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Spectrum auctions in the presence of network externalities

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Abstract. *In recent years governments in a number of countries have auctioned exclusive rights to develop and utilize UMTS ("Universal Mobile Telecommunications System"), some with more success than others. Much theoretical effort has been put into discussing whether or not to auction the rights and the correct design of the auction. However, the authorities obviously perceive that there are some externalities associated with the introduction of this new technology. The fear that the rollout will be slower and less extensive than what is felt optimal from society's point of view have been attacked, in principle, by two different approaches. One is to impose minimum standards for rollout and speed of development and conduct an auction where cash is the bidding variable (e.g. U.K.), the other is to have a "beauty contest" with coverage and speed of development as the bidding variable (e.g. Norway). In this paper focus is on a third alternative, i.e. to auction the rights based on cash bonus bidding and to subsidize coverage. Bidding strategies when auctions are combined with ex post subsidies have received surprisingly little attention in the theoretical literature. The presence of subsidies connected to rolling out the network will increase expected revenue from the auction, whereas the subsidy will not be paid until the network is actually developed. But the subsidy will also have the effect of reduced bid intensity since it increases asymmetry between bidders.*

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I. Introduction

"Third generation" (3G) mobile telecommunication (or UMTS) licences grant the licensees exclusive rights to develop a communications network that will play a key role in creating a mass market for high-quality wireless multimedia communications. UMTS services herald a new, "open" communications universe, delivering high-value broadband information, commerce and entertainment services to mobile users via fixed, wireless and satellite networks. It has been estimated that UMTS will create a mass market for high-quality wireless multimedia communications that will exceed two billion users worldwide by the year 2010, which will be worth over one trillion US dollars to mobile operators over the next ten years.[1] UMTS licences grant the licensees exclusive rights to develop this communications network within a given county.

In the last couple of years, over 100 3G licences have been allocated worldwide. This has primarily been done using cash bonus auctions or, alternatively, so called "beauty contests". For instance, the U.K., Netherlands, Germany, Italy, Austria, Switzerland and Greece used different variants of the simultaneous ascending auction,[2] whereas Denmark used a sealed bid format, all with a cash bonus as the bidding variable.[3] Countries that used a beauty contest include France, Spain, Ireland, Portugal, Luxembourg, Finland, Sweden and Norway.[4] In Norway, for instance, the licensees were selected i.a. on the basis of their offered geographical coverage and roll out, above some minimum requirements.

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[1] See UMTS Forum at http://www.umts-forum.org/what_is_umts.html.

[2] This design was originally proposed by Paul Milgrom, Robert Wilson and Preston McAfee for the allocation of spectrum licences, see e.g. Milgrom (2000).

[3] See Klemperer (2001) for a critical discussion of the auction design, and Klemperer (2002) for an evaluation of the experience from these auctions.

[4] In Norway, the winning licensees had to pay a licensee fee in addition to an annual frequency fee.

Proponents of auctions have argued that the job of picking winners is best left to the market, whereas those in favour of beauty contests argue that winners of the licences should have been chosen on the basis of i.a. which company would guarantee the lowest cost to the consumers and which should install the most cellular infrastructure. British authorities chose to use auction as the allocation mechanism for UMTS licences because “(a)uctions are a fast, transparent, fair and economically efficient way of allocating the scarce resource for radio spectrum. Government should not try to judge who will be innovative and successful”.[5] In an auction, a cash bonus is the bidding variable. The auction process – when it works well – generates information on which of the competing bidders is able to put scarce frequency resources to the best use.

One of the opponents of cash bonus auctions was Professor Nicholas Negroponte, leader of the Media Laboratory at MIT. He argued that a beauty contest should have been used instead: “… winners of the licences should have been chosen on the basis of which company would guarantee the lowest cost to the consumers, which should install the most cellular infrastructure, which would put the most phones in schools and public places, and which would invest the most in creativity”.[6] This view corresponds well with the arguments used in favour of a beauty contest to allocate UMTS licences in Norway. Here, the authorities argued that lack of information on central aspects of UMTS development, i.e. potential market demand, providers’ costs and pricing of services, raised serious problems for the regulatory authorities in their role as planners and organizers, and that official requirements with respect to geographical coverage may lead to an incorrect resource allocation. Furthermore, as the providers themselves are in the best position to assess potential market demand and their own costs, it was concluded that it was appropriate to let

[5] See http://www.spectrumauctions.gov.uk/documents/faq.htm.

[6] *Financial Times*, June 7, 2000.

providers compete on coverage and speed of development when applying for licences.[7]

One important economic feature of the telecommunications industry is the presence of network externalities: a communications network might generate positive externalities between users because existing users benefit when new users join. In addition, there might be wider social benefits of having a telecommunications network that also generates positive external effects, for example, provision to sparsely populated areas. Both types of externalities may have policy implications, the first on pricing structure, and the second on the extension of the network.

The conflict between the two opposing approaches to spectrum allocation seems to a large extent to be caused by the weight attached to such network externalities. In an auction with a cash bonus as the bidding variable – even though the winners typically have been subject to minimum coverage requirements – the authorities have no control over the geographical network coverage the licensees will implement in the end. This may well be below the socially optimal coverage due to positive external network benefits. On the other hand, a beauty contest can be considered as an auction where the winning bidder pays for the licence in the form of i.a. network coverage. However, there is also a "risk" involved in using beauty contests, as a competition according to such criteria as degree of coverage and speed of development might pressure applicants into offering *more* than what is economically feasible even from a sosio-economic point of view. If the resulting coverage in the end corresponds to the socially optimal solution, it will be nothing but an accidental circumstance.

Thus, an important issue is how this controversy can be resolved. First, one must realize that in the presence of externalities, markets – and auctioning rights – can provide only part of the solution to a public problem. In this case, the auction can take care of the allocation problem if the auction is well designed. But the auction must be combined with tools designed specifically to take care of the externalities. One important lesson from economic theory is that externalities can be internalized using

[7] See report from the Norwegian Post and Telecommunications Authority: "Establishing a Regulatory Framework for UMTS in Norway", June 2000. The report can be found at http://www.npt.no/eng/system/html/index.html.

taxes or subsidies, depending on the character of the activity. A major problem with a beauty contest as an allocation mechanism is that it is overburdened with goals, i.e. both to allocate rights *and* achieve a higher geographical coverage than would otherwise result.

In this paper, it is argued that the goals pursued by applying a beauty contest and the benefits of using the market to select winners through an auction can be reconciled if the auction is combined with an *ex post* subsidy. Here, the focus will be on the wider social externalities caused by geographical coverage. Thus, the final total subsidy will be a function of the geographical coverage that the licensees decide to develop in the end.

From the auction literature, it is well known that it is in the seller's interest to condition the bidders' payments on any additional information about the winner's valuation. The bidders will take the presence of the subsidy into consideration in their bidding strategy and formation of optimal bids. We will examine how the presence of the subsidy affects bidding strategies and expected revenue. The points put forward in the paper are based on well-established theoretical results. However, to my best knowledge, their implications in this particular practical context have not been explored. Thus, the paper is not theoretical, but policy oriented. It intends to illustrate the effect of combining a cash bonus auction with a subsidy on bidding strategies using a numerical example.

The structure of the paper is as follows. First, the theoretical background is presented. The model used for this purpose is presented in Section III, numerical results are presented in Section IV, and some concluding remarks appear in Section V.

II. Theoretical background

Auctions[8] have become a frequently used tool when governments allocate exclusive rights to utilize scarce spectrum resources.[9] This has occurred in parallel

[8] A good overview of auction theory can be found in Klemperer (1999).

[9] See McMillan (1995) for an account of why the use of spectrum should be auctioned.

with a significant debate in academia and among practitioners on how to design the auction for this purpose in order to achieve goals with respect to standard design criteria, i.e. efficiency and revenue.[10] A design that has been frequently employed in the last ten years – with different modifications – is the *simultaneous ascending auction*, originally proposed by Preston McAfee, Robert Wilson and Paul Milgrom for the sale of radio spectrum in the United States.[11] Experience with various designs has been followed up by empirical evaluations. For instance, a special issue of the *Journal of Economics and Management Strategy*, which appeared in 1997 (vol. 6, no. 3), was devoted to empirical and experimental evaluation of different institutions for spectrum allocation. Moreover, Klemperer (2002) discusses the experiences with the European UMTS auctions, concluding that auction design is not "one size fits all". Moreover, particular attention had to be given to designing the auction so that entry is attracted and collusive behaviour deterred.

Bidding strategies when the auction is combined with *ex post* subsidies and/or taxation of the winning bidder(s) is, however, an issue that has been little studied in the literature – and to my knowledge not in a practical spectrum auction policy setting. As formulated by Grimm and Schmidt (2000): "auction theory has not been analyzed in the presence of taxation."[12] However, in countries auctioning spectrum licences, we frequently see that the licensees are subject to some form of *ex post* payment. In Norway, for instance, the licensees have to pay an annual fixed spectrum fee in addition to their winning bid at the GSM frequency auctions held by the end of 2001.[13] An important question is why the government chose to combine a cash bonus auction with a fixed fee. The discounted value of the fee will be discounted in the bids and may lead some low value bidders to not participate, thus reducing competition. This issue will not be taken further here.

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[10] See e.g. Cramton (1998). Klemperer (2001) provides a critical discussion.

[11] See McMillan (1994) and Milgrom (2000).

[12] This is not entirely true, but their paper is one of a few exceptions.

[13] Here, a sealed bid auction format was used.

The lack of attention given to the possibility of combining the cash bonus spectrum auction with subsidies related to coverage is surprising, given that it is well known that it is in the seller's interest to *condition* the bidders' payments on any additional information that will be available *after* the auction about the winner's valuation. In a setting with *independent* private values, Hansen (1985) shows that auctions with contingent payoffs, for example, with the profit share as the bidding variable, offer higher expected revenue than cash auctions. This gain occurs because the contingent-payment method allows the seller to capture some portion of the difference between the two highest reservation values. Also in a setting with independent private values, Grimm and Schmidt (2000) consider an auction in the presence of an *ex post* proportional, regressive or progressive tax. They focus on revenue from the auction, and find that the presence of a proportional tax preserves revenue equivalence, whereas a progressive income tax leads to a higher expected revenue in first price than in second price auctions.

McAfee and McMillan (1986) and Riley (1988), on the other hand, explore how the seller can utilize *ex ante* information. McAfee and McMillan assume independent private values in a procurement setting, where the public signal is an unbiased estimate of the winner's private signal. They find that the optimal contract usually is an incentive contract, which makes the total payment depend on the bid and the realized costs: if realized costs exceed the firm's bid, the firm is responsible for some fraction of the cost overrun; if the firm succeeds in holding its costs below its bid, it is rewarded by being allowed to keep part of the cost underrun. An increase in the proportion of the winning bidder's costs that are covered by the principal has an effect similar to a reduction in the variance of the distribution of expected costs among the bidders and forces them to bid lower, i.e. resulting in bid intensification. An implication of their result is that if the government taxes a firm with which it deals, the resulting increase in government expenditure exceeds the tax revenue. As a tentative policy recommendation, McAfee and McMillan indicate that taxes on inputs used by contractors in government projects should be rebated.[14]

[14] Bower and Osband (1991) explore the effect in a defense procurement contracts setting.

Riley (1988) focuses on a setting where the bidders' signals are affiliated, i.e. a buyer with a high estimate then tends to believe that other buyers will have high values as well. Riley examines the effect of introducing positive royalty rates. He shows that when *ex post* information about the object's value to the winner is anticipated by the seller, expected revenue can be increased by making the final payment contingent upon information revealed after the auction. The intuition behind this result is that by introducing a higher royalty rate, the seller reduces the remaining asymmetry in the buyer's valuations net of the royalty payments. This induces the buyers to bid more aggressively and thus increases the expected revenue of the seller. This is the "bid intensification" effect. Thus, the royalty serves to transfer rents from the successful bidder to the seller.

At the outset, if there are positive externalities associated with exploiting spectrum then these benefits of the firm's activities fail to be reflected in prices and bids. There are many reasons commonly cited for making communication services broadly available. Network externalities – the concept that a product's value to a consumer changes as the number of users of the product changes – is one being used to justify some public subsidy to promote more extensive use of a telephone service. Liebowitz and Margolis (1994) provide a critical discussion, arguing that in reality we see many network *effects*, but there is scant evidence of network *externalities.* Here, however, the focus will be on the political and social reasons to promote a geographically widespread communication service. As is well known, these externalities, if they are important, can be internalized through a subsidy.

Thus, the focus in this paper is on bidding strategies in the presence of a subsidy, i.e. a negative *ex post* tax motivated by *externalities* as a function of the geographical extension of the communication network, and how the government can utilize the bid intensification effect through the design of this subsidy. This corresponds to an incentive contract, which makes the total payment depend on the bid and the geographical extension of the network.

III. The model

As is usual, all bidders are assumed to be risk-neutral, maximizing expected profit. For simplicity, it is assumed that the authorities auction only one licence. Each of n buyers receives a signal θ_i, i=1,...,n which can be considered as an unbiased estimate of the constant marginal cost of extending the communication network within the licence with one more unit. This constant marginal cost is assumed to be inclusive of costs of maintaining the transaction-specific assets in operable condition. It is assumed there are no fixed costs above and beyond those sunk into the transaction specific assets.

In addition, we assume that at some date after the auction, before the development of the network starts, the winning lessee(s) observe a further verifiable signal $\theta_0=c$, where c is the 'pure' common value of the costs. Following Klemperer (1999), who presents a tractable example of affiliated information that we will build on here, it is assumed that the signals θ_i are drawn from a uniform distribution on [c-½, c+½]. All values of c are equally likely – i.e. we assume a 'diffuse prior'. Hence, a higher value of θ_i makes a higher value of c more likely, and consequently higher values of the other signals more likely. In addition, we impose – without formality – the usual assumptions of symmetry of signals and symmetry of valuations, see for example, Milgrom and Weber (1982). With θ_0 observable, the authorities can make the winner's payment a function of the signal θ_0 in addition to the bonus payment at the auction. *Ex post* signals is a feature often found in situations where the winning bidder must undertake some activity after the auction, for example, to unveil the actual price of the inputs used to exploit the exclusive right.

Define $c_i = C(\theta_i, \theta_{-i})$ as bidder i's expectation with respect to c, which is conditional on the realized signal θ_i and all the other realized signals θ_{-i}. Then, the expected profit of the lease, with k units installed and any marginal cost c_i considered, is given by:

$$\pi_i=pq(k)-c_ik.$$

The private willingness to pay for the services provided by the network is fixed and represented by p. The number of subscribers reached by the network is a function q of the installed k units of the transaction specific assets, where $q'>0$, $q''<0$ and $K\geq k$, where K represents 100 per cent geographical coverage. The first order condition for optimal investment is:

$$q'(k) = \frac{c_i}{p},$$

that is, the winning bidder will invest until the relation between the marginal cost of extending the network and the marginal revenue equals the marginal effect of the increased coverage. A low c_i/p will result in higher investments and coverage than a high c_i/p. The bidders will bid assuming an optimal k^* for the signals received.

Now, assume that the authorities, due to externalities or specific political goals, expect that the optimal k^* does not correspond to the optimal coverage k^s from society's point of view. Basically, three alternative tools can be considered to achieve a better correspondence: either a subsidy implemented as a fixed percentage ρ covered of *ex post* marginal costs c, so that the bidders perceive $c_i(1-\rho)$; as a fixed amount paid to the licensee per installed base station k; or as a fixed percentage increase in consumers' willingness to pay, i.e. the bidders perceive price as $p(1+\tau)$. In the first case, the corresponding optimal investment, *given* the subsidy is determined by $q'(k) = \frac{c_i(1-\rho)}{p}$. In the last case, the corresponding optimal investment, *given* the subsidy is determined by $q'(k) = \frac{c_i}{p(1+\tau)}$. Only this alternative will be explored here, as the qualitative results will not vary with implementation. Furthermore, it is assumed that the parameters *ex ante* are determined so that the result will be the same coverage for the expected highest bidder, and the subsidy internalizes external benefits.

Two different auction formats will be considered, i.e. an ascending auction and the second-price, or "Vickrey" auction. No formal analysis of bidding strategy and expected revenue under these different formats is presented. This is well covered in the theoretical literature – see, for example, Milgrom and Weber (1982), Riley (1988) and Klemperer (1999). The point here is to develop an illustrative example drawing on existing theory in an actual setting. As Klemperer notes, most examples with affiliated information are very hard to work with. Thus, one contribution of this paper is to illustrate effects that are often hidden in not easily accessible theoretical papers, and to do this in a setting where the results may have practical implications.

IV. Numerical analysis

As mentioned above, Klemperer (1999) develops a tractable example of affiliated information to illustrate revenue ranking under affiliation. The case developed here draws on Klemperer. In his example, bidders receive an affiliated signal with respect to value, and determine the optimal bid as a function of the pure common value of the item. Here the bidders receive a signal with respect to costs, and determine the optimal cash bonus bid on the basis of *ex ante* optimal coverage and expected profit. To ease comparison of outcomes, results will be presented for *a given realization* of the common value. As in Klemperer, we assume that the n bidders receive their (cost) signal as an independent draw from a uniform distribution on [c-½, c+½], where $\theta_0=c$ is the (pure) common value, or *ex post* marginal costs. This means that a lower value of θ_i makes a lower value of θ_0 more likely, according to the formal definition of affiliation.

To find expected revenue from the different auction designs, as well as expected profit and coverage, we need to determine the order statistics of the random sample taken from the given distribution. Let the probability density function be $f(\theta_i|c)$ and the corresponding cdf be $F(\theta_i|c)$. $\theta_{(1)},\ldots, \theta_{(n)}$ denote the order statistics of the random sample $\theta_1,\ldots, \theta_n$, where $\theta_{(1)}< \theta_{(2)}\ldots< \theta_{(n)}$. Then the pdf of $\theta_{(i)}$ can be determined using the following expression:[15]

$$f_{c_{(k)}}(\theta) = \frac{n!}{(k-1)!(n-k)!} f(\theta \mid c)\left[F(\theta \mid c)\right]^{k-1}\left[1-F(\theta \mid c)\right]^{n-k}$$

where the expected k^{th} value can be found by:

$$E(c_{(k)}) = \int_0^{\infty} c f_{\theta_{(k)}}(c \mid y) dc$$

The expected lowest value when, for example, only three bidders (k=1, n=3) draw their value from $f(\theta_i|c)$ is $c_{(1)}=c-1/4$ and when 10 bidders participate, $c_{(1)}=c-9/22$. The corresponding second-lowest values are y and c-7/22, respectively.

We now have everything we need to perform the numerical analysis.[16] The only thing lacking is the specification of some initial parameters: the price received per new customer is p=20 (*10^3),[17] and we assume that the functional relationship between installed base units and potential customers is $.5k^{.5}$ until 100 per cent coverage is reached at K=k=7 (*10^3). At this coverage, it is estimated that 1.32 million customers would subscribe to the services. The relation between installed units and coverage is illustrated in Figure 1 below:

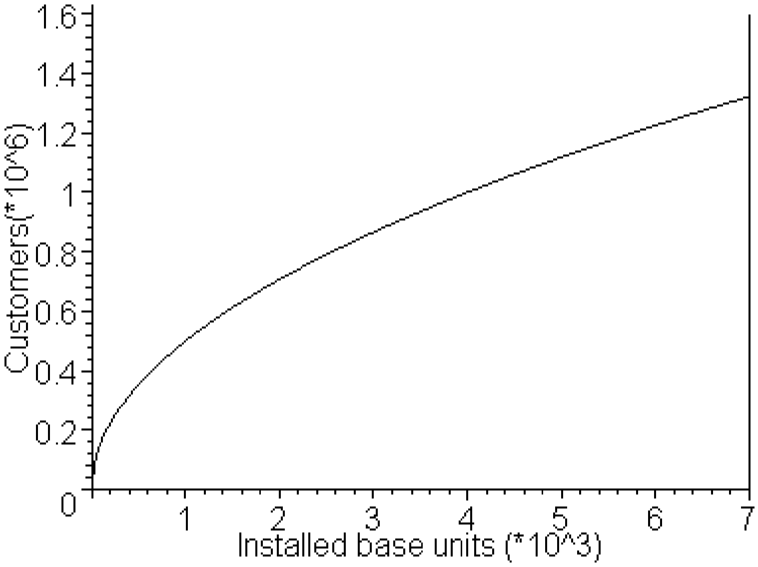

Figure 1. Coverage as a function of installed base units.

Although a very simplistic representation of reality, this captures some essential features of developing a mobile telecommunications network, where the first installed base stations (typically in densely populated urban areas) serve many potential customers, whereas base stations in rural areas have a much lower potential customer

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15 See Casella and Berger (1990), p. 232.

16 The numerical analysis is done using Maple.

17 This is the discounted value of each customer over the concession period. For instance, if each new customer generates expected revenues amounting to p=20 000€, and it turns out that each new base station costs c=4 mill. € , the winning bidder would invest until c/p=200 at the margin, i.e. as long as the new base station at least would generate 200 new customers.

base. The bidder with the lowest cost will, if the lease is won, also develop the most extensive network.

Before we proceed to the expected outcomes under the different auction designs, it is necessary to say something about the socially optimal coverage. Positive externalities imply that $p^s > p$, i.e. the social value is higher than the private willingness to pay for the services. One way to assure correspondence between socially optimal development of the network and the development a private licensee will realize is to subsidize customers so that the licensee perceives the price as $p(1+\tau)=p^s$.[18] For a given *ex post* realization of c, the optimal development will be found at the point where the marginal cost of extending the network further equals the marginal benefit. This is illustrated in Figure 2 below:

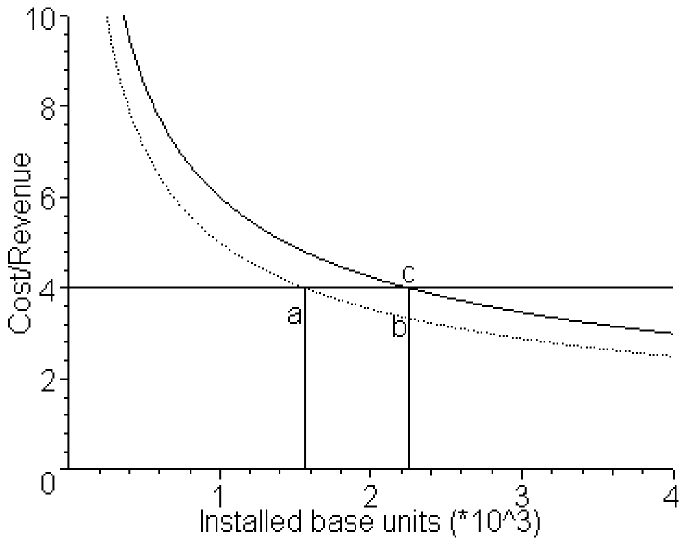

Figure 2. Optimal coverage. The horizontal line represents (constant) marginal costs, while the dotted line represents marginal revenue for the licensee without subsidies. Marginal socio-economic revenue lies above private revenue. Ex post realization of $c=4$ (*10^6).

Without subsidies, the licensee will choose to roll out $k^*=1.56$ (*10^6) base stations. A socio-economic loss, equal to the triangle abc in the figure above, will

[18] Alternative ways would be to subsidize a fixed percentage of costs or to contribute a fixed amount per base station rolled out. This would not alter the results presented here.

occur. The authorities do not, *ex ante*, have more information with respect to c than the bidders, but we assume that they can determine a fair estimate with respect to p^s, and consequently τ. Then, the authorities can rest assured that for any given *ex post* realization of c, the licensee will realize the optimal development. In this example, $\tau=0.20$ and $k^*=k^s=2.25$ (*10^3) and an estimated 750,000 subscribing customers (56%). The question now is how the presence of τ will affect bidding and expected revenue from the auction under the various auction designs.

First, assume that an *ascending auction* design is used. Ten bidders participate, and $\tau = 0$. Based on the order statistics, we find that the expected highest realization of the signals $c_{(10)}=c+9/22$. The expected value of c equals $½(c_{(n)}+c_{(1)})$. k^* is the optimal development given the information this bidder has received.

When the bid level at the auction has reached $pq(k^*)-c_{(10)}k^*-bid=0$, the bidder with the highest cost signal will quit the auction. At this point, the highest-signal bidder is indifferent about winning provided everyone else will quit simultaneously at the same price. The quit provides the other bidders with information with respect to $c_{(10)}$, and bidder i will quit when $pq(k^*)-½(c_{(10)}+c_i)k^*-bid=0$. The bidder with the second lowest cost signal will quit when $pq(k^*)-½(c_{(10)}+c_{(2)})k^*-bid=0$, and this will be the price paid by the bidder with the lowest price signal. Based on order statistics from the uniform distribution, where we found that $c_{(2)}= c-7/22$, we can find expected revenue from the ascending auction as a function of the pure common value c. To ease presentation, the results will be presented for a given c. The corresponding expected profit and expected coverage for the winning bidder can be found based on $c_{(1)}$, which is $c-9/22$.

In a *sealed-bid second-price auction*, the best strategy is to bid sincerely since the price paid not is linked to the bidder's own bid. This implies that each bidder i bids the expected value of the lease, conditional on being tied for winner with one other bidder (Klemperer (1999)). In practice, each bidder i must assume that the value drawn is the highest of n-1 values uniformly drawn from [c-½, c+½] and tied with one other. On average $c_i=c-2/5$, which implies that i's estimate of $c=2/5+c_i$ and the corresponding bid equals $bid = pq(k^*)-(c_i+2/5)k^*$. The price is determined by the bidder with the second-lowest signal $c_{(2)}$, which we know from above *is* $c-7/22$. This can be substituted for c_i in the bid function, and we can find the expected revenue from the auction. The results are presented in Table 1 below.

Table 1. Expected revenue, coverage, subsidy and profit (p=20, c=4, $n=10$).

	Expected revenue from the auction ($*10^9$)	*Ex ante* optimal coverage k^* ($*10^3$)	Expected subsidy ($*10^9$)	Socio-economic loss ($*10^9$)	Expected profit for the highest bidder ($*10^9$)
Second-price					
τ=0	6.12	1.57	0	0.25	0.14
τ=0.20	8.82	2.26	3.01	0	0.20
Ascending					
τ=0	6.18	1.56	0	0.25	0.07
τ=0.20	8.90	2.25	3.0	0	0.10

There are several interesting points to be noticed here. First of all, the ascending auction results in higher expected revenue than the second-price auction. This is a consequence of the affiliation assumption. With affiliation, one bidder's valuation depends on the other bidders' signals. The auction process of the ascending auction provides more information than the second-price auction. In the former case, the winning bidder gets information from the n-1 other bidders, whereas in the second-price auction, the price paid only depends on one other bidder. Consequently, the winner's information rent is lower and hence so is the expected profit for the bidders. In the sealed-bid auction design, no information is conveyed with respect to the other bidders' valuations, thus expected revenue from the auction is even lower. Therefore, the table illustrates a result well known from the auction theory literature (see for example, Klemperer (1999)).

The second, and most important point to be noticed is that the subsidy is discounted in the bid. Expected revenue from the auction increases as the subsidy is introduced. This is, of course, offset by an *ex post* payment from the authorities to the winning bidder, but this payment only turns up in the winning bidder's books as the network is developed. We can also notice that the profit of the winning bidder is an increasing function of the subsidy. The subsidy will gain the bidder who drew the lowest cost signal most, consequently increasing the variance of the distribution of valuations among bidders. This is a result that we recognize from other settings, i.a. McAfee and McMillan (1986) and bidding for contracts and Riley (1988), where the presence of an *ex post* royalty reduces variance and results in a bid intensification effect.

Table 2, below, presents results for the case where n=5, just confirming the theoretical result that expected revenue from the auction is an increasing function of the number of bidders participating.

Table 2. Expected revenue, coverage, subsidy and profit (p=20, c=4, $n=5$)

	Expected revenue from the auction	*Ex ante* optimal coverage	Expected subsidy	Socio-economic loss	Expected profit for the highest bidder
Second-price					
τ=0	6.05	1.59	0	0.25	0.25
τ=0.20	8.71	2.29	3.03	0	0.37
Ascending					
τ=0	6.12	1.56	0	0.25	0.13
τ=0.20	8.82	2.25	3.0	0	0.18

Let us now, just as a contrasting example, assume that the authorities wanted to use a “beauty contest” as the tool to decide who should get the opportunity to develop the network.

For simplicity, it is assumed that the contest is implemented as an ascending auction where coverage is the bidding variable. This corresponds in essence to the way the UMTS-licences were allocated in Norway and Sweden, where coverage and speed of rolling out services were important “bidding variables” in a sealed-bid auction. In addition, more qualitative considerations were used to select winners. Here, the bidder with the highest cost signal will quit when the “coverage” has reached the level where expected profit is zero. This provides the other bidders with information with respect to $c_{(10)}$. Thus, bidder i will quit when the bidding variable k has reached the level where $pq(k)$-½$(c_{(10)}+c_i)k$=0. The bidder with the second lowest cost signal will quit when $pq(k)$-½$(c_{(10)}+c_{(2)})k$=0, and this will be the coverage the bidder with the lowest price signal is obliged to reach. Based on the same order statistics as the previous example, we find that the bidder with the highest cost signal will quit when k=5.14 (for c=4 and n=10). The expected outcome of the auction is a coverage equal to k=6.11 or 94 per cent of all potential customers. The winner of the auction will be the same as when a cash bonus is used as the bidding variable, but the coverage is higher than optimal, both from the firm's and society's point of view. The winning bidder's expected profit will be 0.28 ($*10^9$), which can be contrasted with 0.07, the winning bidder's expected profit under the ascending auction (see Table 1 above). From the authorities point of view, no revenue accrues from the auction. Instead, this is reflected in the higher coverage. Although this is a simplified example, it nevertheless serves to illustrate an important problem with such a design.

V. Summary

Proponents of auctions have argued that the job of picking winners is best left to the market, whereas those in favour of beauty contests argue that winners of the licences should have been chosen on the basis of which company would guarantee the lowest cost to the consumers and promise to install the most cellular infrastructure. Combining a cash bonus auction with *ex post* subsidies related to network externalities reconcile the two views to some extent. The purpose of this paper has been to illustrate bidding outcomes with respect to, inter alia, expected revenue from the auction and expected network development in the presence of an *ex post* subsidy. The analysis has illustrated some points known from the theoretical literature on auctions, and has contributed some new knowledge related to this specific setting. Firstly, in the presence of affiliation, the ascending auction is a superior design from the authorities' point of view since this format reveals more information with respect to the competing bidders' valuation. Secondly, the expected value of the subsidy gets discounted in the bids, thus increasing the expected revenue from the auction. This is, of course, to some extent offset by an *ex post* payment from the authorities to the winning bidder, but this payment only turns up in the winning bidder's books as the network is developed. Finally, the high value bidder will expect to receive more subsidies than the competing bidders in absolute and relative terms. This, however, also leads to a bid *de-intensification* effect caused by an increased asymmetry between bidders. Thus, increasing the subsidy also increases the expected profit of the high value bidder. This point must, however, be considered to be of minor importance relative to the major benefit of combining an auction with a subsidy: The auction is used as a tool to allocate licences efficiently, whereas the subsidy contributes to a better correspondence between the private and social value of developing the network.

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