capacity parameter itself ( $c_4$ ) they are below the initial values, approximately 26 and 18 million tonnes, respectively. I addition, Table 2 list the Akaike Information Criterion (AIC) and the Bayesian (Schwarz) Information Criterion (BIC) statistics measuring the goodness-of-fit. According to these criteria, the initialization of  $c_4$  at 30 million tonnes leads to the best fit.<sup>2</sup>

The initialization of  $c_4$  at 30 million tonnes is favored over that of 20 million tonnes, where the latter put more emphasis on the observations than the former. Further, comparing Figure 1 with Figure 2 shows that the fit of the model is better when initialized at 30 million tonnes; the differences between the measurements and the estimated means are generally within two standard deviations. Furthermore, in the ensemble Kalman filter literature, a traditional tool to evaluate filter and model performance is to inspect plots of root mean squared innovations and root mean squared errors, cf. Figures 5 and 6. Innovations are the corrections or updates which occur in the analysis step. Errors are also called ensemble anomalies, and are the mean difference of the updated ensemble and the estimate (the ensemble mean). Thus, the root mean squared errors are given by

<sup>&</sup>lt;sup>2</sup>The information criteria presented in Table 2 are the averages over five independent parallel runs of the models. What constitutes a substantial difference in criterion values? For BIC Kass and Raftery (1995, p. 777) suggest the following. 0-2: not worth more than a bare mention, 2-6: positive; 6-10: strong, and 10 or more: very strong. In this case, while the average difference was almost 19, the variation is substantial, ranging from a minimum of 0.3 to a maximum of 47. However, it should be noted that out of the 25 possible comparisons only two was below 2.



Figure 7: Forecast two years ahead. White solid line: the median.

 $\mathbf{E}[\mathbf{E}[X^a] - X^a]^3$ . The absolute ensemble mean innovation should in theory decline over the time series and eventually stabilize. A lack of decline in the innovations can be a general sign of model misspecification, filter divergence, too little measurement error, or other more esoteric problems (Kvamsdal and Sandal 2012). Since our parameters are modeled as  $c_k = \exp(\alpha_k)$ , where  $\alpha_k$  is represented by an ensemble, the confidence intervals of the  $c_k$  may not give the correct impression of wether an estimate improves over the time series or not (wether the ensemble converges or not). In comparison, errors decline by construction. Too much error decline is not healthy, however. If the root mean squared error goes to zero, the filter exhibits ensemble collapse and filter divergence.

Figure 7 shows the two year forecast of the model parameterized from the initialization with  $c_4 = 30$  million tonnes. The white curve shows the median forecast, while the shaded area illustrate the forecast uncertainty. The size of the shaded area demonstrates that there are considerable uncertainty in the system. Some observations lie outside the

 $<sup>^{3}</sup>a$  denote the analysis.

shaded area, but most are well within the forecast range. The forecasts seems to be downward biased, however. A reason could be that there is too much uncertainty in the estimated model. Uncertainty induces a downward drag on the drift term (Lund 2002) which would be too large if uncertainty is overestimated. Overestimated stochastic terms are rarely a problem with the ensemble Kalman filter, but with relatively short time series with considerable uncertainty in the parameter estimates, the estimated diffusion terms, which take account of the full uncertainty in the system, could still overestimate uncertainty. Forecasts will then be prone to whatLund (2002) labelled 'stochasticity induced depensation' in the growth equation. For sufficiently long time series, the problem would be smaller, as parameters in a good model would be relatively certain and the diffusion term would be comparatively smaller. That overestimated uncertainty is the culprit of the downward bias in the forecasts is further sentimented by deterministic forecast calculations which simply ignores the diffusion term. Deterministic forecasts are more on par with the observations, at least in terms of absolute levels. All in all, we conclude that the results from comparing two-year forecasts of the estimated, stochastic model with the observations are mixed. While the main, dynamic features seems to be picked up, and most observations are within the forecast range, the forecasts are often too small, and a deterministic forecast has less or no downward bias.

## Conclusion

We have estimated the parameters of a modified logistic ecosystem model of the pelagic fish stocks in the Norwegian Sea with the Ensemble Kalman Filter. Our model only contains four parameters. The model appear to capture much of the dynamics in the system as well as the interactions between the different species. The interactions are competitive, mutually destructive interactions, where NSS herring, mackerel and blue whiting prey upon the same food source(s), thus, limited by a common 'carrying capacity'. Increase in one species' biomass leads to reduced growth for all three species. We could have chosen a more elaborate model, allowing for more detailed intra specific and inter specific interactions. However, when we did this, the parameter values were heavily dependent on the initial values with no dynamics and little convergence over time.

While we use data from 1977 to 2010, after the stock collapse around 1970 the NSS herring stock was only found in Norwegian coastal waters, as the stock became more abundant it started to migrate into the Norwegian Sea again around 1990. In the late 1970s the Norwegian Sea was an important harvest area for the blue whiting fishery, however, the blue whiting catches in the Norwegian Sea declined and the waters west of the British Isles became the most important fishing areas. It was not until the late 1990s that the blue whiting fishery in the Norwegian Sea started to increase again. Similarly, in 2008, the mackerel stock began to migrate into the Icelandic economic zone. Iceland had not previously fished mackerel and blue whiting in any significant amounts, but began doing so when they showed up in Icelandic waters. The degree of overlap and potential interaction between the stocks must be said to have been low or virtually nonexistent for a large part of the analyzed time period. Moreover, the stocks do not interact with each other all the time; part of the year they live in different waters and only overlap during their feeding migrations in the summer. Although the overlaps takes place during an important phase of the fishes' yearly cycle, they all have niches where they do not compete with each other for food.

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