<u>Working Paper No. 40/00</u>

Strategic Informative Advertising in a TV-Advertising Duopoly

by

Tore Nilssen and Lars Sørgard

SNF-project No. 4590
Deregulering, internasjonalisering og konkurransepolitikk

The project is financed by the Research Council of Norway

FOUNDATION FOR RESEARCH IN ECONOMICS AND BUSINESS ADMINISTRATION
BERGEN, AUGUST 2000
ISSN 0803-4028

Strategic Informative Advertising in a TV-Advertising Duopoly*

Tore Nilssen**
Department of Economics
University of Oslo, Oslo, Norway
tore.nilssen@econ.uio.no

and

Lars Sørgard
Department of Economics
Norwegian School of Economics and Business Administration, Bergen, Norway
lars.sorgard@nhh.no

June 13, 2000

JEL No: L82, M37

Keywords: Television industry; Advertising.

Abstract:

We consider a model of strategic informative advertising where the advertising is done on TV and where the TV channels' advertising prices are endogenously determined. We discuss how these prices, and the advertising firms' advertising efforts, vary with the two key parameters of the model: the degree of product differentiation in the product market and a measure of the relative sizes of the TV channels' viewer bases. We find, in particular, that the larger the size difference among the TV channels is, the higher is the advertising price, and thus the less advertising is done.

---

* We have received helpful comments from Miguel Villas-Boas. This research is financed in part by the (i) Norwegian Competition Authority and the Norwegian Ministry of Government Administration and (ii) the Norwegian Research Council (the programme 'Næring, Finans og Marked'), both through SNF - the Foundation for Research in Economics and Business Administration. Nilssen's research was done during a visit at the Haas School of Business at the University of California, Berkeley. Sørgard's research was done during a visit at the Department of Economics at the University of California, Santa Barbara. The hospitality of the two institutions, as well as travel grants from the Research Council of Norway and the U.S.-Norway Fulbright Foundation for Educational Exchange, are gratefully acknowledged.

** *Corresponding author:* Tore Nilssen, Department of Economics, University of Oslo, P.O.Box 1095 Blindern, N-0317 Oslo, Norway. Phone: +47 22 85 49 36. Fax: +47 22 85 50 35. Email: tore.nilssen@econ.uio.no.

1. Introduction

A considerable amount of advertising is channeled through TV.[1] Still, there is little economic research focusing particularly on TV advertising. In the present analysis, we address how advertising firms' demand for advertising is affected by the suppliers being TV channels. We model the advertisers as competing with each other in the product market and demanding advertising on TV in order to increase consumer awareness of their products. The TV market is a duopoly, and we discuss how the two TV channels compete with each other in the advertising market. We envisage the viewers-consumers being divided among the TV channels. We analyze how prices in TV advertising are determined, and how the equilibrium price for TV advertising is affected by the two key parameters: the extent of heterogeneity in consumer preferences in the product market, and the difference in the sizes of the TV channels' viewer bases.

As far as our knowledge goes, the present paper, together with its companion, Nilssen and Sørgard (2000), are the first to consider the market for TV advertising as one where the demand for advertising is based on strategic considerations by the advertising firms.[2] The two papers are also unique in their derivations, albeit in two different models, of an endogenously determined advertising-cost function for firms, while the other literature on product-market advertising takes this function as a given.

Our two papers are differently focused, though. In the present paper, we model in detail how producing firms make their advertising decision, how TV channels price the advertising space they sell, and how this impacts on the way TV channels compete to attract advertising business. In so doing, we choose to disregard any effect from advertising on either TV viewers' behavior or TV channels' programming decisions. In our companion paper, on the other hand, we analyze how imperfect competition among advertising firms affects the way the TV channels compete to attract viewers.

In the analysis below, we first present, in Section 2, a model of product-market competition where advertising decisions are included. This is important in order to understand the

[1]In the US in 1998, for example, TV advertising amounted to $ 41.1 billion, out of a total advertising of $79.5 billion; *i.e.*, more than half of all advertising, in terms of value, was on TV. See the data reported by *Advertising Age* on http://adage.com/dataplace/archives/dp394.html.
[2]Other recent models of the TV industry disregard the strategic interaction among advertisers on TV. See Masson, *et al.* (1990), Motta and Polo (1997), Anderson and Coate (2000), and Gabszewicz, *et al.* (2000).

demand side of the advertising market: In what circumstances will demand be high, and what conditions are there that can suppress such demand for advertising? We focus our analysis on informative advertising and present a model due to Grossman and Shapiro (1984). This model is the basis for the subsequent analysis in this paper.

We go on, in Section 3, to discuss how this model can be extended to take into account the fact that the supply of advertising to a very large extent comes from TV stations. In particular, a TV station is able to price the advertising space it delivers by the variable of chief interest to its customers: the number of viewers. We also discuss the possible implications the presence of TV stations has for the way producers choose their advertising: There are only a few relevant TV stations and thus the choice between them is essential a discrete one. Furthermore, these TV stations may be close to perfect substitutes from the point of view of the producers, as long as records are kept of the number of viewers reached. We take here the view that, not only is advertising priced per viewer, but it is also sold per viewer.[3] Thus, it is possible, in our model, for a TV channel to sell advertising to a client that reaches only a fraction of the total number of the TV channel’s viewers.

Interestingly, there is a sense in which a TV station is capacity constrained: It cannot sell advertising space at a rate higher than 100% of its viewers. Thus, even if a small TV station has the lowest advertising price, it may be necessary for a producer to turn also to higher-priced TV stations when the producer's desired advertising reach goes beyond the low-price TV station's base of viewers. This leads to a softening of competition between the TV stations, as even the higher-priced station may get some advertising business. The equilibrium determination of the TV channels' advertising prices is the topic of Section 4. We find that the equilibrium price of advertising is higher, the more differentiation there is in the product market: An increase in product differentiation increases the producing firms' demand for advertising, and thus the equilibrium price of advertising goes up. In addition, we find that the equilibrium price of advertising increases with an increase in the size difference between the two TV channels. With

[3]The alternative assumption, used in Nilssen and Sørgard (2000), is that advertising is sold per slot, even in cases where the pricing unit is the viewer.

one small and one large TV channel, the larger channel's incentives to undercut the smaller one are low, since it at any rate will get that part of the advertising business that the small channel cannot cater.

Section 5 contains concluding comments.

2. Strategic Informative Advertising

Actual advertising is in part informative and in part persuasive: Firms advertise in order for their products to be known to the public, but they also advertise in order to persuade consumers to buy their product rather than some other product. At present, we want to focus on informative advertising. There are a number of reasons for this focus. First, as the discussion in Tirole (1988) indicates, informative advertising typically leads to an increase in competition, while persuasive advertising has the opposite effect. Although the results from the empirical literature are mixed, there are some empirical studies – notably by Eckard (1991), Leahy (1991), and Gallet (1999) – indicating that TV advertising provides a downward pressure on price. These studies indicate that informative advertising is the more fitting framework. Secondly, we believe that one important effect (both actual and intended) of TV ads, despite the usual character of a message of a certain life style, is to make consumers aware of the advertised product rather than to persuade them to buy this product rather than some other. Thirdly, it is not clear how one should formally model persuasive advertising. Such advertising affects consumers' preferences in some way or the other. But is it by affecting a consumer's perception of what is his favorite product variety, or by affecting his perceived differences among the products on offer, or could it be something else?[4] We sidestep these difficulties associated with the modeling of persuasive advertising by concentrating below on informative advertising.[5]

Our interest in this section is in how the possibility of informative advertising affects the way firms compete in an oligopolistic market. A seminal analysis of this question is by Grossman and Shapiro (1984).[6] Their analysis clarifies the basic effects of firms' informative advertising in

[4]See Fehr and Stevik (1998) for a discussion of these issues.

[5]This differs from the approach in Nilssen and Sørgard (2000), where we choose a model of advertising in the product market with the feature that advertising does not affect the equilibrium price, only how total sales are distributed among the rivals.

[6]We will not review the literature on informative advertising here. However, an early analysis of informative advertising under monopolistic competition is found in Butters (1977). Among more recent studies of informative advertising in oligopoly are McAfee (1994), Stahl (1994), and LeBlanc (1998). None of these studies determine the price of advertising endogenously, as we do here.

an oligopoly: First, the more it advertises, each firm obtains an increase in the number of consumers aware of its product. Thus, effectively, advertising increases the potential market for a firm's product and therefore is a good thing for the firm. Secondly, the more the firms in the market advertise in total, the higher is the number of consumers being aware of two or more products. Such segments of informed consumers are marked by more fierce competition than other segments, since these consumers are able to pick from all the offers they are aware of. Thus, more advertising means more competition, which harms firms' profits. In equilibrium, one will see a trade-off between these two effects of advertising, and the Grossman-Shapiro model is set up to clarify this trade-off.

Consider a Hotelling (1929) style of a market with product differentiation, positing consumers uniformly distributed along the line segment [0, 1].[7] As a matter of normalization, the total number of consumers is measured equal to 1. Each consumer has a unit demand for the product with a gross surplus equal to s. The product differentiation is modeled by way of linear transportation costs: If there is a distance d in product space between a consumer's favorite and the actually consumed product, a utility loss equal to td entails. In order to simplify the subsequent analysis of the market for TV advertising, we need to put a lower bound on the product-differentiation parameter t. In particular, we assume that $t \geq 0.3$. Such a restriction implies a focus on the more likely cases for informative advertising to occur, *viz.*, those cases where the product differentiation in the market is sufficiently high.

On the supply side of the product market, there is a duopoly, with the two firms located at the two extreme positions 0 and 1. A consumer is able to consume a particular firm's product if and only if he receives an ad from this firm. Denote the two firms by A and B. Let φ_i be the fraction of the consumers in the market who receive an ad from firm i. Let, for now, $A(\varphi)$ denote the cost for a firm of reaching a fraction φ of the consumers in the market.

Suppose firm A's advertising reach is φ_A. This is then also the potential demand for firm A's product. This potential demand can be divided into two groups: A fraction φ_B of the φ_A consumers receive, in addition to an ad from firm A, also an ad from firm B and therefore know about both firms. The residual fraction $(1 - \varphi_B)$ of consumers receive an ad from firm A only and do not know about the presence of firm B in this market. We assume that there is no targeting of

[7] This version of the Grossman-Shapiro model, based on the Hotelling set-up, is due to Tirole (1988).

the advertising, nor is there anything else present that could create a correlation between the characteristics of a firm's product and those of the consumers reached by the firm's advertising. Thus, the φ_A consumers aware of firm A's product are evenly spread on the [0, 1]. So is also that subgroup of $\varphi_A\varphi_B$ consumers aware of both products, and so on. It is for the consumers who have received ads from both firms that competition prevails. However, as long as each firm is unable to get information on which consumers have received ads, it will have to offer the same price to all consumers.

The firms' prices interact with consumers' transportation costs to determine each firm's demand in the competitive market segment. If firms set prices p_A and p_B, then consumers on the line segment [0,1] who are below x^* choose firm A while those above x^* choose firm B. The indifferent consumer x^* is determined by:

$$s - p_A - tx^* = s - p_B - t(1 - x^*).$$

Solving this equation, we find that

$$x^* = \frac{1}{2} + \frac{p_B - p_A}{2t} \quad .$$

With the help of this expression, we obtain each firm's demand given both firms' prices and advertising:

$$D_A = D_A(p_A, p_B, \varphi_A, \varphi_B) = \varphi_A[(1 - \varphi_B) + \varphi_B x^*],$$

assuming that, at the prevailing prices, all consumers with information only about firm A actually purchases from this firm, which amounts to assuming that even consumers over at 1 on the [0, 1] line find it worthwhile to "move over" to firm A at 0. Formally, since it takes transportation costs equal to t to move across all the product space (which is of unit length), we assume that, in equilibrium, $p_A \leq s - t$ (and similarly for firm B). Thus,

$$D_A = \varphi_A\left[(1 - \varphi_B) + \varphi_B\left(\frac{1}{2} + \frac{p_B - p_A}{2t}\right)\right], \text{ and}$$

$$D_B = \varphi_B\left[(1 - \varphi_A) + \varphi_A\left(\frac{1}{2} + \frac{p_A - p_B}{2t}\right)\right].$$

An important issue in a formal discussion of the competition between the two firms is the move sequence. Actually, there are two issues here: First, what is the move sequence between pricing decisions and advertising decisions? Secondly, what is the move sequence between the two firms? In many circumstances, price is rightly considered as a short-term decision and therefore modeled as following after more long-term decisions, like characteristics of the products and investments in capacity or R&D. However, it is more doubtful whether advertising is of such a long-term nature that a sequential modeling strategy is called for. We will therefore go on with assuming that prices and advertising are chosen simultaneously. With respect to the other modeling question, we will treat the firms symmetrically so that they take their decisions simultaneously.

With two firms each making two decisions simultaneously, the equilibrium of the game is found as the solution of a system of four first-order conditions, two for each firms: one with respect to price and one with respect to advertising. For firm A, the problem is the following:

$$\max_{\{p_A,\varphi_A\}} \left\{ \varphi_A \left[(1-\varphi_B) + \varphi_B \left(\frac{1}{2} + \frac{p_B - p_A}{2t} \right) \right] (p_A - c) - A(\varphi_A) \right\},$$

and this results in two first-order conditions for the firm, with respect to its price and its advertising level, respectively. After some rearrangements, these first-order conditions can be expressed as:

$$p_A = \frac{p_B + c + t}{2} + \frac{t(1-\varphi_B)}{\varphi_B}, \text{ and}$$

$$A'(\varphi_A) = (p_A - c)\left[1 - \varphi_B + \varphi_B \left(\frac{1}{2} + \frac{p_B - p_A}{2t} \right) \right].$$

From the first of these expressions, we can see that a firm's optimum price is independent of its own advertising but rather depends on the other firm's price and advertising decisions. We also see how the fact that consumers are less than perfectly informed affects the price: The first term on the right-hand side of this expression is the reaction function when consumers are fully informed. We see that firm A's price now will be higher than in the full-information case, since the second term is positive, and more so the less advertising the other firm does. In the second expression above, the marginal cost of advertising, on the left-hand side, is set equal to the marginal benefit on the right-hand side, the latter being the price-cost margin times the probability of a sale.

We now have four equations, two first-order equations for each of two firms. Solving this system is greatly simplified by the observation that the two firms are completely symmetric. We can therefore focus our attention on symmetric equilibria. Let p and φ denote a firm's equilibrium price and advertising level, respectively, *i.e.*, $p = p_A = p_B$, and $\varphi = \varphi_A = \varphi_B$. Applying this symmetry to the above first-order condition with respect to price, we find:

$$p = c + t + 2t\frac{1-\varphi}{\varphi}.$$

If we now go on to insert this expression in the first-order condition with respect to advertising and again make use of symmetry, we obtain:

$$(1) \qquad A'(\varphi) = t\left(1 + 2\frac{1-\varphi}{\varphi}\right)\left(1 - \frac{\varphi}{2}\right).$$

In order to proceed from here, we need to be specific about the advertising cost function $A(\varphi)$. We could, as done in Tirole (1988), assume that $A(\varphi) = \frac{a\varphi^2}{2}$. With such a quadratic advertising cost function, one gets some counterintuitive results, however. In particular, an increase in advertising costs, in the sense of an increase in a, will also *increase* firms' profits. The reason is that, in addition to the direct, negative effect of increased advertising costs on profit, there is also an indirect, positive effect through the equilibrium advertising decisions: Higher advertising costs mean reduced advertising, and thus less competition, higher prices, and increased profit. However, a quadratic advertising cost function may not be the natural choice in our setting. Instead of applying any arbitrary advertising-cost function, we let the advertising costs be endogenously determined. Advertising is in our setting offered by TV channels, which compete on prices of advertising. Thus, the producers' cost of advertising is determined by the rivalry between TV channels.

3. Informative advertising through TV

TV channels regularly use various devices, like Nielsen meters, to keep records of how many viewers they have at any time. These devices make it possible for the TV channels to find out how popular their various programs are, so that they can refine their programming. It also has an effect, however, on how they sell advertising space: Because TV channels keep track of how many viewers they have at any time, it is possible for them to sell advertising space by the viewer. Thus, how much an advertiser will have to pay for its advertising depends on how many viewers this advertising has. In reality, of course, this relationship can be a complicated one.[8] For

[8]See the discussion in Nilssen and Sørgard (2000) of this point.

now, however, we will assume that TV channels sell advertising space at a constant price per viewer of the advertising.[9]

In many markets for TV advertising, there are a limited number of TV channels available to choose from. This is true for all product markets where competition is localized. For example, an advertising firm in Norway has essentially three advertising-financed TV channels available when choosing where to advertise: TV2, TVNorge, and TV3. We will below discuss a case where two TV channels are available. An advertising firm will have to decide not only how much to advertise, but also how to distribute its advertising efforts among the two TV channels. In order to focus on the effects of TV as a medium for advertising, we will disregard any other means for a firm to reach out to its consumers, such as newspapers and magazines.

We will allow for the two TV channels to be of different sizes in terms of their viewer bases. Thus, we let v denote the size, in terms of viewers, of the bigger channel, so that $(1 - v)$ is the size of the smaller channel, with $v \in [\frac{1}{2}, 1)$. As a matter of convention, we will let channel 1 be the bigger one.[10]

Although the two channels may not be of equal size, we do not envisage any systematic difference among the channels' viewers in terms of their preferences for the advertised product. In particular, at each point in the product space [0, 1], a fraction v of the consumers preferring this particular product variety watch the bigger channel while the rest watch the smaller one.

We assume that the only costs of advertising for the firms on the product market are related to what they have to pay the TV channels for the advertising space, or more accurately, for the viewers reached by the advertising.

With two TV channels to choose from, a firm not only will have to decide how much to advertise but also where to advertise. Thus, each firm has a total of three decisions to make: the

[9]Although TV channels may know how many viewers any particular ad attracts, they do not know, in the case of multiple ads on the same product on one TV channel, whether some of the viewers have seen the ad several times. This is an issue that we will not deal with here. It is not necessarily an important limitation of the analysis, however. With detailed knowledge of its viewers at any time, a TV channel should be able in its pricing to account for multiple ads having overlapping groups of viewers.

[10]Note that we treat a TV channel's number of viewers as an exogenous variable. An alternative assumption, pursued in Nilssen and Sørgard (2000), would be that viewers respond negatively to an increase in a TV channel's advertising.

price of its product; advertising in the big TV channel 1; and advertising in the small channel 2. Let φ_{ij} denote firm j's advertising reach among the viewers of channel i, $i \in \{1, 2\}$, $j \in \{A, B\}$. Firm j's total advertising reach is: $v\varphi_{1j} + (1 - v)\varphi_{2j}$. The price of advertising per viewer is b_1 in channel 1 and b_2 in channel 2. The advertising costs depend on how many viewers are reached by advertising on each channel: $A(\varphi_{1j}, \varphi_{2j}) = b_1\varphi_{1j}v + b_2\varphi_{2j}(1 - v)$, $j \in \{A, B\}$.

The informed consumers' choices are not affected by how they get informed. Therefore, in this market segment, the indifferent consumer is determined as in the previous section. Firms' various decisions are again taken simultaneously. Thus, we can state firm A's problem as follows:

$$\max_{\{p_A, \varphi_A^1, \varphi_A^2\}} \pi_A = \left\{\varphi_A\left[(1-\varphi_B)+\varphi_B\left(\frac{1}{2}+\frac{p_B - p_A}{2t}\right)\right](p_A - c) - b_1\varphi_{1A}v - b_2\varphi_{2A}(1-v)\right\}.$$

The first-order condition with respect to price is as before:

$$p_A = \frac{p_B + c + t}{2} + \frac{t(1-\varphi_B)}{\varphi_B}.$$

Each firm has now two first-order conditions with respect to advertising, one for each TV channel. Note, however, that the way advertising pricing is made by TV channels in this model creates a linear relationship between a firm's marginal costs of doing advertising on a particular TV channel and its marginal benefit from it. The marginal benefit from advertising is independent of which channel the marginal advertising is put on, since both channels' viewers are evenly distributed as consumers in product space. Denote this marginal benefit for firm A by K_A, *i.e.*:

$$K_A = (p_A - c)\left[(1-\varphi_B)+\varphi_B\left(\frac{1}{2}+\frac{p_B - p_A}{2t}\right)\right]$$

The net marginal profit for firm A from advertising on the big channel 1 can now be expressed as:

$$\frac{d\pi_A}{d\varphi_{1A}} = K_A v - b_1 v = [K_A - b_1]v.$$

The corresponding expression for advertising on channel 2 is:

$$\frac{d\pi_A}{d\varphi_{2A}} = [K_A - b_2](1-v).$$

The firm will spend the additional funds on advertising on the channel where the net marginal profit is the greater. Thus, it chooses channel 1 if:

$$[K_A - b_1]v > [K_A - b_2](1 - v),$$

i.e., if:

$$b_1 < \left(2 - \frac{1}{v}\right)K_A + \left(\frac{1}{v} - 1\right)b_2 .$$

A corresponding condition holds for firm B.

Again, we make use of symmetry, focusing on symmetric equilibria. We let $p_A = p_B = p$, $\varphi_A = \varphi_B = \varphi$, and, therefore, $K_A = K_B = K$. Also, $\varphi_{iA} = \varphi_{iB} = \varphi_i$, $i \in \{1, 2\}$. A firm will choose the cheaper TV channel for the whole of its basis of viewer, if necessary, before considering making use of the other channel's services. It may have to use the latter channel, however, if the desired level of advertising cannot be covered through one channel. For example, if the small channel 2 is the cheaper one but each firm wants to obtain an advertising reach in excess of $(1 - v)$, the number of channel 2's viewers, then they both will have to turn to the other channel.

In equilibrium, the marginal benefit of advertising is set equal to marginal costs of advertising for the firms. Since the latter is equal to either b_1 or b_2, we have that, either $K = b_1$, or $K = b_2$. In both these cases, however, the condition for a firm preferring channel 1 over channel 2, which with symmetry reads:

$$b_1 < \left(2 - \frac{1}{v}\right)K + \left(\frac{1}{v} - 1\right)b_2 ,$$

reduces to:

$$b_1 < b_2.$$

Symmetry also simplifies the expression for the marginal benefit of advertising:

$$K = \frac{t(2-\varphi)^2}{2\varphi},$$

which is identical to the right-hand side of Eq. (1) above.

In order to resolve a technicality associated with the possibility of one of the two TV channels having to choose its price from an open set, we assume that, at equal prices, *i.e.*, when $b_1 = b_2$, the small channel 2 serves all the advertising demand up to its full capacity $(1 - v)$ of viewers. We now have the following result on the firms' decisions to advertise:

Proposition 1: Given TV channels' advertising prices b_1 and b_2, firms' advertising decisions are as follows:

(i) If $\frac{(2-v)^2 t}{2v} < b_1 < b_2$, then:

$$\varphi_1 = \frac{1}{tv}\left[b_1 + 2t - \sqrt{b_1^2 + 4b_1 t}\right] \text{ and } \varphi_2 = 0.$$

(ii) If $b_1 \leq \frac{(2-v)^2 t}{2v} < b_2$, then:

$$\varphi_1 = 1, \text{ and } \varphi_2 = 0.$$

(iii) If $b_1 < b_2$ and $b_2 \in \left(\frac{[(1+v)t-v]^2}{2[(1-v)t+v]}, \frac{(2-v)^2 t}{2v}\right]$, then:

$$\varphi_1 = 1, \text{ and } \varphi_2 = \frac{1}{t(1-v)}\left[b_2 + 2t - v - \sqrt{b_2^2 + 4b_2 t}\right].$$

(iv) If $b_1 < b_2 \leq \min\left\{\frac{[(1+v)t-v]^2}{2[(1-v)t+v]}, \frac{(2-v)^2 t}{2v}\right\}$, then:

$$\varphi_1 = \varphi_2 = 1$$

(v) If $b_1 \geq b_2 > \frac{(1+v)^2 t}{2(1-v)}$, then:

$$\varphi_1 = 0, \text{ and } \varphi_2 = \frac{1}{t(1-v)}\left[b_2 + 2t - \sqrt{b_2^2 + 4b_2 t}\right].$$

(vi) If $b_1 > \frac{(1+v)^2 t}{2(1-v)} \geq b_2$, then:

$$\varphi_1 = 0, \text{ and } \varphi_2 = 1.$$

(vii) If $b_1 \geq b_2$ and $b_1 \in \left(\frac{[(2-v)t-(1-v)]^2}{2(tv+1-v)}, \frac{(1+v)^2 t}{2(1-v)}\right]$, then:

$$\varphi_1 = \frac{1}{tv}\left[b_1 + 2t - (1-v) - \sqrt{b_1^2 + 4b_1 t}\right], \text{ and } \varphi_2 = 1.$$

(viii) If $\min\left\{\frac{[(2-v)t-(1-v)]^2}{2(tv+1-v)}, \frac{(1+v)^2 t}{2(1-v)}\right\} \geq b_1 \geq b_2$, then:

$$\varphi_1 = \varphi_2 = 1.$$

Proof: Suppose first that $b_1 < b_2$. In order for each firm's equilibrium advertising to be so low that $\varphi < v$, implying that only the cheaper channel 1 is used, it must be true that: $K = b_1$, or, inserting for K,

$$\frac{t(2-\varphi)^2}{2\varphi} = b_1.$$

Solving for the only root that takes value between 0 and 1, we obtain:

$$\varphi = \frac{1}{t}\left[b_1 + 2t - \sqrt{b_1^2 + 4b_1 t}\right]$$

In order for this to satisfy the premise $\varphi < v$, calculations reveal that we must have:

$$b_1 > \frac{(2-v)^2 t}{2v}.$$

Likewise, in order to have $\varphi > v$, we must have:

$$\frac{t(2-\varphi)^2}{2\varphi} = b_2,$$

or:

$$\varphi = \frac{1}{t}\left[b_2 + 2t - \sqrt{b_2^2 + 4b_2 t}\right]$$

In order for this to satisfy $\varphi > v$, we must have:

$$b_2 < \frac{(2-v)^2 t}{2v}.$$

In the intermediate case, we have $\varphi = v$. In addition, the solution must obey the restriction $\varphi \leq 1$. Together, this gives parts (i)-(iv) of the Proposition.

Suppose next that $b_1 \geq b_2$. If $\varphi < 1 - v$, then it is given by $K = b_2$, or:

$$\varphi = \frac{1}{t}\left[b_2 + 2t - \sqrt{b_2^2 + 4b_2 t}\right]$$

In order to satisfy $\varphi < 1 - v$, we must have:

$$b_2 > \frac{(1+v)^2 t}{1-v}.$$

The proof for parts (v)-(viii) now continues as for the case of $b_1 < b_2$. *QED.*

An illustration of firms' advertising behavior, for various combinations of b_1 and b_2, is provided in Figure 1. Notice that, if both TV channels raise their prices, so that we move in a northeast direction in the Figure, then advertising goes down and therefore also competition is weakened in the product market. If one TV channel alone raises its price, so that we move horizontally or vertically in the Figure, then that channel gets less advertising.

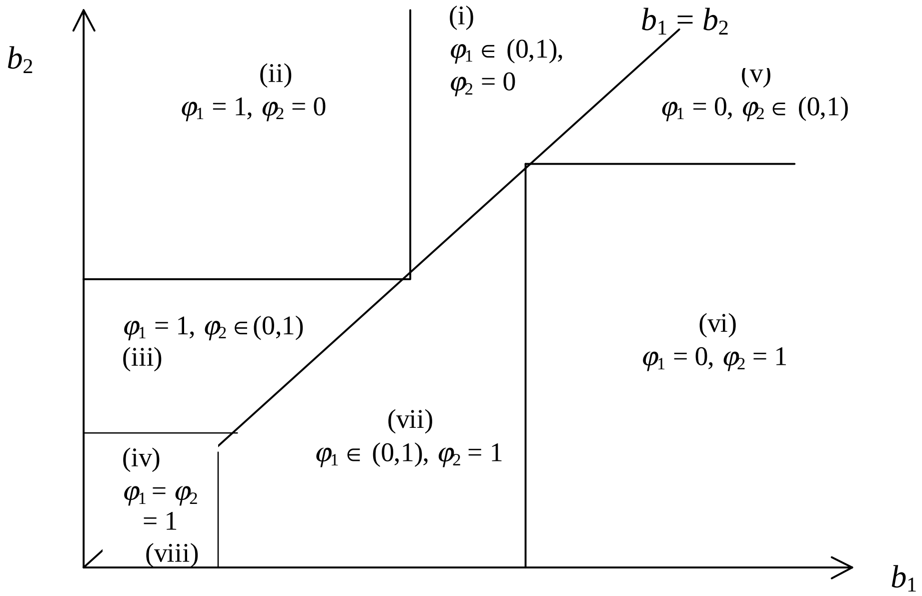


Figure 1

4. The market for advertising on TV

In the analysis so far, the prices that the TV channels charge for advertising have been treated as exogenous. In this Section, we extend the analysis to include the TV channels' decisions on advertising prices. Under the assumption that the numbers of viewers in the two channels are constant, there is no interaction between advertisers in different product markets. We will also assume that a TV channel is able to price discriminate among various product markets but not among firms in the same product market (*i.e.*, to treat advertising for cereals as a service distinct from advertising for pharmaceuticals, and so on). This way, we can concentrate the analysis on one product market and the interaction between the rivalry in this product market and the rivalry between the TV channels offering advertising space. Finally, we also assume, quite simplistically, that there are no costs for TV channels associated with the provision of advertising space. With these assumptions, we can study the equilibrium determination of advertising prices for one product market at the time.

We envisage a two-stage game, in which the TV channels simultaneously determine their respective prices of advertising at stage 1, while the advertising firms' price and advertising decisions are made simultaneously, as in the previous Section, at stage 2.

It is clear from Proposition 1 and the illustration of it in Figure 1 that a channel's viewer base works as a form of capacity: A TV channel cannot sell more advertising reach than it has viewers. Thus, even the higher-priced channel may get some of the advertising business, precisely because the lower-priced channel cannot provide the full service that the advertising firms want. However, in contrast to other models of capacity constraints, notably Kreps and Scheinkman (1983), this does not give rise to non-existence of a pure-strategy equilibrium. In the present model, each TV channel's best-response price is a continuous function of the other channel's price: If the other channel's price is low, then it pays to keep one's own price a bit higher. Otherwise, the best response is to undercut. In particular, we have the following useful result:

Lemma: Let ε be a small, positive number. There exists a $t^* > 0.3$ such that:[11]

(i) channel 1's best response, $B_1(b_2)$, is:

$B_1(b_2) = \beta_1$, if $b_2 < \beta_1$, and

$B_1(b_2) = b_2 - \varepsilon$, otherwise,

where

$$\beta_1 = \frac{1}{8}\left[(1-v-20t)+(4t+1-v)\sqrt{1+\frac{16t}{1-v}}\right], \text{ if } t < \frac{3-v}{2(1-v)}; \text{ and}$$

$$\beta_1 = \frac{(1+v)^2 t}{2(1-v)}, \text{ if } t \geq \frac{3-v}{2(1-v)};$$

(ii) channel 2's best response, $B_2(b_1)$, is:

[11] The precise expression for t^* is:

$$t^* = \frac{v}{16}\left[\frac{1}{9}\left(2-v+\sqrt[3]{H}+\frac{v^2-4v+7}{\sqrt[3]{H}}\right)^2-1\right],$$

where: $H = 206 - 3v + 6v^2 - v^3 + 3\sqrt{4677 - 72v + 222v^2 - 24v^3 - 3v^4}$.

$B_2(b_1) = \beta_2$, if $b_1 < \beta_2$,

$B_2(b_1) = b_1$, otherwise,

where

$$\beta_2 = \frac{[(1+v)t - v]^2}{2[(1-v)t + v]}, \text{ if } t \in [0.3, t^*] \text{ or}$$

$$\text{if } v \geq 4\sqrt{2} - 5 \text{ and } t \in \left(\frac{v\left(3v - 1 - \sqrt{v^2 + 10v - 7}\right)}{4(1-v)^2}, \min\left[\frac{v}{2v-1}, \frac{2+v}{2v}\right] \right);$$

$$\beta_2 = \tfrac{1}{8}\left[(v - 20t) + (4t + v)\sqrt{1 + \tfrac{16t}{v}}\right], \text{ if } v < 4\sqrt{2} - 5 \text{ and } t \in \left(t^*, \frac{2+v}{2v}\right), \text{ or}$$

$$v \geq 4\sqrt{2} - 5 \text{ and } t \in \left(t^*, \frac{v\left(3v - 1 - \sqrt{v^2 + 10v - 7}\right)}{4(1-v)^2} \right); \text{ and}$$

$$\beta_2 = \frac{(2-v)^2 t}{2v}, \text{ if } t \geq \min\left[\frac{v}{2v-1}, \frac{2+v}{2v}\right].$$

Proof: We sketch here a proof of part (i). Part (ii) can be proved the same way. Channel 1's profit is:

$$\Pi_1 = b_1 v \varphi_1.$$

Whenever $\varphi_1 = 1$, the channel's profit increases with an increase in b_1. Whenever $\varphi_1 = 0$, profit is zero. Thus, there are two cases left to consider, in which $\varphi_1 \in (0,1)$.

Consider first case (i) of Proposition 1, where $\varphi_1 = \frac{1}{tv}\left[b_1 + 2t - \sqrt{b_1^2 + 4b_1 t}\right]$. Inserting this into the profit expression, the channel's marginal profit with respect to price is, after some rearrangement, found to be equal to:

$$\frac{d\Pi_1}{db_1} = \left[\sqrt{b_1^2 + 4b_1 t} - b_1\right]\left[\frac{b_1 + 2t}{\sqrt{b_1^2 + 4b_1 t}} - 1\right] > 0,$$

where the inequality holds because both terms inside brackets are positive. Thus, it pays for channel 1 to increase the price in case (i), which means that the optimum response to a high price by channel 2 is to slightly undercut it.

Consider next case (vii) of Proposition 1, where $\varphi_1 = \frac{1}{tv}\left[b_1 + 2t - (1-v) - \sqrt{b_1^2 + 4b_1 t}\right]$, creating scope for an interior solution. In particular, the first-order condition of channel 1 in this case is:

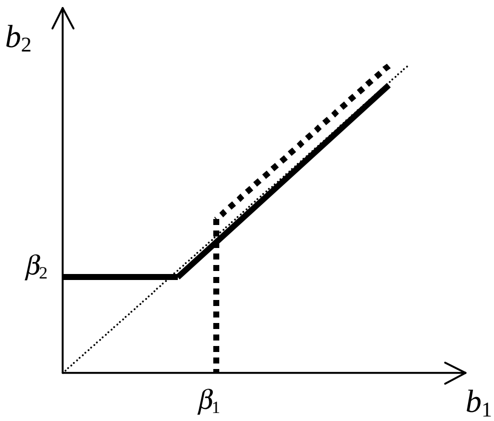
$$\left[\sqrt{b_1^2 + 4b_1 t} - b_1\right]\left[\frac{b_1 + 2t}{\sqrt{b_1^2 + 4b_1 t}} - 1\right] = \frac{1-v}{t}.$$

The solution to this equation has two roots, one of which is always negative and thus can be excluded. The other root is:

$$b_1 = \frac{1}{8}\left[(v - 20t) + (4t + v)\sqrt{1 + \frac{16t}{v}}\right].$$

However, we have to check for combinations of t and v such that either case (vii) is empty or the interior solution is outside the bounds of case (vii). The result of this check is the definitions of β_1 and β_2 in the Lemma. *QED.*

The interesting message coming out of this Lemma is that, when the rival's price on advertising reach is sufficiently small, it does not pay for a TV channel to undercut it. Rather, the optimum price in such a case is higher than the rival's price, but otherwise independent of it. The equilibrium prices are always equal and determined by which of the two, β_1 or β_2, is the smaller. The interesting issue, thus, is whether $\beta_1 > \beta_2$ or the opposite. It turns out that, despite the expressions for β_1 and β_2 being quite ugly, we have that $\beta_1 > \beta_2$ under the maintained restriction that $t \geq 0.3$. An illustration of the best-response functions of the two TV channels is provided in Figure 2.



TV channel 1's best response ▪▪▪▪▪▪▪▪▪
TV channel 2's best response ▬▬▬

Figure 2

Thus, we now have:

Proposition 2: In equilibrium, the two TV channels set the same price on advertising reach: $b_1^* = b_2^* = b^* = \beta_1$. In particular:

(i) If $0.3 \le t < \frac{3-v}{2(1-v)}$, then $b^* = \frac{1}{8}\left[(1-v-20t)+(4t+1-v)\sqrt{1+\frac{16t}{1-v}}\right]$.

(ii) If $t \ge \frac{3-v}{2(1-v)}$, then $b^* = \frac{(1+v)^2 t}{2(1-v)}$.

Proof: It follows from the Lemma that, with ε negligibly small, in equilibrium, $b_1^* = b_2^* = b^* = \max[\beta_1, \beta_2]$. A case-by-case comparison of the two reveals that, under assumption that $t \ge 0.3$, $\beta_1 > \beta_2$. Now, the Proposition follows from part (i) of the Lemma. *QED.*

We see from this Proposition how the introduction of a market for TV advertising has made the price of advertising, and therefore the advertising costs of the advertisers, endogenous. While there is some competitive pressure towards low prices, this pressure is limited by the fact that the advertising reach obtained through one single channel is limited by the size of this channel's viewer base. Even if one channel prices advertising lower than the other does, it may still be necessary for advertising firms to make use of both channels in order to reach the desired fraction of consumers. This is the main mechanism by which the competition between the TV channels is softened, with the implication that advertising prices always are greater than their marginal costs.

Note that, although the two TV channels have the same advertising price, their marginal profits are not the same: The price is for advertising per viewer, and since channel 1 has more viewers than channel 2, it also has a greater marginal profit in equilibrium.

The equilibrium in Proposition 2 has essentially the feature that the larger channel 1 finds it unprofitable to undercut the other channel and therefore lets it have as much advertising as it can take. This leaves a residual demand for channel 1 that, at the equilibrium price of case (i) of the Proposition, exactly balances the profit that it could have had by undercutting. In case (ii), the decision by channel 1 not to undercut is restricted by the advertising firms' behaviour: It cannot choose an advertising price greater than $\frac{(1+v)^2 t}{2(1-v)}$ or it will be totally without business, according to Proposition 1(vi).

It is also straightforward to get out some comparative statics in this case:

Proposition 3:

(i) The equilibrium price of advertising, b^*, increases with either an increase in the differentiation in the product market, or the size of the larger TV channel, *i.e.*,

$$\frac{db^*}{dt} > 0, \text{ and } \frac{db^*}{dv} > 0.$$

(ii) Firms' advertising, φ, decreases with the size of the larger TV channel, while the effect of an increase in the product differentiation is ambiguous, *i.e.*,

$$\frac{d\varphi}{dt} < or > 0, \text{ and} \frac{d\varphi}{dv} < 0.$$

Proof: (i) Follows from differentiating the expressions for b^* in Proposition 2. (ii) Total advertising equals $v\varphi_1 + (1 - v)$, since, in equilibrium, we are in case (vii) of Proposition 1. Substituting the expressions for b^* in Proposition 2 into the expression for φ_1 given in Proposition 1(vii) and differentiating the total give the result. *QED.*

An increase in the product differentiation makes the products more different in the view of consumers and firms get more interested in advertising since now competition is more relaxed. This increased demand for advertising, in turn, accounts for the increased price of advertising. But it also explains the ambiguous total effect of an increase in t on advertising: In addition to the positive direct effect, there is also a negative effect through the increase in advertising price.

Under the restriction we use here, that t is not very low, it is the smaller TV channel that has the higher incentives to set a low price. An increase in the (relative) size of the larger channel means that a larger fraction of total advertising will have to be made through the high-price large channel. Of course, in equilibrium, there is a negligible price difference between the two channels. However, as the larger channel gets even larger, it has even lesser incentives to compete with the aggressive small channel, and the equilibrium price of advertising increases. This feeds into less advertising being sold.

We thus find that asymmetry between TV channels dampens price competition. The more limited the small TV channel's audience, the higher are the prices on advertising. A natural question, then, is whether it can be profitable for the 'small' TV channel to deliberately act so that its audience is limited. One natural interpretation is that the 'small' TV channel is an entrant. If it invests only a limited amount in program quality, its audience is limited. By such a strategy, it both saves investment in program quality and dampens price competition.

If we, as above, interpret our game as an entry game, our result shares some similarities with what was labeled *judo economics* in Gelman and Salop (1983). They showed that a producer might have incentives to restrict its sales in order to stop its rival from cutting its price: A producer with a large market share would rather maintain high prices to serve all its existing consumers than cut prices to compete with the small producer. However, in their model with sequential price setting, the small firm sets a lower price than the large firm in equilibrium. Given that prices are typically flexible, we find it more natural to assume simultaneous price setting, and we reproduce the main result in Gelman and Salop (1983) - capacity limitation dampens price competition - although equilibrium prices are now identical.

5. Concluding remarks

Our starting point was that the price of advertising on TV is determined by the rivalry between TV channels. In line with this observation, the producers' cost of advertising on TV should be endogenously determined. Surprisingly, though, no studies have as far as we know followed such an approach when modeling the advertising technology. The purpose of this paper has been to help fill in this gap. Our approach has led us to develop a model where prices on advertising are linear and thus each producer faces a constant marginal cost of advertising. Then TV channels' price setting affect the level, rather than the slope, of the producing firms' marginal costs of advertising. In contrast, in Tirole (1988), advertising costs are by assumption quadratic and thus the marginal cost of advertising is increasing. In the latter setting, it has been shown that an increase in the cost of advertising *increases* the producers' profit. However, in our model with endogenous prices of advertising and thus linear advertising costs, there is no longer any adverse relationship between advertising prices and advertising firms' profits.

Although our model is highly stylized, we have pointed to one mechanism suggesting that price rivalry on TV advertising may not result in cut-throat competition. The driving force is that the advertising firms may need to use more than one TV channel to reach its consumers. The larger the asymmetry between TV channels concerning the number of viewers, the less is the rivalry on prices on advertising. The large TV channel sets a high price and lets the small TV channel slightly undercut its price and therefore lets the small channel have as much advertising as it can. The large channel then becomes the residual supplier of advertising, setting a high price to maximize its profit from the residual demand for advertising. This suggests that it can be more beneficial to be a small scale TV channel than to challenge a dominant TV channel. A small-scale TV channel saves on investments in program quality to attract viewers, and it dampens the rivalry on prices of advertising slots. One interpretation might be that an entrant has incentives to enter on a small scale in order to keep the post-entry advertising price high.

References

Anderson, S.P. and S. Coate, 2000, "Market Provision of Public Goods: The Case of Broadcasting", Working Paper 7513, National Bureau of Economic Research.

Butters, G.R., 1977, "Equilibrium Distributions of Sales and Advertising Prices", *Review of Economic Studies* **44**, 465-491.

Eckard, E.W., 1991, "Competition and the Cigarette TV Advertising Ban", *Economic Inquiry* **29**, 119-133.

Fehr, N.-H.M.v.d. and K. Stevik, 1998, "Persuasive Advertising and Product Differentiation", *Southern Economic Journal* **65**, 113-126.

Gabszewicz, J., D. Laussel, and N. Sonnac, 2000, “TV-Broadcasting Competition and Advertising”, Discussion Paper 00/6, CORE, Université Catholique de Louvain; available at http://server.core.ucl.ac.be/services/psfiles/dp00/dp2000-6.pdf.

Gallet, C.A., 1999, “The Effect of the 1971 Advertising Ban on Behavior in the Cigarette Industry”, *Managerial and Decision Economics* **20**, 299-303.

Gelman, J. R. and S. C. Salop, 1983, "Judo Economics: Capacity Limitation and Coupon Competition", *Bell Journal of Economics* **14**, 315-325.

Grossman, G.M. and C. Shapiro, 1984, "Informative Advertising with Differentiated Products", *Review of Economic Studies* **51**, 63-81.

Hotelling, H. 1929, "The Stability of Competition", *Economic Journal* **39**, 41-57.

Kreps, D.M. and J.A. Scheinkman, 1983, "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes", *Bell Journal of Economics* **14**, 326-337.

Leahy, A.S., 1991, "The Effect of Television Advertising on Prices", *Advances in Econometrics, Vol. 9: Econometric Methods and Models for Industrial Organization*, G.F. Rhodes, ed., Greenwich, CT: JAI Press, 255-265.

LeBlanc, G., 1998, "Informative Advertising Competition", *Journal of Industrial Economics* **46**, 63-77.

McAfee, R.P., 1994, "Endogenous Availability, Cartels, and Merger in an Equilibrium Price Dispersion", *Journal of Economic Theory* **62**, 24-47.

Masson, R.T., R. Mudambi, and R.J. Reynolds (1990), ”Oligopoly in Advertiser-Supported Media”, *Quarterly Review of Economics and Business* **30**, 3-16.

Motta, M. and M. Polo, 1997, "Beyond the Spectrum Constraint: Concentration and Entry in the Broadcasting Industry", manuscript, Universitat Pompeu Fabra and Universita' Bocconi; available at http://www.iue.it/Personal/Motta/broadcasting.pdf.

Nilssen, T. and L. Sørgard, 2000, "TV Advertising, Programming Investments, and Product-Market Oligopoly", manuscript, University of Oslo and Norwegian School of Economics and Business Administration; available at http://www.uio.no/~toreni/TVQual.pdf.

Stahl, D.O., 1994, "Oligopolistic Pricing and Advertising", *Journal of Economic Theory* **64**, 162-177.

Tirole, J., 1988, *The Theory of Industrial Organization*, Cambridge, MA: MIT Press.