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**A new approach of fitting biomass
dynamics models to data**

by

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Abstract

A non-traditional approach of fitting dynamic resource biomass models to data is developed in this paper. A variational adjoint technique is used for dynamic parameter estimation. In the variational formulation, a cost function measuring the distance between the model solution and the observations is minimized. The data assimilation method provides a novel and computationally efficient procedure for combining all available information, i.e., the data and the model in the analysis of a resource system. This technique will be used to analyze data for the North East Arctic Cod Stock.

Two alternative population growth models: the logistic and the Gompertz model are used for estimating parameters of simple bioeconomic models by the method of constrained least squares. Estimates of the parameters of the models dynamics are reasonable and can be accepted. The main inference from the work is that the average fishing mortality is found to be significantly above the maximum sustainable yield (MSY) value.

1 Introduction

In spite of the growing criticisms of the biomass dynamics models or the surplus growth models (Clark, 1990), they remain the biological basis for most bio-economic analysis. Parameter estimation has been the greatest source of difficulty in applying the generalized biomass dynamics models in management schemes (Rivard and Bledsoe, 1978). The bulk of the research in this area has been done by fishery biologists in the past. Several methods have been developed for fitting these models to observed data. Three approaches have been commonly used to fit surplus production models to observations: effort averaging methods, process-error estimators, and observation-error estimators (see Polacheck et al., 1993).

The technique applied in this paper is a variational method for dynamic parameter estimation (Lawson et al., 1995; Smedstad and O'Brien, 1991). The variational adjoint technique determines an optimal solution by minimizing a cost function, which measures the difference between the model solution and the available observations. This method has many advantages compared to the traditional approaches used in fitting surplus production models. First, the technique is a dynamic approach while the ordinary regression used previously is static. Second, it can be applied to both linear and nonlinear models and does not require that nonlinear models be transformed into linear models, as is often the case in traditional regression. It is also computationally more efficient compared with the finite difference method. Recent applications of data assimilation techniques to biological and ecosystem models are Lawson et al. (1995), Spitz et al. (1998) and Matear (1995). To emphasize the advantages, we further point out that this technique can easily be used to estimate systems of dynamic equations that include feedback for discrete or continuous economic models. Interested reader may refer to the works of Lawson et al. (1995) and Spitz et al. (1998), which are applications to complex biological and ecosystem system models and Ussif et al. (2001).

Two forms of the existing growth models (Clark, 1990) in combination with a simple proportional exploitation rule will be used to estimate the biological and economic input parameters, such as the fishing mortality using, real data for the North-East Arctic Cod stock (NEACs). Notice that the biomass model is used only as an example and could be replaced by any model. The bioeconomic model employed in this analysis is simple. It combines surplus growth models with a linear harvest

function to analyze the data. The biological functions contain parameters that are very crucial in determining certain important quantities of interest to fisheries managers and researchers. Estimates of parameters such as the intrinsic growth rate and the environmental carrying capacity of the population are rare for some important fish stocks around the world. Accurate measurement of these parameters is in fact very difficult if not impossible. As a consequence, quantities of considerable importance to management, such as the maximum sustainable yield (MSY), are unreliable.

The goals of this paper are to demonstrate the potential of the variational adjoint technique in the analysis of natural resource systems and to apply the technique to the NEACs for two different growth models. Thus our focus is on the use of variational assimilation in bioeconomics. The paper is organized as follows. Section 2 is a brief discussion of the methodology used in the analysis. In section 3, we present the results of some twin experiments and then discuss an empirical application of the model. Section 4 concludes the paper.

2 Data Assimilation Methods

In this paper, the variational adjoint technique will be employed to fit the dynamic resource models to the observations. It consists of three components: the forward model with the definition of a cost function, the adjoint or backward model and an optimization procedure (Lawson et al., 1995). The forward model is the bioeconomic model e.g., an open access or a sole owner fishery model. The adjoint model consists of equations that provide a method of calculating the gradient of the cost function. The gradients are then used in a line search using standard optimization procedures to find the minimum of the cost function.

2.1 The cost function

One of the major components of data assimilation techniques is the choice of the estimator. Many attractive estimators exist, however, the least squares estimator has been the most popular among

researchers mainly due to its simplicity and mathematical convenience. The least squares fitting criterion is defined as

$$J = (x - x^{obs})^T W_x (x - x^{obs}) + (p - p_0)^T W_p (p - p_0) \quad (1)$$

where \mathbf{x} is the prediction of the model, x^{obs} is the observed or measured quantity, p and p_0 are the parameter and its estimated vector respectively, the \mathbf{W} 's are the inverse of the measurement error covariance matrices, i.e., the weighting matrices which are assumed to be positive definite and symmetric, and T denotes the transpose operator. We further assume that these matrices are $W_i = w_i I$, where I is the identity matrix. The penalty term on the parameters will increase the chance that the cost function becomes convex. This may be helpful in reducing the flat regions in the cost function and may improve the convergence to the minimum (see Smedstad and O'Brien 1991). In all the experiments we will set $w_x = 1.0$ which means that all we have to choose are the weights w_p on the parameters. If the model parameters are more uncertain than the observations then greater penalty is imposed, i.e., they are given less weight and vice versa. Complete lack of prior knowledge of the parameters implies $w_p = 0$.

2.2 Derivation of the adjoint model

Minimization of the cost function J subject to the dynamics is a constrained optimization problem (Luenberger, 1984; Bertsekas, 1982). An efficient¹ technique for the minimization of the cost function is the variational adjoint method. It consists of transforming the constrained problem into an unconstrained optimization problem via the use of the undetermined Lagrange multipliers. The adjoint equations are used to compute the gradients of the cost function. It is then possible to use a gradient search method to find model parameters that yield predictions which are as close as possible to the observations. To illustrate the numerical procedure, we use the discrete dynamics. The reason is to avoid using the more complex language of functional analysis. Not that this has the advantage that the equations derived are simply the equivalent of what is in the computer code.

The dynamics are thus

¹ In comparison with calculating the gradients using finite difference approximation.

$$x_{n+1} = x_n + g(p; x_n)dt, \quad (2)$$

$$x_0 = u, \quad 0 \leq n \leq N-1 \quad (3)$$

where N is the number of observations in time, \mathbf{p} a vector of parameters to be estimated and dt is the time step. The discretization scheme used is a simple forward difference scheme. The discrete form of the Lagrange functional is constructed as follows

$$L = \sum_{n=1}^N (x_n - x_n^{obs})^2 + w_p \sum_{i=1}^{N_p} (p_i - \hat{p}_i)^2 + \sum_{n=1}^{N-1} \lambda_n (x_{n+1} - \{x_n + g(p; x_n)dt\}) \quad (4)$$

where λ_n is the value of the multiplier at time step n and N_p is the number of parameters. The first order conditions for the minima are

$$\frac{\partial L}{\partial \lambda_n} = 0 \quad (5)$$

$$\frac{\partial L}{\partial x_n} = 0 \quad (6)$$

$$\frac{\partial L}{\partial p_i} = 0 \quad (7)$$

From these equations, we obtain

$$x_{n+1} - \{x_n + g(p; x_n)dt\} = 0 \quad (8)$$

$$\frac{\partial J}{\partial x_n} - \lambda_n (1.0 + dt \frac{\partial g}{\partial x_n}) + \lambda_{n-1} = 0 \quad (9)$$

$$\frac{\partial L}{\partial p_i} = \frac{\partial J}{\partial p_i} - \sum_{n=1}^{N-1} \lambda_n dt \frac{\partial g}{\partial p_i} \quad (10)$$

where $\frac{\partial L}{\partial p_i}$ is the derivative with respect to the i^{th} parameter and $\frac{\partial g}{\partial x_n}$ is the partial with respect to x_n .

It is immediately seen that equation (8) recovers the model dynamics, i.e., the forward model, equation (9) gives the backward model forced by the model-data misfits and equation (10) is the gradient with respect to the parameters. To find the model parameters that give model forecasts that are as close as possible to the observations using the classical search algorithms, correct values of the gradients are required. Methods for verifying the correctness of the gradient are available both numerically and analytically where possible (see, Spitz et al., 1998; Smedstad and O'Brien, 1991). We have in this paper checked all gradient calculations to ensure reliable parameter estimates. The optimization procedure used for the minimization is the quasi-Newton procedure developed by Gilbert and Lemarechal (1991).

3 The Dynamics of the Biomass

Most of the fisheries economics literature has been based on the simplified population dynamics models of the Schaefer type (Sandal and Steinshamn, 1997; Clark, 1990), and it is apparent that these models will continue to be used for some time. While efforts are underway in the development of more complex models, it is appropriate to explore techniques of identifying the inputs of the existing models. The surplus production model, though very simple, can represent quite good approximations of the complex dynamics. A continuous surplus production model is applied in this analysis. The basic form of the mathematical equation is

$$\frac{dx}{dt} = g(x) - h \tag{11}$$

where $x(t)$ is the biomass at time t , $h(t)$ is the rate of depletion of the population due to human activities, e.g., commercial and recreational fishing, g is the natural additions to the biomass.

3.1 The net growth models

Two variants of the growth model are considered in this paper. The structure of their growth is quite complicated requiring sophisticated mathematical functions to adequately model them. Fortunately, there are simpler models that reasonably and approximately represent the intricate growth models. The two growth functions used in this paper are

$$g(x) = \begin{cases} rx(1 - \frac{x}{K}) \\ rx \ln(\frac{K}{x}) \end{cases}$$

where x is as defined previously, r per year is the intrinsic growth rate of the population, K in kilotons is the maximum population level² of the biological species. The first is the logistic growth which is a special case of the modified logistic when the exponent is unity (Clark, 1990, Eggert, 1998) and the second is the Gompertz growth function.

The production function h for a resource industry can be assumed to depend on the stock biomass and the effort expended in fishing (see Clark, 1990; Eggert, 1998). The simplest form of the exploitation rate is the Gordon-Schaefer type of production function where the rate of removal of the stock is assumed to be linearly related to the effort and stock size. That is, $h = qex$, where $e(t)$ is the fishing effort and q is the catchability coefficient. The fishing mortality (f)³ is in general a function of time but for this analysis, we will assume that it is constant. This assumption corresponds to the assumption of constant effort in bioeconomics. Let $f = qe$ be "loosely defined as" the instantaneous average fishing mortality rate, then the simple rule takes the form

$$h(x) = fx, \quad x \geq x_0 \tag{12}$$

² The carrying capacity is assumed constant in this application which is a simplification in our models. It may be more realistic to allow it to vary with time.

³ Note that this is a mathematical simplification. Ideally f should be a variable since efforts are not constant.

where x_0 is the least observed or estimated stock. The dynamics thus contain three parameters, r , K and f . To further explain the above assumption, we point out that, $f(t) = h(t)/x(t)$ by definition. These values derived from the actual harvest and the estimated stock (see Anon. 1998) have quite been stable and do not show any significant trend. Hence, we assume that, this ratio is approximately constant i.e. f is constant. This assumption may be quite reasonable because, it leads to a more parsimonious model⁴ that includes the key variable i.e. the stock.

In Figure 1 below, we show plots of the growth functions.

The graph of the logistic is symmetric about one half the carrying capacity while the Gompertz is asymmetric and is skewed towards the left. For the same K and r , the latter predicts lower (MSY) biomass $x_{MSY} = (K/e)$, where $e \cong 2.71$ and a corresponding higher MSY. In practical applications, the Gompertz growth function seems inappropriate for less resilient species. This is because, the combination of high MSY and low x_{MSY} prescribed by this model can result in an unpardonable mistake on the side of management in case of recruitment failures.

⁴ Note that $h = \alpha + \beta x$ where $\alpha = 0$.

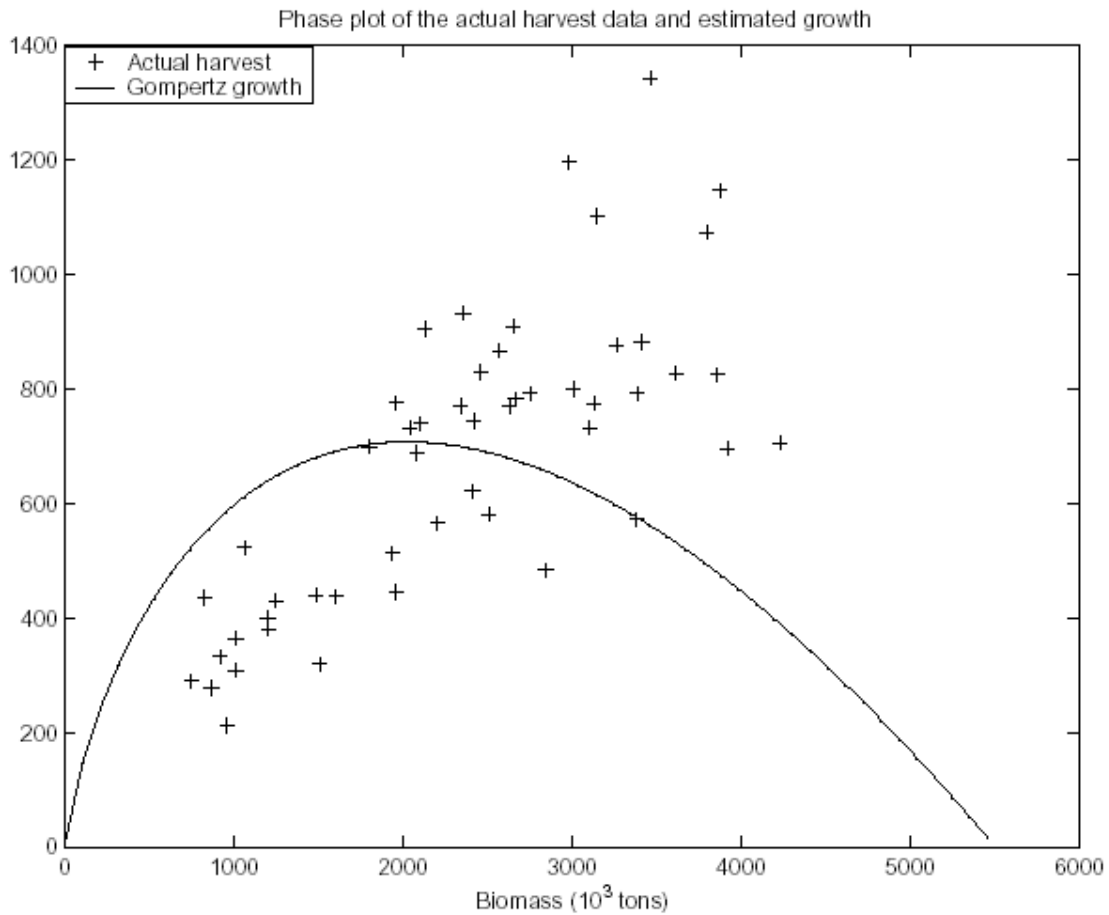


Figure 1: The biomass size is in percent of the carrying capacity and the growth functions are rescaled by a factor of r times K .

3.2 Twin experiments

To assess the performance of the variational adjoint method, several identical twin experiments are performed, i.e., data generated from the model itself using known parameters of the model are used. The results are discussed below. Note that, the purpose of the twin experiments is to test the adjoint computer code to ensure that reliable estimates are obtained in the real life application.

For all the experiments in this paper the convergence criterion for the optimization is $\|\Delta J\|/\|\Delta J_1\| \leq 10^{-8}$, where, ΔJ and ΔJ_1 are the gradients of the current and initial points respectively and $\|\cdot\|$ is the norm operator (see Gilbert and Lemarechal, 1991). For this paper, we will use the value of $r = 0.35$ estimated in Nakken (1988). This leaves us with two parameters to estimate and provides a more parsimonious model. Thus, reducing the problem of overfitting and identification.

First clean data, i.e., data without any type of noise, are used in the numerical experiments and the parameters are recovered (Tables 1 and 2). To test the robustness of the algorithm, we used several different initial guesses. In all the cases the parameter recovery was accurate to more than four orders of magnitude (Tables 1 and 2). The only difference was in the number of iterations, which increased as we depart from the true values. The optimal value of the cost function in these experiments is zero because the simulated observations are exactly consistent with the model solution. However, in a more realistic situation the data may not be consistent with the model solution and the minimum of the cost function can be expected to be greater than zero.

Parameters	Logistic		
	$\sigma = 0.0$	$\sigma = 0.001$	$\sigma = 0.01$
K	1.000000	0.992734	0.931131
F	0.174999	0.174099	0.165910

Table 1: Results of twin experiments with $r = 0.35$. The true parameters are $K = 1.0$ and $f = 0.175$. The weights used are $w = 1.0$ and $w_p = 0.0$ and σ is the standard deviation of the measurement errors.

Parameters	Gompertz		
	$\sigma = 0.0$	$\sigma = 0.001$	$\sigma = 0.01$
K	0.999999	1.000094	1.000917
f	0.349999	0.349739	0.347374

Table 2: Results of twin experiments with $r = 0.35$. The true parameters are $K = 1.0$ and $f = 0.35$ and σ ⁵ is the standard deviation of the measurement errors. The weights used are $w = 1.0$ and $w_p = 1.0$ that is we are assuming that our prior knowledge of the data and the parameters is equal. Note that the Gompertz function is highly nonlinear and hence has nonquadratic cost function. By penalizing the parameters, we increase the chance that the cost function becomes convex.

For the experiments above, the data input for the assimilation were assumed to be perfect, i.e., random and measurement noise were not included. This is however the most ideal situation one may assume. Next we investigate the effect of imperfect measurements because real data are inherently noisy. Similar experiments as in the case of noise free data are performed; this time with random noise included in the data. The data are generated by $x^{obs} = x + \varepsilon$, where ε is assumed to be normally distributed with zero mean and constant variance. The level of noise was varied in order to evaluate the degree to which the noise affects the recovery of the true parameters. The recovery of the parameters was quite good except that the precision of the recovered parameters and the convergence of the conjugate gradient method were slightly affected (see Tables 1 and 2). The accuracy decreased while the number of iterations increased with increasing amount of noise. The results are not surprising because the model and the data are somewhat consistent.

3.3 An Application to North-East Arctic Cod Stock

The NEACs is the most important demersal species along the coast of Norway and Northern Russia. This fishery has played an important economic role within the coastal communities for the past thousand years. The NEACs has for the past half century experienced large variations which result in corresponding variations in the annual harvest quantities. The stock size fell from its

⁵ Note that random variates from unit normal can take values between negative and positive infinity. Thus, that a smaller sigma may help keep the simulated values between zero and one.

highest level of 4.1 million tons in 1946 to its lowest level of 0.75 million tons in 1981. Time series plot of the history of the stock indicates signs of recovery from its worst state around 1990, but recent reports show that the fishery is again in deep trouble (see Figure 2 below).

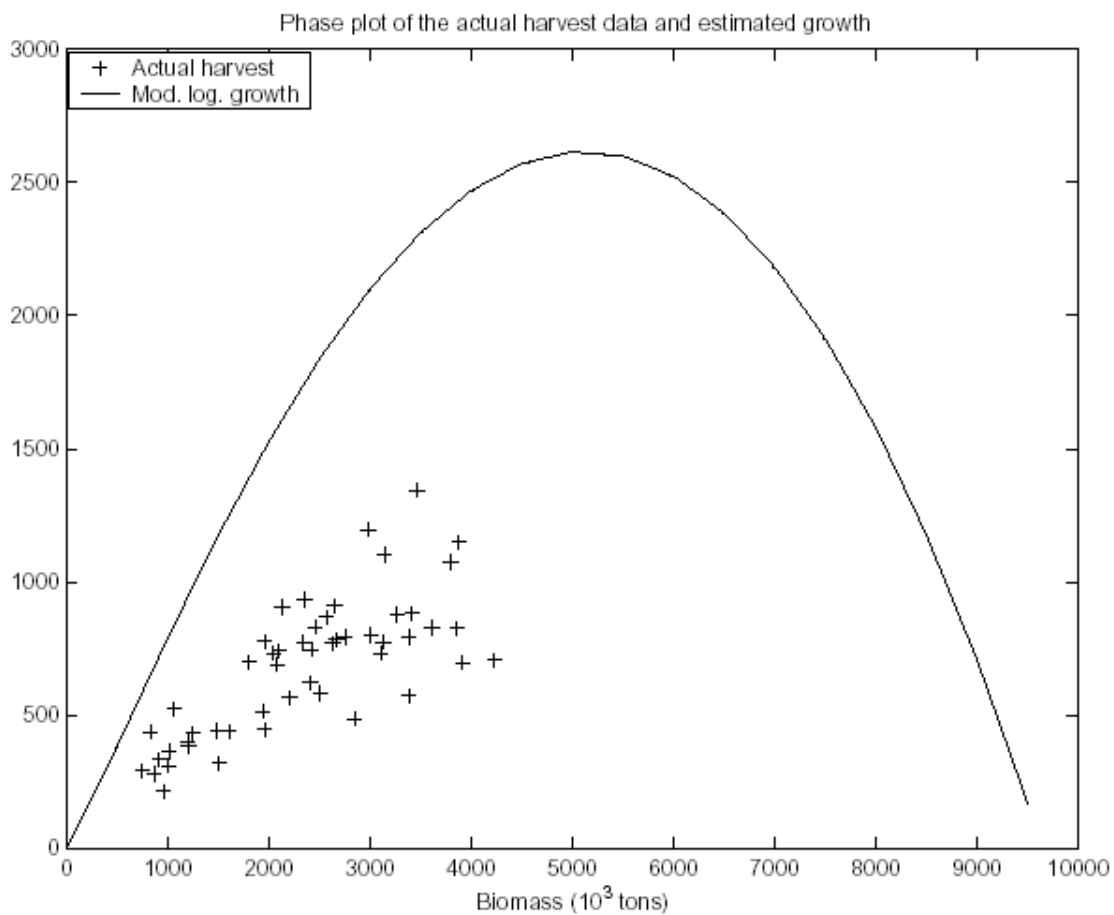


Figure 2: Source: ICES 1998. Graph of actual harvest and the stock biomass. Note that this shows the assumed relationship between the harvest and the stock. The data is for the period of 1946 - 1996.

In this study, a time series of observations from 1946 to 1996 is used. The data set is obtained from the ICES 1998 report (see Anon., 1998). The variational adjoint method is used to fit the hypothesized dynamics to the observations. The NEACs provides a good example for testing the data assimilation method because biomass models have previously been used to analyze this fishery (see Naken, 1988).

The optimization was started by randomly generating reasonable initial guesses for the parameters using a uniform random deviate intrinsic function. By seeding the generator, different initial guesses were used to check for the presence of local extrema. Note that the use of the uniform distribution is for computational ease.

A surface plot of the logarithm of the cost function for the logistic model in space of the two estimated parameters for the logistic model is shown in Figure 3.

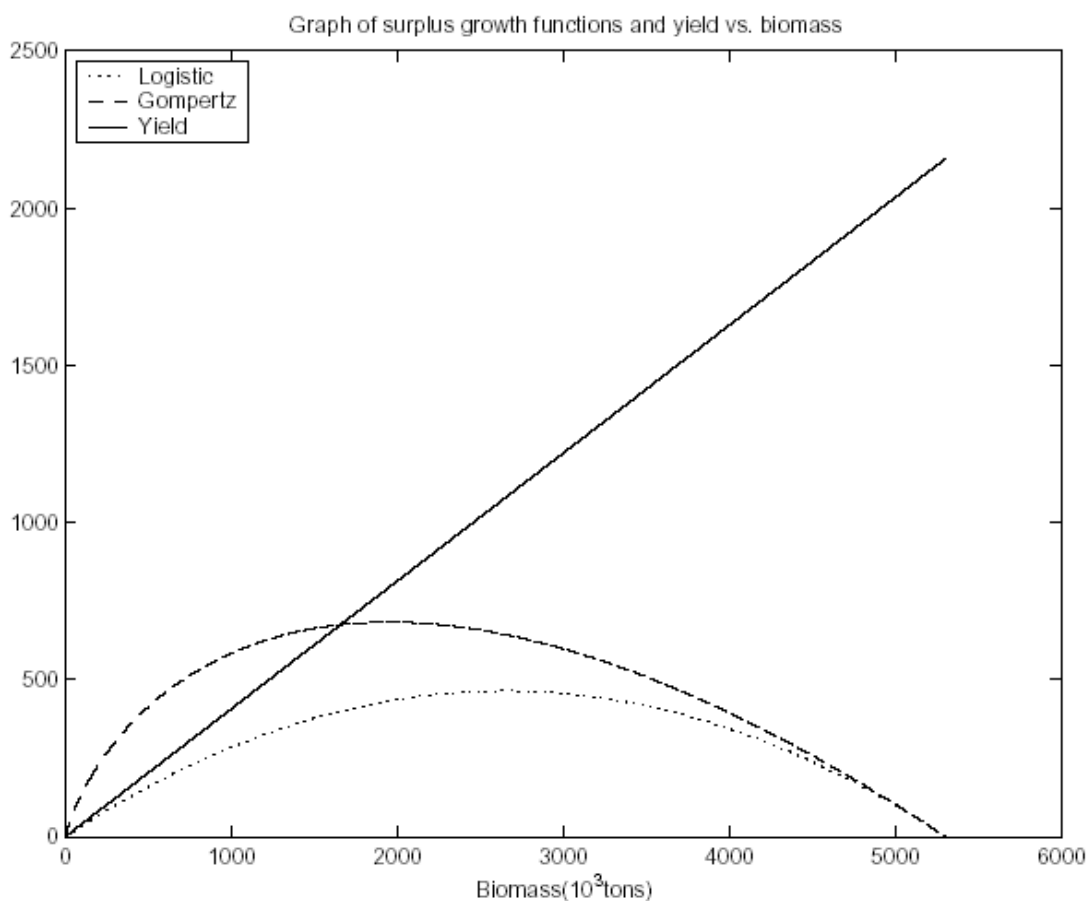


Figure 3: A plot of the logarithm of the cost function in space of the estimated parameters (K, f) .

The graph is obtained by perturbing the parameters around their estimated values to check if a minimum is reached. It appears from Figure 3 that the cost function is convex, at least locally, and

a minimum has been reached. The best fit parameters, estimates of the MSY and x_{MSY} , are shown in Table 3 below.

Parameters	Logistic	Gompertz
K	5268.5 (868.3)	5499.99 (1322.7)
F	0.4076 (0.0579)	0.4964 (0.1044)
X^*_{MSY}	2634.5	2023.33
MSY^*	460.9	707.96

Table 3: Model parameters for the biomass dynamics models. The units are: f (1/yr) and the rest are in kilo-tons. The stars represent calculated quantities and standard errors are in parenthesis.

A star in Table 3 indicates that the values were calculated, and the standard errors are given in parentheses. The logistic and the Gompertz functions tend to give reasonable estimates of the parameters for the NEACs because they lie within acceptable bounds for this stock.

Point estimates of parameters are usually useful, but, the usefulness is enhanced if error bounds are provided. Several methods for calculating these uncertainties exist (see, Polacheck et al., 1993 and Thacker, 1989). The approach used here is a Monte Carlo simulation method where we randomly generated values for the intrinsic growth rate within a given range and repeated the experiment many times and the standard deviations were calculated. For this stock, some scientists believe that the intrinsic growth rate lies between 0.25 and 0.45. It may also be reasonable to argue that the carrying capacity is at least the size of the estimate of the stock biomass (4.1 million tons) at the beginning of the assimilation period which is 1946. At that time commercial fishing had not taken place for some years due to World War II. For this reason, we assume that the carrying capacity may reasonably lie between 4.1 and 6.5 million tons. In fact, there should be no reason why these bounds should be absolutely correct. There may be a certain probability that the true parameters lie outside the estimated bounds. The fishing mortality has at least been equal to the natural growth rate of the stock since the stock have been declining and its upper bound can be assumed to be no less than 0.55. These arguments are based on our experience and intuition.

In figure 4, time series of the actual stock and the model results using the estimated parameters are displayed. The fit to the data is quite good for both models (Figure 4) with the logistic model explaining about 55.0 % of the data while the Gompertz function explains about 53 % of the data. It is observed that the estimates for the latter model are relatively higher than for the former.

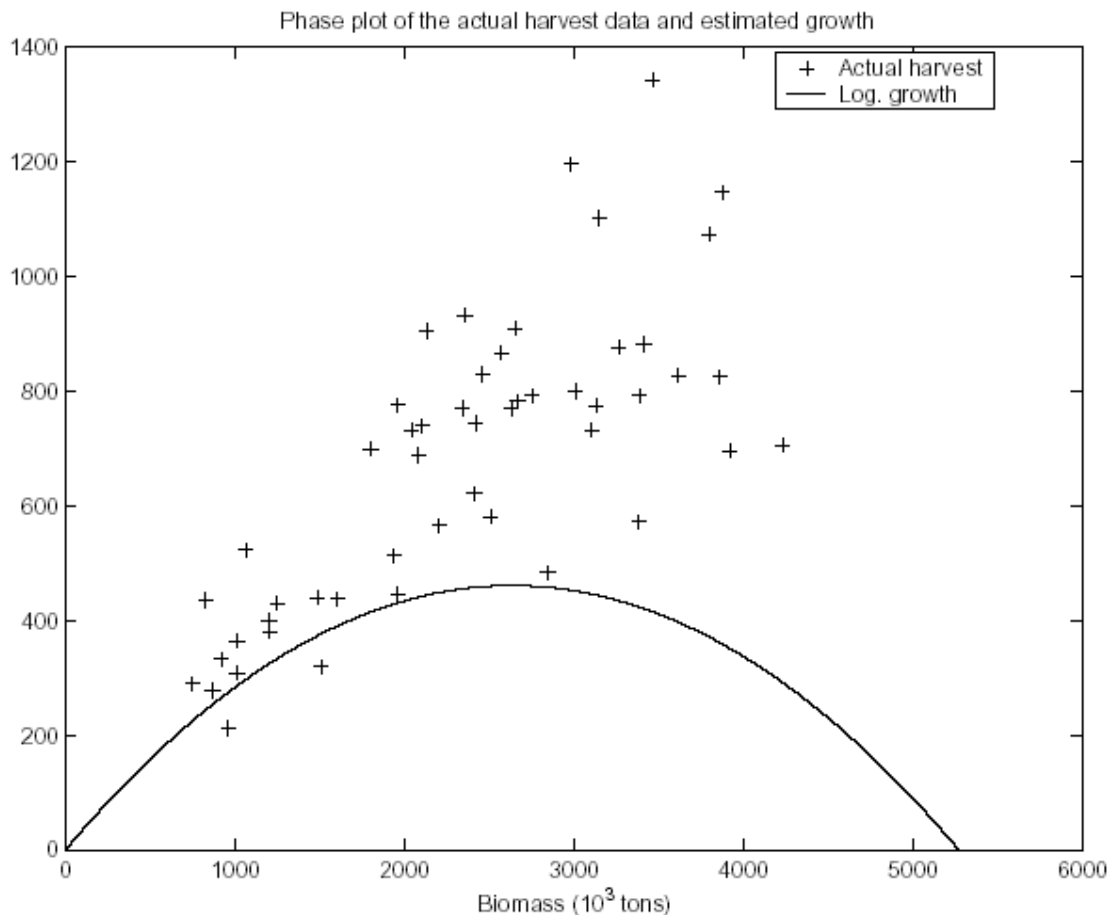


Figure 4: A time series plot of the actual and estimated stock using the optimal parameters.

The plot shows which component of the data is resolved by the models and which is not. As can be seen from Figure 4, the model has been able to capture the trend in the data but not the stochastic component. The reason is obvious here because the models are highly aggregated and deterministic. Next, the growth functions using the estimated parameters are presented on the same graph as the actual harvest data. Note that the growth models are important components of the bioeconomic models. By comparing the estimates of the natural productivity of the stock to the

actual rate of harvesting, we may be able to infer whether the general decline in biomass has been captured in the growth component of our models. That is, the stock has been exploited at an unsustainable rate leading to the alarming state of the fishery. Figures 5-6 show plots of the actual harvest and growth curves against biomass. The plus sign represents the actual harvest while the solid line represents the estimated net growth for the Gompertz. The logistic growth model predicts that the harvest rate has been persistently above the net growth curve (see Figure 5 below). At the lower end of the graph, we notice that the actual harvest is close to the growth curve and is below it on a few occasions. One interesting observation is that several points tend to cluster around the MSY. Recall that most real-world fisheries have until recently been managed based on the concept of MSY. The NEACs is not an exception but most of the data period is characterized by open access. In most real world fisheries, the observation is that the stock biomass has generally been declining over time which is an indication that the natural productivity of the stock has been less than the rate of harvesting and hence declining as shown in Figure 4.

The estimates of the latter model, i.e., the Gompertz model, are quite similar to that of the logistic model because both models have predicted that the fishing mortality has been far too high⁶ and that the stock size has generally been declining. It is observed in Figure 6, however, that the actual harvest has for a large part been less than the estimated natural growth which according to our model, should lead to an increase in the stock, this is opposite of what has been observed. The reason is that the Gompertz function is skewed to the left i.e., the growth is faster when the size of the stock is low.

⁶ That is above the rate that will lead to MSY. These are $f = r/2$ for Logistic and $f = r$ for Gompertz.

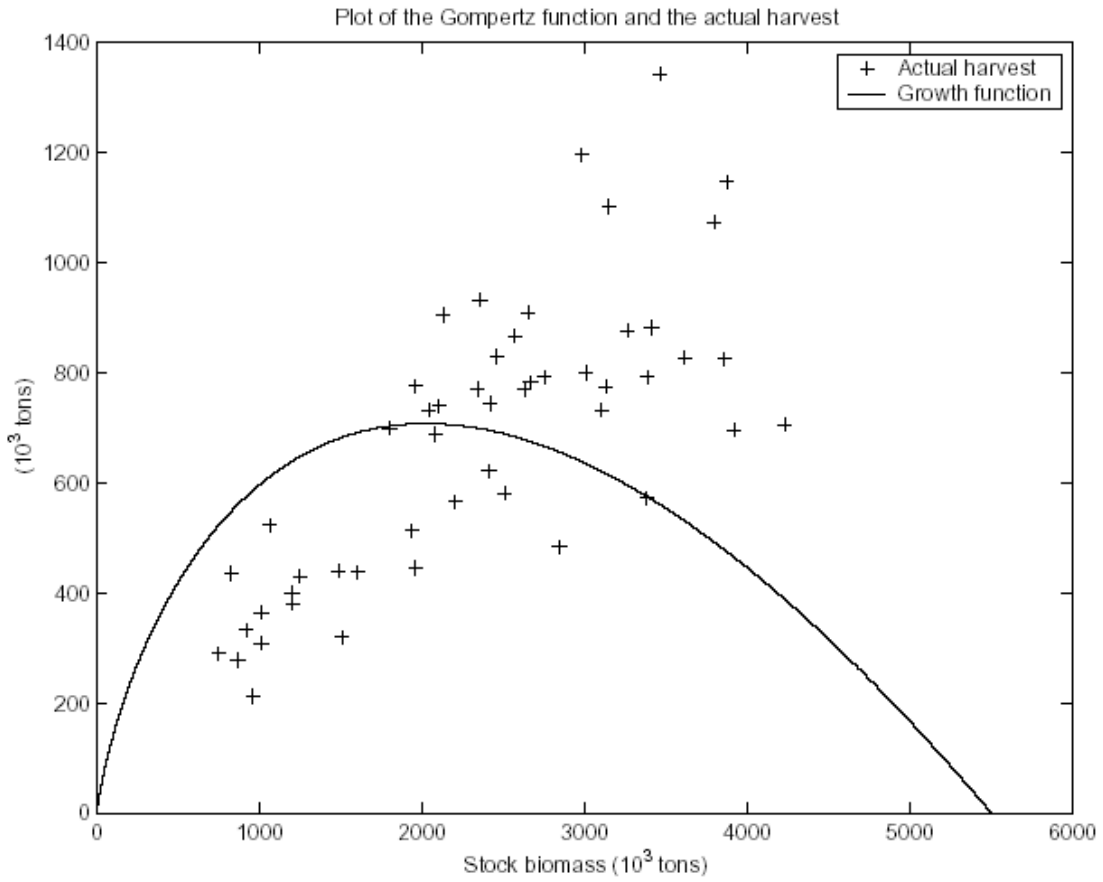


Figure 5: The graph of the logistic growth model using the estimated K and $r = .35$ vs. the stock biomass.

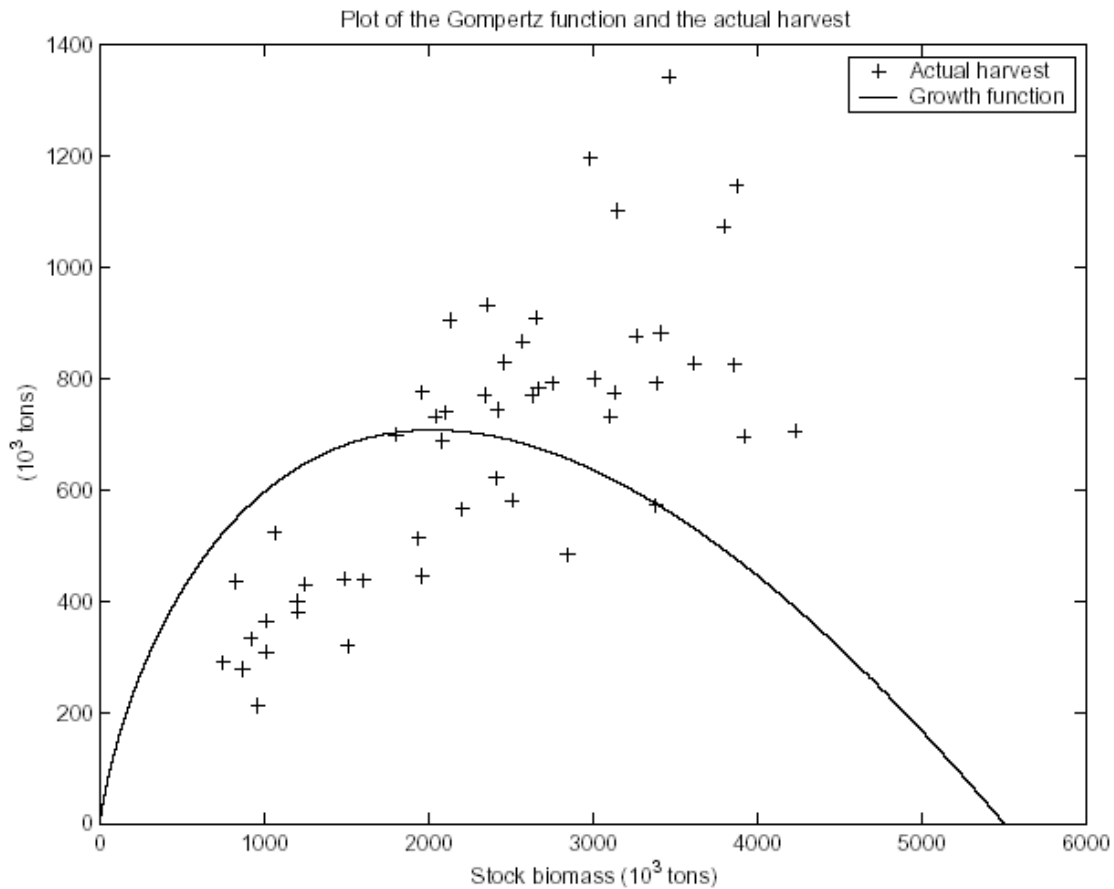


Figure 6: The graph of the Gompertz growth model using the estimated K and $r = .35$ vs. the stock biomass.

Note that the Gompertz model seems to overestimate the productivity of the stock biomass and underestimates the x_{MSY} (see Table 3). An important caveat however is that, while these values may have some empirical significance, we should not overestimate the scientific significance. The estimates may not accurately reflect the true values due to the large errors in the data and the simplistic nature of the models.

To further discuss the results of the paper, we provide estimates (e.g., the MSY) that might be of considerable interest to managers of the NEACs.

Estimates of x_{MSY} and MSY quantities are shown in rows 4 and 5 of Table 3. The Schaefer-logistic model seems to outperform the Schaefer-Gompertz model. The former gave a lower MSY estimate but a higher value for the optimal standing biomass. These estimates are quite appealing and are more acceptable than the estimates of the Gompertz. The 2001 quota recommendation from ICES is around 263 kilotons which is lower than the MSY estimated for the logistic. The MSY for the Gompertz is around the values of the Total Allowable Catches (TACs) that have been suggested for the NEACs in the late 90's (Nakken 1998). These values have turned out to be too high as the stock biomass has continued to decline. The estimated sustainable biomass level of around 2.0 million tons for the Gompertz function may be too low. This is because this value is lower than the estimates of the stock in the mid to the late 90s which is still considered low by some scientists. It is obvious that the Gompertz growth function does not seem to work well for NEACs because it overestimates the MSY and underestimates the x_{MSY} .

4 Conclusions

The NEACs is analyzed using constrained optimization for dynamic model parameter estimation. Two alternative growth models are used in the analysis. The production relation for the fishery is assumed to be linear in the biomass and constitute a simple feedback rule. A quite restrictive assumption of constant fishing mortality is made which yields a proportional fishing policy. The model dynamics are nonlinear in the parameters and quadratic in the stock. A cost function measuring the discrepancy between the data and its model equivalent was minimized subject to a dynamic constraint. The variational adjoint method is used to efficiently estimate the parameters. Parameter estimates from the Schaefer-logistic and the Schaefer-Gompertz models are reasonable, i.e., they are within acceptable ranges. Both models have about the same explanatory power, i.e., $R^2=0.55$. This seems quite reasonable since the models were able to capture the trend in the data but failed to capture the periodic oscillations (Figure 4). It is obvious that the models are not sophisticated enough to explain the random events inherent in the system. Ecosystem effects and environmental variability are very important variables and ought to be included in the model. Predictions from these models are consistent with recent experiences in fisheries and other natural resource stocks. Both the stock biomass and the amount harvested have been declining while

fishing mortality is increasing due to technical innovations. More efficient boats are being developed and other advanced fishing equipment is available making the stock more vulnerable to exploitation. This has led to the collapse of some fisheries such as the Canadian Northern cod fishery (Grafton et al., 2000).

This paper has demonstrated the usefulness of the data assimilation methods in dynamic parameter estimation. It exposes some of the strengths and weaknesses of the simplified biomass dynamics models and provides model parameters that are in close agreement with the observations. The methods have numerous additional capabilities that are worth exploring in the future. For example, more sophisticated models that include spatial and other ecological interrelations can be calibrated using this technique. It can also be used to perform sensitivity analysis (Navon, 1997). Bioeconomists may find these methods indispensable when the questions that interest managers most have to be answered and when more realistic models become available.

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