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**Using Local Gaussian Correlation in a
Nonlinear Re-examination of Financial Contagion**

by
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Using Local Gaussian Correlation in a Nonlinear Re-examination of Financial Contagion

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Abstract

This paper examines financial contagion, that is, whether the cross-market linkages in financial markets increase after a shock to a country. We use a new measure of local dependence (introduced by Tjøstheim and Hufthammer (2011)) to study the contagion effect. The central idea of the new approach is to approximate an arbitrary bivariate return distribution by a family of Gaussian bivariate distributions. At each point of the return distribution there is a Gaussian distribution that gives a good approximation at that point. The correlation of the approximating Gaussian distribution is taken as the local correlation in that neighbourhood. By examining the local Gaussian correlation before the shock (in a stable period) and after the shock (in the crisis period), we are able to test whether contagion has occurred by a bootstrap testing procedure. We illustrate our approach by re-examining the Mexican crisis of 1994, the Asian crisis of 1997-1998 and the financial crisis of 2007-2009. We find evidence of contagion based on our new procedure and are able to describe the nonlinear dependence structure of these crises.

JEL classification: G100; F3

Keywords: Financial crises; Nonlinear dependence; Contagion

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1 Introduction

In the past decades, the international financial markets have become ever more interlinked. Of particular interest is the spread of crises in these markets, that is, large falls in asset values in one country are quickly followed by falls in other countries. The existence of such an effect is called contagion if these falls cannot be explained by interdependence in trade or common macroeconomic factors. This effect is important for risk management and the performance of international portfolios. An increased dependence in financial markets during a crisis period, will imply that the diversification effect could be less than anticipated.

Some of the first studies on testing for contagion focused on the cross-market correlations before a shock (that is, in a 'stable period') and after the shock (in a 'turmoil period'); see for instance Bertero and Mayer (1989) and King and Wadhvani (1990), and references therein. Attention has often been focused on correlation of certain segments of returns (e.g. exceedance correlation). The appellation 'conditional correlation' has been used for these correlations, since they are computed conditionally on returns belonging to selected segments of the returns. If the conditional correlation increases significantly in the turmoil period, this suggests that contagion has occurred (often referred to as 'correlation breakdown'). However, conditioning the correlation by considering a subset of the sample space introduces a bias effect that makes interpretation difficult. Thus these 'correlation breakdowns' can be generated by data whose correlation coefficient is constant (see Boyer et al. (1999)). Further, as also noted by Boyer et al. (1999) and Forbes and Rigobon (2002), since the market volatility usually increases in unstable time periods, these correlations will be larger than in stable periods, and conclusions of contagion may thus be considered incorrect if synchronisation in volatility is not considered a contagion effect. Forbes and Rigobon (2002) explicitly adjust for the increase in correlation due to increase in volatility, and conclude that no contagion had occurred during the 1987 crash, the Mexican devaluation in 1997 and the East Asian crisis. However, they noted that there was a high degree of interdependence in all the periods studied.

There exist several alternative methods of testing for contagion. For example, Longin and Solnik (2001) and Bae et al. (2003) have used extreme value theory models, while Ramchand and Susmel (1998), Ang and Bekaert (2002) and Gallo and Otranto (2008) have based themselves on Markov switching models. Recently, Rodriguez (2007) has proposed

to use copula models with switching-parameters to study contagion effects, and that paper suggests that contagion may be a nonlinear phenomenon. Especially, since the author notes that 'patterns of change in tail behavior differ widely across markets, with tail dependence being more prevalent in times of financial turmoil'. Thus, the correlation coefficient is not appropriate in such cases, because it is designed to only measure the linear association between two markets. For a review of different tests of contagion we refer to Dungey et al. (2005) and Pesaran and Pick (2007).

In this paper we return to the correlation concept, but we introduce the use of a new measure of local correlation, see Tjøstheim and Hufthammer (2011) (cf. also Teräsvirta et al. (2010) p. 58-60), to study the contagion effect. The central idea of the new approach is to approximate an arbitrary bivariate return distribution by a family of Gaussian bivariate distributions. At each point of the return distribution there is a Gaussian distribution that gives a good approximation at that point. The correlation of the approximating Gaussian distribution is taken as the local correlation in that neighbourhood. This results in a nonlinear dependence measure, which is inherently local. By comparing the local correlation for the stable and crisis period, we are able to test whether contagion has occurred by introducing a test procedure.

The traditional global correlation analysis implicitly assumes a Gaussian distribution for financial returns, which is just a local assumption in our approach. Further, the local Gaussian correlation does not suffer from the bias problem of the conditional correlation described above. Another advantage of using local Gaussian correlation is that this approach is able to detect more complex, nonlinear changes in the dependence structure, that the global correlation may mask. It thus allows us to get a better understanding of the dependence between markets in the tails of the distribution, as well as in other segments. This is in contrast to the most common tests for contagion. Note that the copula method can also reveal nonlinear dependence and tail-dependence, but one then uses one or more parameters that have a rather indirect interpretation as a measure of dependence. In this respect our procedure has a more natural basis and retains a correlation interpretation based on local Gaussian approximation.

A related approach to our procedure is presented in Bradley and Taqqu (2004, 2005*a,b*). This approach is used by Inci et al. (2011). In these papers the authors also look at a local correlation measure, but without the local Gaussian aspect. They rather require a specific

regression-like relationship to hold between the returns X and Y in two markets, and it introduces asymmetry in the sense that it is not invariant to interchanging X and Y in computing the local correlation. There is an adjustment, cf. Bjerve and Doksum (1993), where this can be corrected for, but it leaves a heuristic impression. Note further that the definition of contagion used by Bradley and Taqqu (2004, 2005*a,b*) and Inci et al. (2011) is 'there is contagion from market X to market Y when the dependence becomes higher at the loss tail distribution of X than at its center'. This implicitly leads to a restriction in the local dependence contagion pattern, whereas we do not impose such a constraint. We can measure if there is stronger positive dependence between markets for any part of the distribution of returns.

The organization of the paper is as follows. Section 2 summarizes the new measure, and we present a nonparametric bootstrap test for contagion in section 3. In section 4 we present the different crises, the data used, and perform the analysis. Section 5 offers some conclusions. A few technical details regarding the local Gaussian correlation measure and the estimation of it, are given in the appendix.

2 Local Gaussian approximation and local correlation

The correlation ρ is primarily meaningful for a bivariate Gaussian density,

$$\phi(u, v, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)} \left[\left(\frac{u-\mu_1}{\sigma_1}\right)^2 + \left(\frac{v-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{u-\mu_1}{\sigma_1}\right)\left(\frac{v-\mu_2}{\sigma_2}\right) \right]\right\}. \quad (1)$$

There are several precise interpretations of the correlation ρ or its estimate (see, for example, Rodgers and Nicewander 1988; Rovine and von Eye 1997), but, most importantly, it completely describes the dependence structure of a pair of random variables (U, V) having ϕ as its density, and the Gaussian property is invariant to linear transformations.

In this paper, we will look at pairs of equity indices (log) returns, say X and Y , and study their dependence. The bivariate return density f for the two returns, X and Y , is (almost) never Gaussian, at least when looking at returns in a relatively small interval (daily

or smaller), see e.g. Rydberg (2000). In the conditional correlation approach one tries to take care of this by computing the ordinary (sample) correlation restricted to a set A , but as mentioned in the introduction, this introduces a bias effect. We therefore believe that it is better to start with the density f itself, and approximate it, not the correlation, locally. This local approximation is done with a family of Gaussian distributions such that at each point (x, y) , the density $f(x, y)$ is approximated (in a sense which will be made clear later) by a Gaussian bivariate density,

$$\begin{aligned} \phi_{x,y} &= \phi(u, v, \mu_1(x, y), \mu_2(x, y), \sigma_1(x, y), \sigma_2(x, y), \rho(x, y)) \\ &= \frac{1}{2\pi\sigma_1(x, y)\sigma_2(x, y)\sqrt{1-\rho(x, y)^2}} \exp\left\{-\frac{1}{2(1-\rho(x, y)^2)} \right. \\ &\quad \left. \times \left[\left(\frac{u-\mu_1(x, y)}{\sigma_1(x, y)}\right)^2 + \left(\frac{v-\mu_2(x, y)}{\sigma_2(x, y)}\right)^2 - 2\rho(x, y)\left(\frac{u-\mu_1(x, y)}{\sigma_1(x, y)}\right)\left(\frac{v-\mu_2(x, y)}{\sigma_2(x, y)}\right) \right] \right\}, \quad (2) \end{aligned}$$

where the parameters depend on (x, y) , and in such a way that $\phi_{x,y}$ is close to f in a neighbourhood A of (x, y) , but not necessarily elsewhere. As we move to another point (x', y') of f , another Gaussian $\phi_{x',y'}$ is required to approximate f in a neighbourhood A' of (x', y') . The correlation $\rho(x, y)$ completely characterises the dependence structure of $\phi_{x,y}$, and since $\phi_{x,y}$ is close to f in A , it approximates the complete dependence structure of f in A (but again not necessarily elsewhere). In this way the dependence in f is described (given the appropriateness of the local fitting) by the family of Gaussian distributions $\{\phi_{x,y}\}$ and the associated correlations $\{\rho(x, y)\}$.

To do this in practice, we require a method for fitting a Gaussian $\phi_{x,y}$ to f in a neighbourhood of (x, y) , and one such method is local likelihood, as described in Hjort and Jones (1996), and which was applied to local Gaussian approximations in Tjøstheim and Hufthammer (2011). In the appendix we give a brief outline of this method, and refer to the cited publication for further details.

The derivations in the appendix is based on the assumption that the pairs (X_t, Y_t) , $t = 1, \dots, T$ are independent and identically distributed. This is not always realistic, and especially the volatility may exhibit dependence in time. In this paper we therefore apply a GARCH(1,1) filtering to come closer to this assumption (Bollerslev et al. 1992). The asymptotic theory in the appendix can be extended to the stationary case under some regularity

conditions, see Tjøstheim and Hufthammer (2011). The purpose here is to introduce the local concepts in the context of financial returns and to demonstrate their usefulness when studying financial contagion.

We note the following advantages of applying the local Gaussian correlation model as a method for testing for financial contagion.

1. The dependence measure is based on a family of Gaussian distributions, and describes the dependence relation for $\phi_{x,y}$ and hence for f at the point (x, y) , since $\phi_{x,y}$ approximates f at that point. It thus gives a complete characterization of dependence locally. Moreover, properties that are true for global Gaussian dependence holds locally in a neighbourhood of (x, y) .
2. Unlike the conditional correlation and similar local dependence measures, in the case that f itself is Gaussian, $\rho(x, y) \equiv \rho$ everywhere, where ρ is the ordinary correlation of f . Thus ρ does not suffer from the bias problem of the conditional correlation mentioned in the introduction. For the local correlation measure used by Bradley and Taqqu (2004, 2005*a,b*) and Inci et al. (2011), the local correlation does not reduce to ρ , but to a function of ρ .
3. As will be seen, the local Gaussian correlation $\rho(x, y)$ is capable of detecting and quantifying nonlinear dependence structures. Moreover, a quantitative interpretation can be given in terms of the strength of the correlation of the approximating local Gaussian distribution.

We note that local Gaussian correlation is also capable of detecting asymmetric dependence between financial assets, in particular for modelling multivariate dependencies. In Støve et al. (2011) these topics will be studied in detail.

3 A bootstrap test for contagion

As in Forbes and Rigobon (2002) by contagion we understand: ‘a significant increase in cross-market linkages after a shock to one country (or a group of countries)’. By using the setup in the previous section, we will now introduce a test for contagion, and we will use the measure of local Gaussian correlation to study whether cross-market linkages have increased. The

proposed test is a bootstrap procedure, and similar tests are often used in a nonparametric setting, e.g. for testing difference between quantities in nonparametric regressions, see e.g. Hall and Hart (1990) and Vilar-Fernandez et al. (2007).

Let $Y_t, t = 1, \dots, T$ be the stock market return in the crisis country and $X_t, t = 1, \dots, T$ be the stock market return in another market. A filtration of the data is performed to remove dependence over time, and to remove volatility effects as is done by Forbes and Rigobon (2002). The standardised returns are denoted $d_t = (X'_t, Y'_t)$. The data are then split up in a stable period (NC) and a turmoil period (C) (note that such a predefined split of observations will naturally impact the outcome, see Dungey and Zhumabekova (2001), thus the test can be performed on a number of splits, to ensure robustness of the results). Contagion is present if the local correlation function for the turmoil period is significantly above the local correlation function for the stable period, i.e. we want to test

$$\begin{aligned} H_0 &: \rho_{NC}(x_i, y_i) = \rho_C(x_i, y_i) \quad (\text{no contagion}) \\ H_1 &: \rho_{NC}(x_i, y_i) < \rho_C(x_i, y_i) \quad (\text{contagion}), \end{aligned}$$

where (x_i, y_i) is the data-grid of interest.

The bootstrap method works as follows. From the observations $\{d_1, \dots, d_T\}$, draw at random and with replacement a resample $\{d_1^*, \dots, d_T^*\}$. Divide this resample into time periods NC and C and compute $\hat{\rho}_{NC}^*(x_i, y_i)$ and $\hat{\rho}_C^*(x_i, y_i)$ on a grid $i = 1, \dots, n$. In this section and section IV, we will use a diagonal grid, i.e. $x_i = y_i$. Next calculate the test variable

$$D_1^* = \frac{1}{n} \sum_{i=1}^n [\hat{\rho}_C^*(x_i, x_i) - \hat{\rho}_{NC}^*(x_i, x_i)] w(x_i, x_i),$$

where w_i is a weight function to screen off parts of the local correlation or to concentrate on a certain region. By repeated resampling, D_1^* is computed for these resamples and its distribution constructed (i.e. the distribution under H_0). Finally, from the real filtered observations $\{d_1, \dots, d_T\}$ calculate $\hat{\rho}_{NC}(x_i, x_i)$, $\hat{\rho}_C(x_i, x_i)$ and the test statistic $D_1 = \frac{1}{n} \sum_{i=1}^n [\hat{\rho}_C(x_i, x_i) - \hat{\rho}_{NC}(x_i, x_i)] w(x_i, x_i)$. The p-value in terms of the D_1^* distribution is found, and imply a rejection of H_0 if it is below a chosen significant level α .

Unlike the test of Bradley and Taqqu (2004, 2005*a,b*), this test focuses not only on

possible changes in the tail, but overall differences in the correlation structure. In the next sections we perform two simulation studies to check the finite sample performance of the bootstrap test in terms of both size and power of the test. As described in the appendix, the local Gaussian correlation estimator, $\hat{\rho}(x, y)$ depends on two smoothing parameters, the bandwidths $h = (h_1, h_2)$, and to a lesser degree on the kernel used. In the simulations and the empirical analysis we use the Gaussian kernel, and choose the bandwidths using a simple rule of thumb – the global standard deviation times a constant. We also prefer to oversmooth rather than undersmooth, that is, we smooth slightly towards a constant local correlation (equal to the global correlation). This approach gives reasonable results, see also Tjøstheim and Hufthammer (2011) for further discussions regarding bandwidth selection.

3.1 Study of error in significance level

The set-up for the simulation study for checking the significance level of the test is as follows. The same data generating process is used for both the NC period and the C period, thus H_0 is true. We use a Clayton copula with parameter $\theta = 2$ with two Gaussian marginal distributions, each with a mean equal to zero and a standard deviation equal to 4. This is a typical model for bivariate equity returns, see e.g. Okimoto (2008). We generate $M = 500$ independent sets of data from this model, where each data set is on the form $\{d_1, \dots, d_T\}$. We set $T = 400$ and let the NC period consists of 300 observations while the C period consists of 100 observations, since usually the turmoil period will be smaller than the stable period. For a given set of data, the test statistic D_1 is calculated. Bootstrap test of nominal level 0.01, 0.05 and 0.10 are conducted based on $B = 500$ bootstrap samples from each of the given data sets, as described above. The null hypothesis is rejected if the proportion of bootstrap statistics exceeding D_1 is less than or equal to the appropriate nominal level. Note that the same simulations were used to check all three nominal levels. The weight function is in this case chosen as $w(x_i, x_i) = 1$ if $x_i \in [-10, 9]$ and 0 elsewhere. The reason for this choice is that there are almost none observations outside this interval.

In Figure 1 the local Gaussian correlations are shown for one of the simulated data sets, the left plot shows the local correlations for the NC period, the right plot for the C period. As expected, we see that the two plots are relatively similar. Note that the local correlations are largest in the lower left hand-side in both plots, due to the fact that the Clayton copula

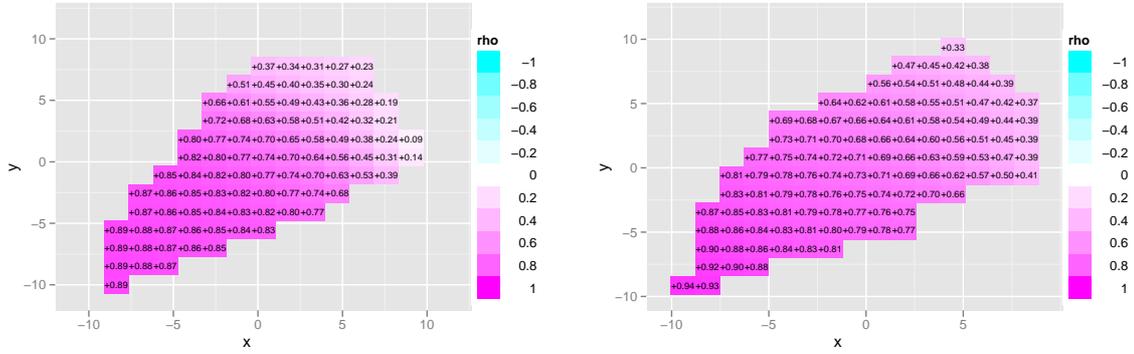


Figure 1: Local Gaussian correlation maps estimated from one simulation from the level error study. Left plot based on data from the NC period - right plot based on data from the C period.

exhibit lower tail dependence. This shows that the local correlations is capable of detecting nonlinear dependencies.

For clarity we restrict ourselves to looking at the local Gaussian correlation at values at the diagonal, i.e. the two plots in Figure 1 are turned into Figure 2. As mentioned in the last section, the bootstrap test uses the local correlations computed on this diagonal grid, but of course we could have used a larger and more general grid (x_i, y_i) to take account for any differences in local correlations outside the diagonal.

The empirical significance level of the test is reported in Table 1. The results show that the empirical level of the bootstrap test is consistently close to the nominal level.

Table 1: Results from the level study.

Nominal level (α)	0.01	0.05	0.10
Number of rejected null (of 500)	8	26	51

3.2 Study of power

When studying the power of the bootstrap test, we use the same setup as for the level study (both in terms of gridpoints and weight functions), but the data generating process is different. For the NC period, we now use a Gaussian copula with parameter equal to 0.5 (i.e. the global correlation is 0.5), and two Gaussian marginals with mean equal to zero and

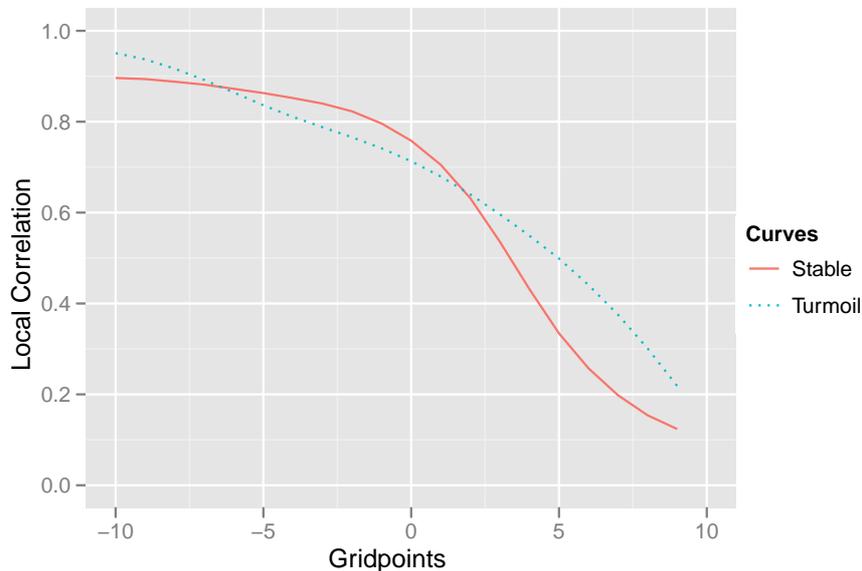


Figure 2: Diagonal local Gaussian correlations for the NC and C periods for one simulation from the level study.

a standard deviation equal to 4. For the C period we use a Clayton copula with parameter $\theta = 2$ with two Gaussian marginals with mean zero and standard deviation equal to 4 (i.e. the global correlation is around 0.7). This means that H_1 is true. Again, we generate 500 independent sets of data, and proceed as the above level study, and we obtain the empirical power for three nominal levels (0.01, 0.05 and 0.10). In Figure 3 the local correlation curves for the NC period and C period are shown for one of the simulated data set. Clearly, the local correlation curve for the C period is significantly higher than the local correlations for the NC period.

For the power study we also let $B = 500$ and $T = 400$. In Table 2 the empirical power is shown (i.e. number of accepted H_1 out of the 500 tests). Overall the power is acceptable, for the nominal level 0.05, the power of the test is around 0.70. Based on the results from both the level and power study, we conclude that the proposed bootstrap test performs as expected for these experiments.

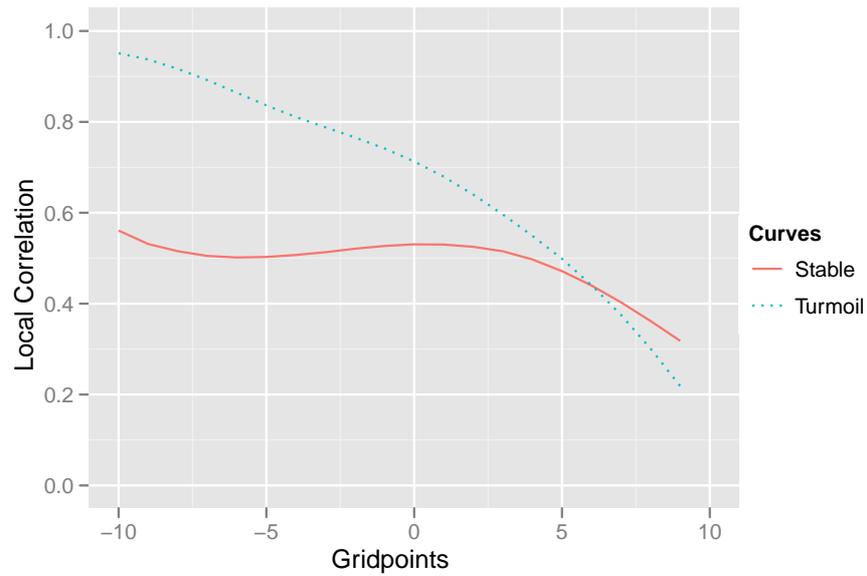


Figure 3: Diagonal local Gaussian correlations for the NC and C periods for one simulation from the power study.

Table 2: Results from the power study.

Nominal level (α)	0.01	0.05	0.10
Number of accepted alternatives (of 500)	210	342	396
Power in %	42.0	64.8	79.2

4 Empirical results

In this section, three crises are re-examined; the Mexican Peso crisis of 1994, the East Asian crisis of 1997 and the recent financial crisis 2007-2009. We use daily data (mainly in US dollars) of stock indices, where the equity price indices are retrieved from DataStream (see the appendix for the mnemonics). We calculate the returns as 100 times the difference in the log of the price indices.

The first crisis studied is the Mexican Peso crisis of 1994 (see e.g. Forbes and Rigobon (2002) for background information of the crisis). We use daily data (in US dollars) of stock indices from Mexico, Argentina, Brazil and Chile from January 1st 1993 to December 31st 1995, as Rodriguez (2007). The price indices, where we normalise the indices to 100 on 1st January 1993 (by dividing later index values by the values at this start date) and the daily log returns are shown in Figure 4 and 5, respectively. For obvious reasons, Mexico is chosen as the source country for the crisis.

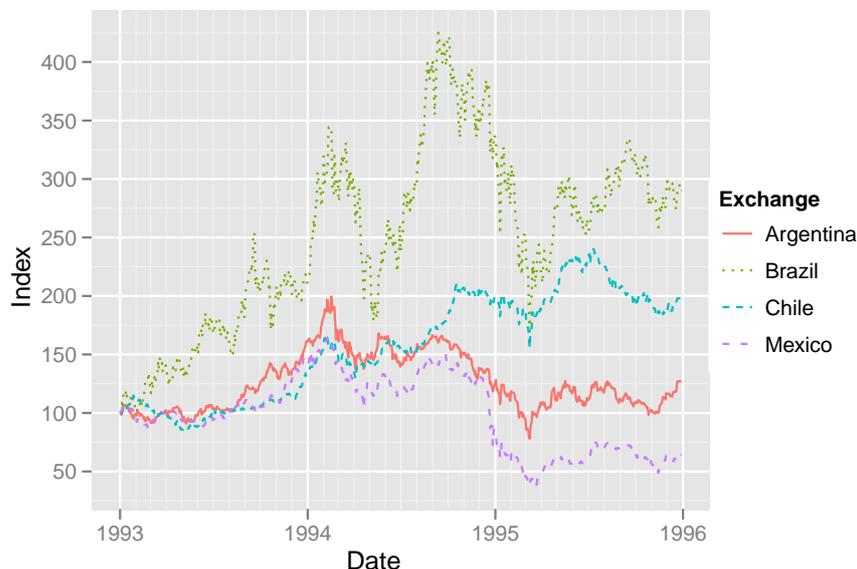


Figure 4: Market indices for Argentina, Brazil, Chile and Mexico, normalised to 100 at the start date.

We define the stable period from January 1st 1993 to December 18th 1994, and the crisis period is thus starting on December 19th 1994 (the day the exchange rate regime in Mexico

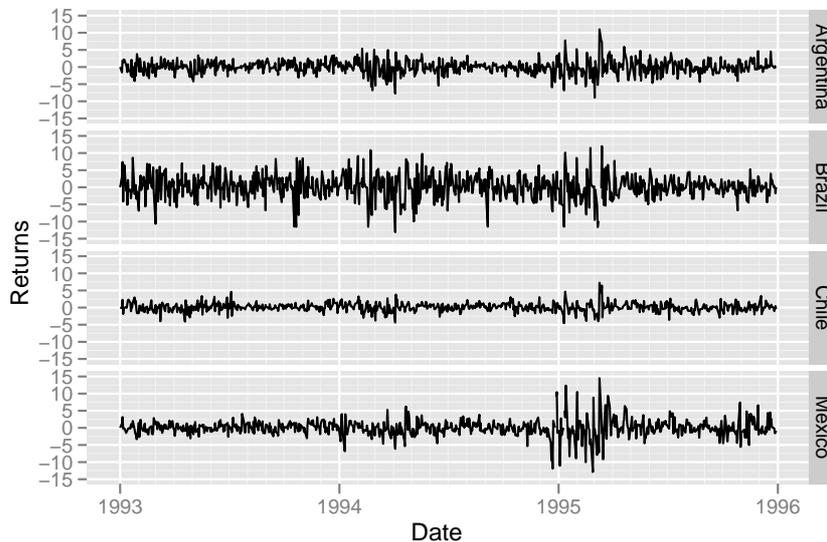


Figure 5: Returns for the market indices for Argentina, Brazil, Chile and Mexico (in this order, from top to bottom).

was abandoned). This is the same starting date for the crisis as Forbes and Rigobon (2002). We set the end period to December 31st 1995. One might dispute this long crisis period, but in order to ensure enough observations when estimating the local correlations, we choose to use a longer crisis period. The descriptive statistics for the stable period and the crisis period series are shown in Table 3. The data exhibit non-normality, as is seen from the skewness and kurtosis coefficients. Also, the Jarque-Bera test rejects normality for all series.

As earlier stated, the observations of each variable should be independent over time in order to use the bootstrap. We thus filter each return series by a univariate GARCH(1,1) model with a Student t error distribution, and then compute the standardised residuals, which are used in the further analysis.¹ We then also remove possible effects due to synchrony in volatility mentioned in the Introduction. That is, for each series, we have the

¹Note that we do not fit separate GARCH models for the stable and the turmoil period; doing so has little effect on the results, however.

Table 3: Descriptive statistics for the Latin American returns.

Stable period	Mexico	Argentina	Brazil	Chile
Mean	0.035	0.063	0.262	0.136
Standard deviation	1.592	1.743	3.701	1.162
Minimum	-6.823	-7.753	-18.923	-4.364
Maximum	6.253	5.378	10.853	4.610
Skewness	-0.118	-0.425	-0.499	-0.182
Excess kurtosis	4.867	4.436	4.865	4.540
Jarque-Bera	75.4	59.3	95.4	53.3
Crisis period				
Mean	-0.231	-0.030	-0.101	-0.009
Standard deviation	3.861	2.442	3.605	1.470
Minimum	-21.797	-8.955	-11.625	-4.580
Maximum	14.523	11.018	23.325	7.268
Skewness	-0.789	0.324	0.906	0.636
Excess kurtosis	9.290	5.255	10.277	6.592
Jarque-Bera	473.2	61.9	632.7	163.4

following model for log-return r_t :

$$\begin{aligned}
 r_t &= \mu + a_t, \\
 a_t &= \sigma_t \epsilon_t, \\
 \sigma_t^2 &= \omega + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2,
 \end{aligned}$$

where the notation is self explanatory. The standardised residuals are calculated as $\hat{a}_t = (r_t - \hat{\mu})/\hat{\sigma}_t$. The diagnostic output of the GARCH filtering is shown in Table 4. The shape parameter of the Student t error distribution is also given. The Ljung-Box statistics calculated for squared residuals indicate that the fitted models are adequate.

As the global correlations may mask more complex, nonlinear changes in the dependence structure, which the local Gaussian correlation may be able to detect, we estimate the local Gaussian correlations for the three country pairs based on the standardised residuals. The estimated local Gaussian correlations between the standardised returns from Mexico and the three other countries for both the stable and crisis period are shown in Figure 6. The local Gaussian correlations between Mexico and Argentina seem not to have changed in the tails from the stable to the crisis period, but in the middle of the distribution a larger

Table 4: GARCH parameter estimates for the Latin American markets. ** and * mean that we reject that the parameter is equal to zero at the 1% and 5% levels, respectively. The p-values of the Ljung-Box statistics for tests of lack of correlation of squared residuals from the model are shown for different lags (10, 15 and 20).

Market	μ	ω	α	β	shape	Q(10)	Q(15)	Q(20)
Mexico	-0.05	0.21*	0.15**	0.81**	5.19**	0.11	0.26	0.54
Argentina	0.1	0.06	0.09**	0.90**	10.0**	0.84	0.71	0.79
Brazil	0.2	0.36	0.10**	0.88**	7.45**	0.28	0.39	0.32
Chile	0.09*	0.27	0.23**	0.62**	5.91*	0.73	0.29	0.07

local Gaussian correlation is estimated in the crisis period. The local Gaussian correlation curves for Mexico-Argentina and Mexico-Chile are in agreement with the findings of Rodriguez (2007); there is no evidence of changes in tail dependence from the stable period to the turmoil period in these cases. However, in the Mexico-Brazil case, the local Gaussian correlation curves do indicate changes in tail dependence, as well as in other segments of the data, which is in possible contrast to Rodriguez (2007), where he finds that a normal copula describe the dependence in the crisis period best.² This would correspond to the two curves being horizontal in Figure 6, which they approximately are over quite large segments of the data.

In order to test for contagion, we use the proposed bootstrap test. As in the simulation studies, a diagonal grid ($x_i = y_i$) is used with a weight function equal to one from -2.5 to 2.5 and zero elsewhere. Further, the number of bootstrap draws (B) is set equal to 1000. The p-values from the bootstrap tests are shown in Table 5. Choosing a significance level of 5% we have contagion from Mexico to Brazil. This is also observed by Rodriguez (2007). Using a test functional more concentrated on moderate values of return would probably result in a lower P-value, also for the pair of Mexico and Argentina.

The second analysis is the East Asian crisis of 1997, see e.g. Forbes and Rigobon (2002) for some background information of this crisis. We use daily data (in US dollars) of stock indices from five Asian countries; Thailand, Malaysia, Indonesia, Korea and Philippines. The observation span is from January 1st 1996 to June 30th 1998, in total 652 observations.

²We note that although Rodriguez (2007) do not specify a stable and turmoil period as we do (he resort to a Markov switching model), we still compare our results with his, as we are as he, interested in any nonlinear changes in the dependence structure.

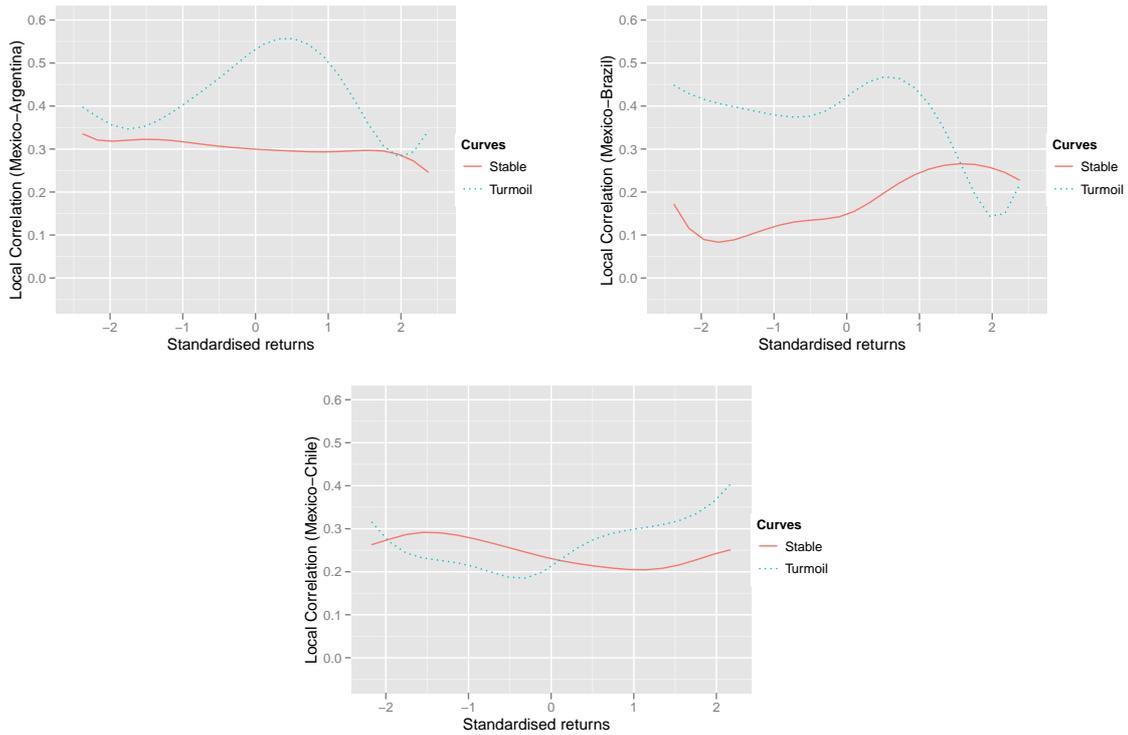


Figure 6: Local Gaussian correlation curves estimated from the Latin American equity indices in the stable and crisis period.

Table 5: Results from the bootstrap test for the Latin American markets. Testing for contagion from Mexico.

Market	p-value	Contagion (on 5% level)?
Argentina	0.106	No
Brazil	0.023	Yes
Chile	0.331	No

The price indices and the daily log returns are shown in Figure 7 and 8, respectively.

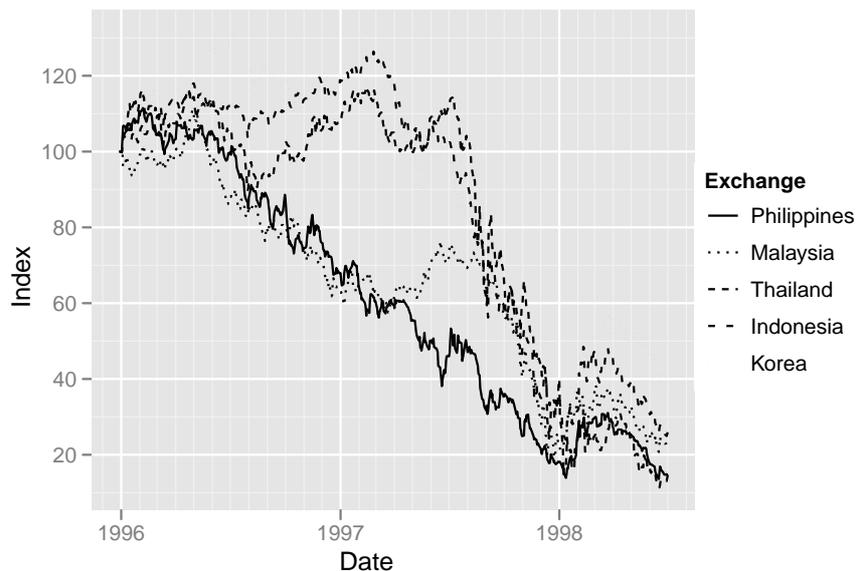


Figure 7: Market indices for Indonesia, Korea, Malaysia, Philippines and Thailand, normalised to 100 at the start date.

We define the stable period from January 1st 1996 to July 1st 1997, from Chiang et al. (2007), and choose Thailand as the source of the crisis (due to the fact that the Thai government gave up defending the value of its currency on July 2nd 1997). The crisis period is thus from July 2nd 1997 to June 30th 1998, where the end date is from Rodriguez (2007)³. The descriptive statistics for the stable period series and the crisis period series are shown in Table 6. The data exhibit non-normality, as is seen from the skewness and kurtosis coefficients. Also, the Jarque-Bera test rejects normality for all series.

As for the Mexican crisis, we filter the data using the same GARCH model. The diagnostic output of the GARCH filtering is shown in Table 7. The shape parameter of the Student t error distribution is also given. The Ljung-Box statistics calculated for squared residuals indicate that the fitted models are adequate.

The estimated local correlations between the standardised returns from Thailand and the four other countries for both the stable and crisis period are shown in Figure 9. Clearly, there is evidence of non-linear local correlations, and the local correlation curves for the crisis

³In fact, we use the same data span as Rodriguez (2007).

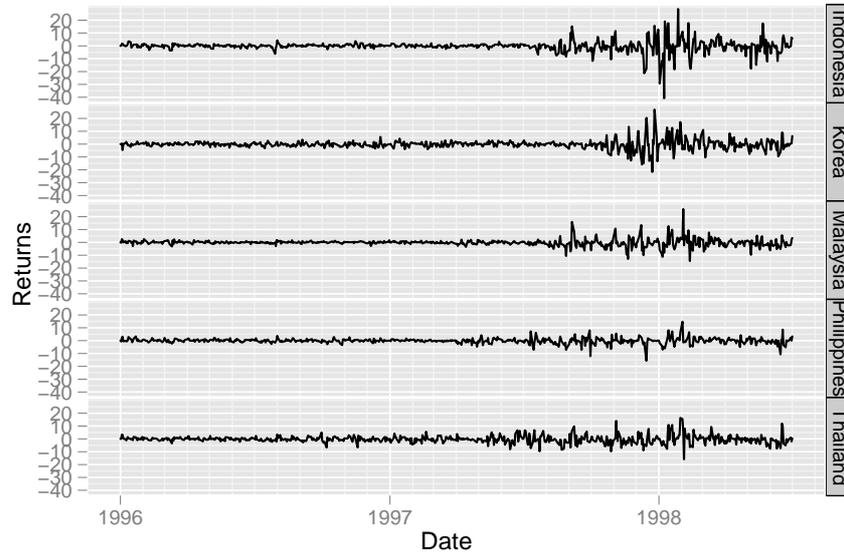


Figure 8: Returns for the market indices for Indonesia, Korea, Malaysia, Philippines and Thailand (in this order, from top to bottom).

Table 6: Descriptive statistics for the Asian returns.

Stable period	Thailand	Malaysia	Indonesia	Korea	Philippines
Mean	-0.199	0.005	0.031	-0.081	-0.004
Standard deviation	1.753	0.859	1.110	1.419	1.123
Minimum	-8.105	-3.512	-6.226	-4.613	-4.784
Maximum	6.723	2.558	4.334	5.663	4.188
Skewness	-0.068	-0.360	-0.416	0.072	-0.095
Excess kurtosis	3.005	1.752	3.334	0.630	1.937
Jarque-Bera	150.5	59.2	196.0	7.2	63.3
Crisis period					
Mean	-0.439	-0.519	-0.780	-0.414	-0.347
Standard deviation	4.169	4.250	6.916	5.287	3.134
Minimum	-15.893	-14.553	-40.821	-21.651	-15.732
Maximum	16.352	25.887	28.706	26.865	14.754
Skewness	0.695	1.103	-0.596	0.576	-0.148
Excess kurtosis	2.470	6.804	6.573	4.825	4.715
Jarque-Bera	89.9	568.2	496.1	274.2	248.9

Table 7: GARCH parameter estimates for the Asian markets. ** and * mean that we reject that the parameter is equal to zero at the 1% and 5% levels, respectively. The p-values of the Ljung-Box statistics for tests of lack of correlation of squared residuals from the model are shown for different lags (10, 15 and 20).

Market	μ	ω	α	β	shape	Q(10)	Q(15)	Q(20)
Thailand	-0.23**	0.10	0.17**	0.85**	4.01**	0.04	0.12	0.23
Malaysia	0.002	0.02	0.14**	0.88**	4.16**	0.90	0.98	0.98
Indonesia	0.02	0.11*	0.25**	0.80**	3.18**	0.25	0.37	0.23
Korea	-0.13*	0.07	0.14**	0.86**	7.14*	0.55	0.51	0.44
Philippines	-0.03	0.04	0.13*	0.88**	3.62**	0.85	0.83	0.93

periods are above the curves from the stable period. In most cases the local correlation curves also seem to have changed shapes, i.e. from almost symmetric curves in the stable period to asymmetric curves in the crisis period. For Thailand-Malaysia the whole local correlation curve for the crisis period has moved up by almost the same amount, and is clearly above the curve for the stable period. In the Thailand-Indonesia case, the local correlation curve for the crisis period is higher only in the left-tail, raising the almost uncorrelated tail in the stable period to a positive local correlation of about 0.3. For the last cases, Thailand-Korea and Thailand-Philippines the local correlation curves for the crisis period are above those of the stable period. In particular, it is noted a quite strong increase in the local correlation for high returns, which could be due to similar readjustments in the crisis period. In conclusion, the plots seem to provide some evidence that contagion has occurred, but in order to examine whether contagion is significant in these four cases, we again use the proposed bootstrap test.

The diagonal grid ($x_i = y_i$) is used with a weight function equal to one from -2 to 2 and zero elsewhere (for the Thailand-Philippines test, the interval is from -1.5 to 1.5 due to few observations outside this interval). Further, the number of bootstrap draws (B) is set equal to 1000. The p-values from the bootstrap tests are shown in Table 8. Choosing a significance level of 5% we have contagion to Korea and Malaysia. Moreover, a weighted test function more concentrated on low values would probably lead to contagion at the 5% level also for Indonesia. However, the p-values for the other two markets would imply contagion on the 10 % level, but with rather different nonlinear structures. In conclusion, these findings do partly support Rodriguez (2007). In that paper, mixture copulas with high tail dependence

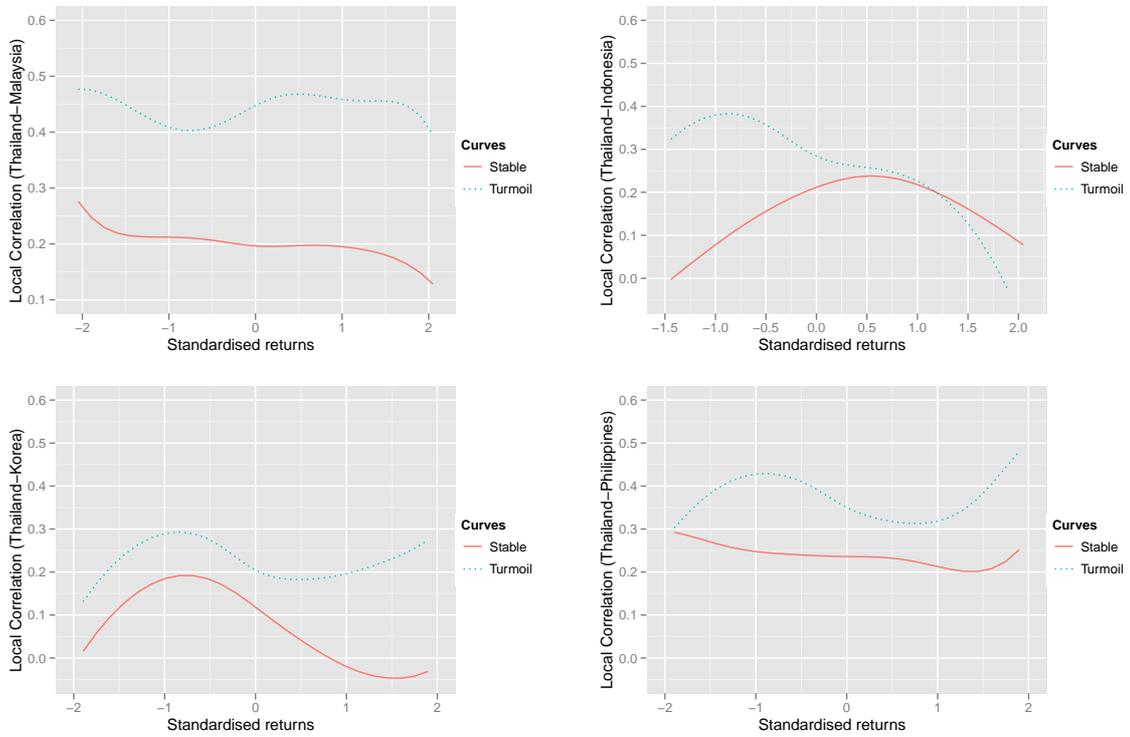


Figure 9: Local Gaussian correlation curves estimated from the Asian equity indices in the stable and crisis period.

where found to model best the dependencies in turmoil times, whereas mixture copulas with less tail dependence were best to model dependencies in calm periods.

Table 8: Results from the bootstrap test for the Asian markets. Testing for contagion from Thailand.

Market	p-value	Contagion (on 5% level)?
Malaysia	0.006	Yes
Indonesia	0.064	No
Korea	0.048	Yes
Phillipines	0.072	No

We finally examine the financial crisis of 2007–2009, and try to assess whether contagion has occurred from the US to other countries. We use daily log returns of stock market indices for five countries, the United States, where the crisis originated, the United Kingdom, France, Germany and Norway. The data span the time period from 1st January 2005 to 7th August 2009, a total of 1200 observations. We define the stable period to be from 1st January 2005 to 8th August 2007 (678 observations), and the turmoil period from 9th August 2007 to 7th August 2009 (522 observations), as the liquidity crisis emerged 9th August 2007 in the US⁴. Note that the returns are based on local currency, although returns in US dollar are most frequently used in such studies. However, the analyses conducted in Forbes and Rigobon (2002) do indicate that this will likely only have minor impact on the results. A related analysis of this crisis, based on a copula approach, has been conducted by Horta et al. (2010). See that paper for some background information on the crisis.

The descriptive statistics for the stable period series and the crisis period series are shown in Table 9. The data exhibit non-normality, as seen from the skewness and kurtosis coefficients. Also, the Jarque-Bera test rejects normality for all series.

Figure 10 shows market indices for the five countries. The corresponding log returns are given in Figure 11.

As for the Mexican and Asian crisis, we filter the data using a GARCH(1,1) model with Student t distributed error terms, before further analysis. The diagnostic output of the GARCH filtering is shown in Table 10. The Ljung-Box statistics calculated for squared residuals indicate that the fitted models are adequate.

⁴See NY Times: http://www.nytimes.com/2007/08/10/business/10liquidity.html?_r=1

Table 9: Descriptive statistics for the US and European return series.

Stable period	US	UK	France	Germany	Norway
Mean	0.031	0.042	0.060	0.086	0.094
Standard deviation	0.675	0.713	0.838	0.893	1.302
Minimum	-3.534	-3.197	-3.227	-3.463	-6.015
Maximum	2.386	2.604	2.505	2.605	6.916
Skewness	-0.374	-0.431	-0.347	-0.393	-0.395
Excess kurtosis	1.992	2.104	1.145	0.887	3.416
Jarque-Bera	129.6	147.9	51.5	40.3	351.0
Crisis period					
Mean	-0.075	-0.058	-0.094	-0.064	-0.092
Standard deviation	2.180	1.978	2.128	2.036	2.823
Minimum	-9.469	-9.266	-9.472	-7.433	-11.276
Maximum	10.957	9.384	10.594	10.797	11.016
Skewness	-0.086	0.008	0.212	0.333	-0.426
Excess kurtosis	4.275	3.987	4.423	5.247	2.489
Jarque-Bera	403.4	350.5	434.8	615.7	152.9

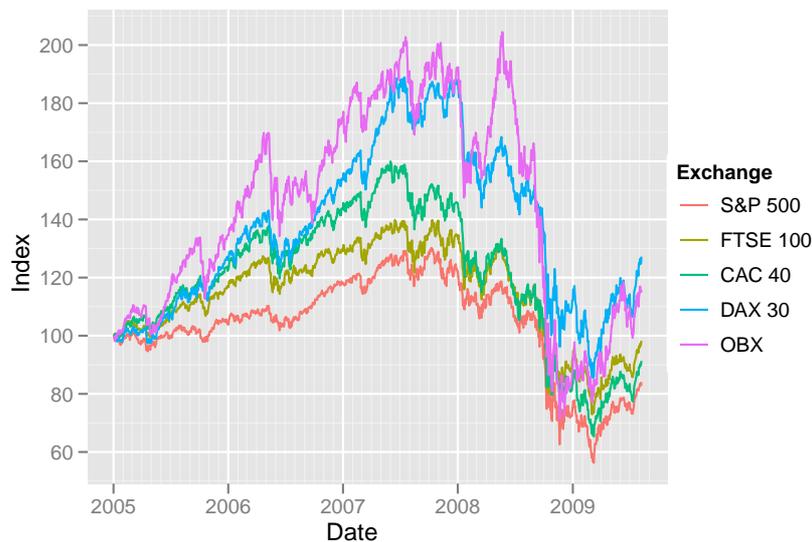


Figure 10: Market indices for Norway, Germany, France, the United Kingdom and the United States (in this order, from top to bottom), normalised to 100 at the start date.

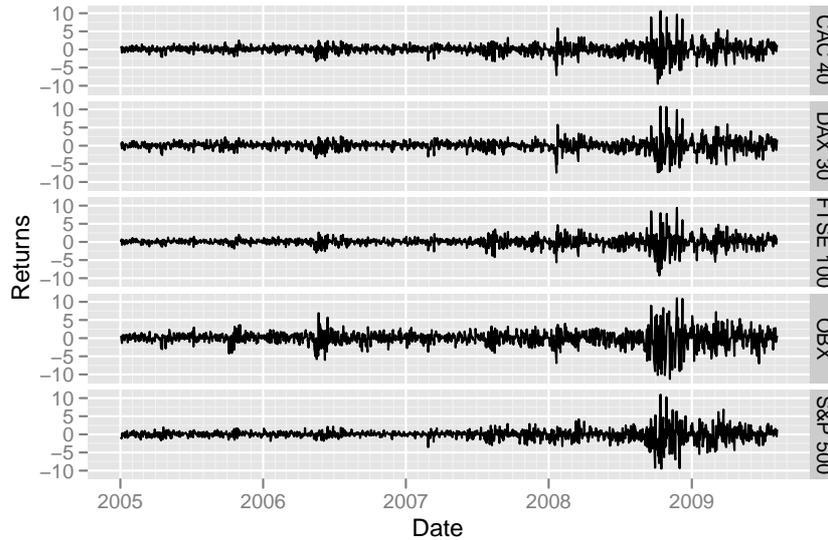


Figure 11: Non-standardised returns for the market indices for Norway, Germany, France, the United Kingdom and the United States (in this order, from top to bottom).

Table 10: GARCH parameter estimates for the US and European markets. ** and * mean that we reject that the parameter is equal to zero at the 1% and 5% levels, respectively. The p-values of the Ljung-Box statistics for tests of lack of correlation of squared residuals from the model are shown for different lags (10, 15 and 20).

Market	μ	ω	α	β	shape	Q(10)	Q(15)	Q(20)
US	0.06*	0.006	0.09**	0.91**	5.09**	0.17	0.45	0.66
UK	0.01	0.013*	0.13**	0.87**	7.87**	0.40	0.06	0.11
France	0.07**	0.02*	0.10**	0.89**	7.80**	0.52	0.04	0.09
Germany	0.11**	0.02*	0.11**	0.89**	5.72**	0.66	0.30	0.40
Norway	0.13**	0.05**	0.15**	0.85**	10.00**	0.50	0.80	0.92

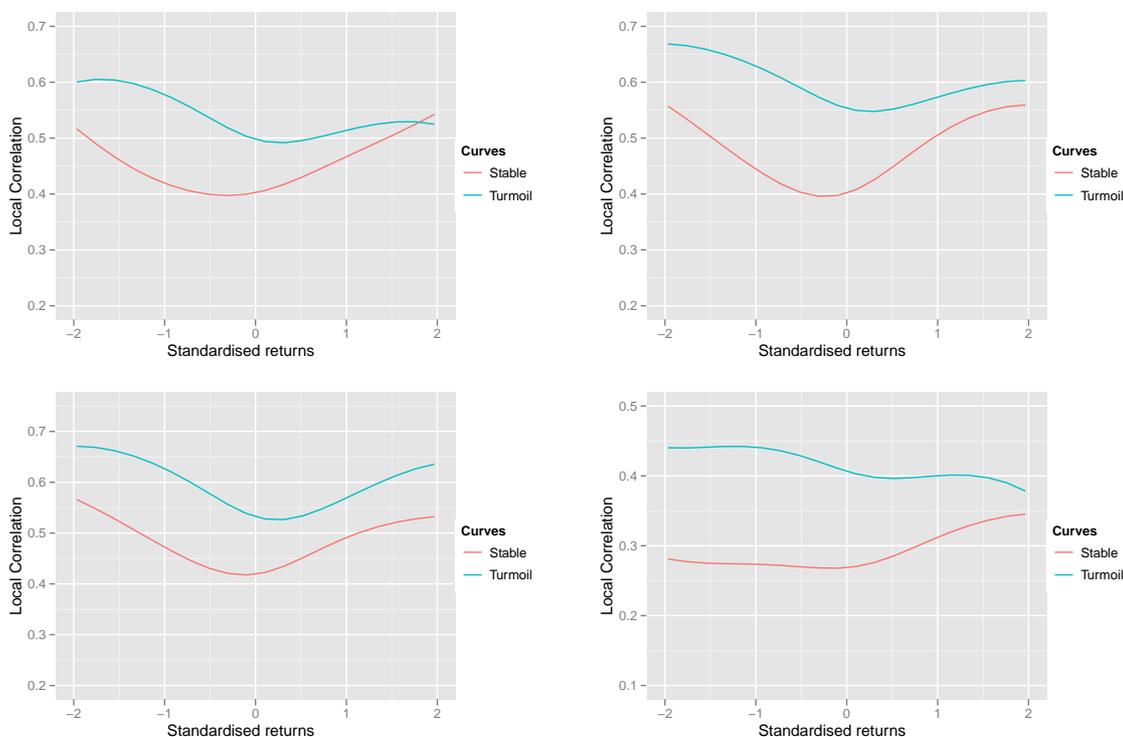


Figure 12: Local Gaussian correlation curves estimated between the US and European equity indices in the stable and crisis period (upper left: US-UK, upper right: US-France, lower left: US-Germany and lower right: US-Norway).

The estimated local Gaussian correlations for the four country pairs based on the standardised residuals are shown as diagonal plots in Figure 12.

The plots provide some evidence that the dependence between the US and the European markets have increased during the 2007–2009 crisis compared to the stable period from 2005 to August 2007. Most of the increase happens for (slightly) negative returns (evidence for asymmetric dependence), but it should be noted that there is nearly a uniform increase in local correlation over large segments of return values, not at all only for the tails. This means that once GARCH effects, producing strong correlations between large returns, have been eliminated, there is still stronger dependence between the US market and European markets during the period of turmoil for negative returns. The shape of the curves is the same across the various countries, with the stable periods tending to have more of a U-shape. One exception is the US–Norwegian market, where the curves are more flat, indicating closer

to constant dependence.

We next use the bootstrap test from section 3 on the GARCH-filtrated returns. We use a diagonal grid $x_i = y_i$, choose $B = 1000$, and $w_i = 1$ if $x_i \in [-2.5, 2.5]$ and 0 elsewhere. The p-values from the bootstrap tests are shown in Table 11. Choosing a significance level of 5%, we reject the null of no contagion for all countries, except the UK (although it is significant at the 10% level and indeed close to the 5% level). We therefore claim that contagion has occurred from the US to France, Germany and Norway during the crisis of 2007-2009.

Table 11: Results from the bootstrap test

Market	p-value	Contagion?
UK	0.053	No
France	0.009	Yes
Germany	0.015	Yes
Norway	0.031	Yes

Summing up and taking into consideration the shapes of the correlations curves, we have presented evidence of contagion during several crises by considering local Gaussian correlation instead of ordinary correlation. In particular, we notice that most dependence structures are non-linear, and that large changes of the curves happen most frequently in the tails of the distributions, but in general there may be substantial increase in local correlation over large segments of returns. We have only used diagonal grid values of the local Gaussian correlations to test for contagion, but in practice a more general grid can be applied.

5 Conclusions

In this paper we have applied a new measure of dependence, called the local Gaussian correlation, to re-examine financial contagion between international stock markets. In addition, a bootstrap test procedure for contagion is proposed. We note two advantages of using the local Gaussian correlation; first, for bivariate normal data, the local Gaussian correlation avoids the bias of the conditional correlation, and second, for non-normal data it provides a way of describing any non-linear changes in dependence and the departure from global normality. This is in contrast to the Pearson correlation coefficient, which only measures linear dependence. The local Gaussian correlation also differs from the related measure in-

troduced by Bjerve and Doksum (1993), and applied by Bradley and Taqqu (2004, 2005*a,b*) and Inci et al. (2011) to test for financial contagion. That measure is also a local measure of dependence, but it requires a regression relationship to hold, it is not a symmetric measure in (X, Y) , and it does not have a local interpretation in terms of a Gaussian correlation.

The results from our contagion analysis indicate that contagion has occurred both in the East Asian crisis of 1997 and the present financial crisis. For the Mexican crisis of 1994, the results are mixed, but the most important observation, may be that the dependence between markets is not linear, and that increased dependence during crises (increased local correlation) often happens for small (and in particular negative) and moderate standardised returns. On average the effects are strongest in the lower tail, but there are substantial effects over large segments of returns. This imply that contagion is a nonlinear phenomenon. Further, this observation also has implications for asset management.

One additional assumption we have used is that of independent pairs of variables. In the present paper we have used a GARCH model to come closer to this assumption, but the theory of local Gaussian correlation can in principle be extended to the general time series case. Moreover, a parametric model for the local Gaussian correlation can be useful for extreme events. We have only considered bivariate problems here, but a multivariate extension is possible under certain simplifying assumptions, as outlined in Støve et al. (2011).

Acknowledgement

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References

- Ang, A. and Bekaert, G. (2002), ‘International asset allocation with regime shifts’, *Review of Financial Studies* **15**(4), 1137–1187.
- Bae, K., Karolyi, G. and Stulz, R. (2003), ‘A new approach to measuring financial contagion’, *Review of Financial Studies* **16**, 717–763.
- Bertero, E. and Mayer, C. (1989), Structure and performance: Global interdependence of stock markets around the crash of october 1987. CEPR Discussion Papers.
- Bjerve, S. and Doksum, K. (1993), ‘Correlation curves: measures of association as functions of covariate values’, *The Annals of Statistics* **21**(2), 890–902.
- Bollerslev, T., Chou, R. Y. and Kroner, K. F. (1992), ‘ARCH modeling in finance: A review of the theory and empirical evidence’, *Journal of Econometrics* **52**(1-2), 5–59.
- Boyer, B. H., Gibson, M. S. and Loretan, M. (1999), Pitfalls in tests for changes in correlations, Discussion paper 597.
URL: <http://www.federalreserve.gov/pubs/ifdp/1997/597/default.htm>
- Bradley, B. and Taqqu, M. (2004), ‘Framework for analyzing spatial contagion between financial markets’, *Finance Letters* **2**(6), 8–15.
- Bradley, B. and Taqqu, M. (2005a), ‘Empirical evidence on spatial contagion between financial markets’, *Finance Letters* **3**(1), 77–86.
- Bradley, B. and Taqqu, M. (2005b), ‘How to estimate spatial contagion between financial markets’, *Finance Letters* **3**(1), 64–76.
- Chiang, T. C., Jeon, B. N. and Li, H. (2007), ‘Dynamic correlation analysis of financial contagion: Evidence from asian markets’, *Journal of International Money and Finance* **26**, 1206–1228.
- Dungey, M., Fry, R., González-Hermosillo, B. and Martin, V. L. (2005), ‘Empirical modelling of contagion: a review of methodologies’, *Quantitative Finance* **5**, 9–24.
- Dungey, M. and Zhumabekova, D. (2001), Testing for contagion using correlations: Some words of caution, Pacific Basin Working Paper Series 01-09, Federal Reserve Bank of San Francisco.
URL: <http://www.frbsf.org/publications/economics/pbcpapers/2001/pb01-09.pdf>
- Forbes, K. J. and Rigobon, R. (2002), ‘No contagion, only interdependence: Measuring stock market comovements’, *The Journal of Finance* **57**(5), 2223–2261.

- Gallo, G. M. and Otranto, E. (2008), ‘Volatility spillovers, interdependence and comovements: A markov switching approach’, *Computational Statistics & Data Analysis* **52**, 3011–3026.
- Hall, P. and Hart, J. D. (1990), ‘Bootstrap test for difference between means in nonparametric regression’, *Journal of The American Statistical Association* **85**, 1039–1049.
- Hjort, N. L. and Jones, M. C. (1996), ‘Locally parametric nonparametric density estimation’, *The Annals of Statistics* **24**(4), 1619–1647.
- Horta, P., Mendes, C. and Vieira, I. (2010), ‘Contagion effects of the subprime crisis in the european nyse euronext markets’, *Portugese Economic Journal* **9**, 115–140.
- Inci, A., Li, H. C. and McCarthy, J. (2011), ‘Financial contagion: A local correlation analysis’, *Research in International Business and Finance* **25**, 11–25.
- King, M. and Wadhvani, S. (1990), ‘Transmission of volatility between stock markets’, *Review of Financial Studies* **3**, 5–33.
- Longin, F. and Solnik, B. (2001), ‘Extreme correlation of international equity markets’, *The Journal of Finance* **56**(2), 649–676.
- Okimoto, T. (2008), ‘New evidence of asymmetric dependence structures in international equity markets’, *Journal of Financial and Quantitative Analysis* **43**(3), 787–816.
- Pesaran, M. H. and Pick, A. (2007), ‘Econometric issues in the analysis of contagion’, *Journal of Economic Dynamcis & Control* **31**, 1245–1277.
- Ramchand, L. and Susmel, R. (1998), ‘Volatility and cross correlation across major stock markets’, *Journal of Empirical Finance* **5**(4), 397–416.
- Rodgers, J. L. and Nicewander, W. A. (1988), ‘Thirteen ways to look at the correlation coefficient’, *The American Statistician* **42**(1), 59–66.
- Rodriguez, J. C. (2007), ‘Measuring financial contagion: A copula approach’, *Journal of Empirical Finance* **14**(3), 401–423.
- Rovine, M. J. and von Eye, A. (1997), ‘A 14th way to look at a correlation coefficient: Correlation as the proportion of matches’, *The American Statistician* **51**(1), 42–46.
- Rydberg, T. H. (2000), ‘Realistic statistical modelling of financial data’, *International Statistical Review* **68**, 233–258.

Støve, B., Hufthammer, K. O. and Tjøstheim, D. (2011), Measuring asymmetries in financial returns: A new approach using local Gaussian correlation. In preparation.

Teräsvirta, T., Tjøstheim, D. and Granger, C. W. (2010), *Modelling Nonlinear Economic Time Series*, Oxford University Press.

Tjøstheim, D. and Hufthammer, K. O. (2011), Local Gaussian correlation: A new measure of dependence. Unpublished manuscript.

URL:

<http://folk.uib.no/st11188/local-gaussian-correlation/local-gaussian-correlation.pdf>

Vilar-Fernandez, J., Vilar-Fernandez, J. and Gonzalez-Manteiga, W. (2007), ‘Bootstrap tests for nonparametric comparison of regression curves with dependent errors’, *Test* **16**, 123–144.

A Local likelihood theory

The following is a brief summary of the theory of local Gaussian correlation. Much more details concerning e.g. existence, uniqueness, asymmetry and tail properties can be found in Tjøstheim and Hufthammer (2011).

We use the same notation as in Section 2. Given the observations $\{(X_1, Y_1), \dots, (X_T, Y_T)\}$, the ordinary (standardised) log likelihood for a Gaussian density ϕ is given by

$$L^* = \frac{1}{T} \sum_{t=1}^T \log \phi(X_t, Y_t).$$

With ρ as in Equation (1), the maximum likelihood estimate is

$$\hat{\rho} = \frac{\sum (X_t - \bar{X})(Y_t - \bar{Y})}{\left(\sum (X_t - \bar{X})^2 \sum (Y_t - \bar{Y})^2\right)^{1/2}}.$$

We introduce kernel functions $K_{h_1}(X_i - x)$ and $K_{h_2}(Y_i - y)$ to describe a neighbourhood A around (x, y) . Here $K_{h_1} = h_1^{-1}K(h_1^{-1}(X_i - x))$, and similarly for K_{h_2} , where h_1 and h_2 are the bandwidths in the x and y direction, respectively. One might think that the

appropriate local likelihood associated with (2) would be given by

$$L' = \frac{1}{T} \sum_{t=1}^T K_{h_1}(X_t - x) K_{h_2}(Y_t - y) \log \phi_{x,y}(X_t, Y_t),$$

using the kernel function to localise the log likelihood, but it turns out (cf. Hjort and Jones (1996)) that an adjustment is needed, resulting in the local log likelihood

$$\begin{aligned} L &= \frac{1}{T} \sum_{t=1}^T K_{h_1}(X_t - x) K_{h_2}(Y_t - y) \log \phi_{x,y}(X_t, Y_t) \\ &\quad - \int K_{h_1}(u - x) K_{h_2}(v - y) \phi_{x,y}(u, v) du dv. \end{aligned}$$

Letting $\theta(x, y) = [\mu_1(x, y), \mu_2(x, y), \sigma_1(x, y), \sigma_2(x, y), \rho(x, y)]^T$ be the parameter vector of $\phi_{x,y}$, and letting $w_j(u, v, \theta)$ denote the derivative $\partial \log \phi_{x,y}(u, v, \theta) / \partial \theta_j$, it is easily seen that

$$\frac{\partial L}{\partial \theta_j} = \frac{1}{T} \sum_{t=1}^T K_{h_1}(X_t - x) K_{h_2}(Y_t - y) w_j(X_t, Y_t, \theta) \tag{3}$$

$$- \int K_{h_1}(u - x) K_{h_2}(v - y) w_j(u, v, \theta) \phi_{x,y}(u, v, \theta) du dv. \tag{4}$$

Letting $T \rightarrow \infty$ for fixed h_1 and h_2 and using the law of large numbers on the first term of Equation (4) the expression for $\partial L / \partial \theta_j$ converges towards

$$\int K_{h_1}(u - x) K_{h_2}(v - y) w_j(u, v, \theta) [f(u, v) - \phi_{x,y}(u, v, \theta)] du dv.$$

For small bandwidths, under appropriate smoothness conditions, and requiring $\partial L / \partial \theta_j = 0$ for all j , we have

$$w_j(x, y, \theta(x, y)) [f(x, y) - \phi_{x,y}(x, y, \theta(x, y))] + O(h^T h) = 0,$$

and the local likelihood estimates, satisfying $\partial L / \partial \theta_j = 0$, constrain $\phi(u, v, \theta(x, y))$ to be close to $f(x, y)$ when (u, v) is close to (x, y) . This is the sense in which the family $\phi_{x,y}$ approximates f as the neighbourhood defined by the bandwidth $h = [h_1, h_2]$ shrinks.

In practice we obtain the estimates $\hat{\theta}_{h,T}(x, y)$ by requiring Equation (4) to be zero, and solving the resulting 5-dimensional set of equations numerically. Note that we then obtain

not only a local correlation estimate $\hat{\rho}_h(x, y)$, but also local mean estimates $\hat{\mu}_{1,h}(x, y)$, $\hat{\mu}_{2,h}(x, y)$, and local variances $\hat{\sigma}_{1,h}^2(x, y)$ and $\hat{\sigma}_{2,h}^2(x, y)$, where the latter can be used to obtain local covariance estimates.

Letting h be fixed and T tend to infinity, $\hat{\theta}_h(x, y)$ converges in distribution to $\theta_h(x, y)$, satisfying

$$\int K_{h_1}(u - x)K_{h_2}(v - y)w_j(u, v, \theta_h(x, y)) \times [f(u, v) - \phi(u, v, \theta_h(x, y))] du dv = 0.$$

In practice we can check the quality of the Gaussian approximation by comparing $\phi(x, y, \hat{\theta}_h(x, y))$ to $\tilde{f}(x, y)$, the kernel estimate of f . (In fact in Hjort and Jones (1996) the main point of the local likelihood analysis is to derive alternatives to the kernel estimate of f .) Arguments in Hjort and Jones (1996), Tjøstheim and Hufthammer (2011) demonstrate that $\hat{\theta}_h(x, y)$ is asymptotically normal such that

$$(Th_1h_2)^{1/2}[\hat{\theta}_{T,h}(x, y) - \theta_h(x, y)] \xrightarrow{d} \mathcal{N}(0, J_h^{-1}M_h(J_h^{-1})^T),$$

where

$$\begin{aligned} J_h = & \int K_{h_1}(u - x)K_{h_2}(v - y)w(u, v, \theta_h(x, y)) \\ & \times w^T(u, v, \theta_h(x, y))\phi(u, v, \theta_h(x, y)) du dv \\ & - \int K_{h_1}(u - x)K_{h_2}(v - y)\nabla w(u, v, \theta_h(x, y)) \\ & \times [f(u, v) - \phi(u, v, \theta_h(x, y))] du dv \end{aligned} \quad (5)$$

and

$$\begin{aligned} M_h = & h_1h_2 \int K_{h_1}^2(u - x)K_{h_2}^2(v - y)w(u, v, \theta_h(x, y)) \\ & \times w^T(u, v, \theta_h(x, y))f(u, v) du dv \\ & - h_1h_2 \int K_{h_1}(u - x)K_{h_2}(v - y)w(u, v, \theta_h(x, y))f(u, v) du dv \\ & \times \int K_{h_1}(u - x)K_{h_2}(v - y)w^T(u, v, \theta_h(x, y))f(u, v) du dv. \end{aligned} \quad (6)$$

These expressions are somewhat deceptive, since $J_h^{-1}M_hJ_h^{-1}$ is of order $(h_1h_2)^{-2}$, so that

$\text{Var}[\widehat{\theta}_{T,h}(x, y)]$ is of order $(Th_1^3h_2^3)^{-1}$, a considerably slower convergence rate than the traditional nonparametric rate of $(Th_1h_2)^{-1}$.

The leading term in the covariance expression $J_h^{-1}M_hJ_h^{-1}$ is quite problematic to evaluate, and in practice we have used two alternative methods. The obvious alternative is to use the bootstrap since we have assumed iid observations. The other alternative is to estimate (5) and (6) directly using a mixture of numerical integration and empirical averages, with estimates of the parameters inserted. The latter method has the advantage that it can be extended to the case of stationary observations. The block bootstrap would be another alternative for handling the stationary non-iid case.

B Data sources

The data consist of daily price indices from different equity markets. The returns are computed as the difference of the natural logarithms of the daily price indices multiplied by 100. The price indices are from Datastream. The mnemonics are: Mexico - (MXIPC35~U\$), Argentina - (TOTMAR\$), Brazil - (BRBOVES~U\$), Chile - (TOTMCL\$), Thailand - (TOTMTH\$(PI)), Malaysia - (DJMALY\$(PI)), Indonesia - (DJINDS\$(PI)), Korea - (TOTMKO\$(PI)), Phillipines - (DJPHIL\$(PI)), US - (S&PCOMP(PI)), UK - (FTSE100(PI)), Germany - (DAXIND(PI)), France - (FRCAC40(PI)) and Norway - OSLOOBX(PI).

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