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Indexing of Technical Change in Aggregated Data

Sturla Furunes Kvamsdal

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by

Sturla Furunes Kvamsdal

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Indexing of Technical Change in Aggregated Data

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Abstract

The Baltagi-Griffin general index of technical change for panel data has earlier been applied to aggregated data via the use of period dummy variables. Period dummies force modeling into estimation of the latent level of technology through choice of dummy structure. Period dummies also do not exploit the full information set because the order of observations within periods is ignored. To resolve these problems, I suggest estimating the empirical equation for all possible structures of the dummy variables. The average over the different estimates provides an index of technical change. I demonstrate the method with both simulated and real data.

Keywords

Technical change; Baltagi-Griffin general index; period dummies

JEL-classification

C13;C43;O33

Introduction

Economists hold technological progress to be an important source of growth, but its latent nature makes measurement difficult. Measurement is, however, a natural first step towards an understanding of the role of progress in growth beyond the normative. Much of the empirical literature focuses on the estimation of technical change in industry panels or cross sections. In a number of situations, however, only aggregated, industry-wide data are available. Examples are historical data, data from poorly monitored or informal industries, and data from developing countries. Faced with such data, economists can try to apply methods like state space modelling or nonparametric estimation. Alternatively, economists can turn to crude but simpler measures. One such crude measure is to introduce time period dummies into a regression of output on inputs to estimate an index of technical change. Period dummies have a number of intuitive and methodological issues. Perhaps the most striking issue is that the estimated index is a step function with a relative coarse resolution compared to the observation frequency. In most cases, estimates at the observation frequency are desirable. Further, a regression with period dummies is not information efficient and serial correlation is almost certain to occur. As it turns out, most issues with period dummies can be resolved with a quite simple procedure.

When introducing time period dummies into a regression of output on inputs, some choices have to be made. One is of period length and whether all periods should be of equal length (possibly except for a last, residual period). Periods of various lengths would require a fair amount of motivation and I will not consider various period lengths here. Once the period length, or analogously the number of periods, is decided, one is presumably forced to commit to a given structure of the period dummy variables. Embodied in this dummy structure are arbitrary period shifts decided *ad hoc* by the period length. The problem of arbitrary shifts is limited in that one should not interpret the step function literally, but remain because estimates are invariable to the order of observations within periods. That is, time period dummies do not exploit the full information set. Further, estimates of input coefficients are sensitive to the idiosyncratic choice of period length, and finally, period dummy regressions tend to struggle with serial correlation (Hannesson *et al*. 2010).

The procedure I suggest consist of repeated estimations of the empirical equation, where the period shifts and hence the dummy variables are shifted one observation at the time. If, say, the period length is l such that each dummy variable covers l observations (with a potentially shorter residual period), one needs to shift the dummy variables l times before they have cycled through all possible configurations. For each observation, one then has l

equally relevant estimates of the level of technology. The average over the l different estimates provides an index of technical change resolved at the observation frequency. The averaged index exploits the full information set in the sense that it is sensitive to the order of observations. In comparison, each of the l different period dummy estimates are invariable to the order of observations within periods of the given dummy structure. Further, the averaged index improves goodness of fit and reduces serial correlation. In some examples, all traces of serial correlation are removed with a careful choice of the period length.

The key point is that with the average index, one is not forced to commit to any given period dummy structure. Rather, all possible, and at least *a priori* equally relevant, period dummy structures are invoked to avoid influence from *ad hoc* period shifts.

Hannesson *et al.* (2010) studied technological change in the Norwegian Lofoten cod fishery with time series on inputs (effort and stock levels) and output (catch). The data contained only aggregated, industry-wide data, and rather than pursuing advanced methods and a more demanding analysis, for example in state space (Harvey *et al.* 1986), they introduced time period dummies and ran ordinary least squares. They had in mind the general index approach of Baltagi and Griffin (1988) and related work, but without panel or cross-sectional data. However, their period dummies essentially generated an artificial panel structure in the data. The estimated index became a step function with a coarse resolution relative to the observation frequency, while a finer resolution was desirable (Hannesson *et al.* 2010, p. 757). (Obviously, they correctly interpreted their estimates as period averages, and insisting on the step function is admittedly pedantic, but is nevertheless what they estimated. On another note, they undoubtedly considered other options and probably chose period dummies because their relative ease of implementation compensated for the eventual loss in methodological sophistication and, one may speculate, the additional insight gained.)

Measurement of productivity and efficiency more generally is a long-standing topic in economics, and a plethora of methods and ideas have been explored. An early impulse to the literature was the seminal contribution by Solow (1957), who conceived of the notion of measure (shifts in the production function) pursued in much subsequent work; a notion that also lie at the heart of the approach I pursue here. While I cannot provide a full overview of the literature, I will mention a few interesting contributions. On a general level, Grilliches (1995) provide an insightful discussion on *inter alia* separability of production functions, relevance of data and models, and the link between public policy influence on research and development and the importance of economic and empirical understanding. Dorfman and Koop (2005) and related papers – their paper introduces a special issue of *Journal of*

Econometrics – draw up what may still be perceived as approximately the research front. Focusing on panel data, Stern (2004; 2005) discusses a number of different empirical methods. The unobservable nature of technical change invites state space approaches, and a number of studies have followed the lead of Harvey *et al.* (1986). An application to panel data is Slade (1989). State space models are now mostly applied in macroeconomics (see, for example, Fuentes and Morales 2011).

After Hannesson (1983) laid out the bioeconomic production function in fisheries economics and subsequent work by Squires (1992; 1994), Kirkley *et al.* (1995), and others, technical change in fisheries and other renewable industries has attracted increasing interest (see, for example, Jin *et al.* 2002, Fox *et al.* 2003, Kirkley *et al.* 2004, Hannesson 2007). Nevertheless, one may still argue that the topic has gained too little attention in the resource economics literature, in particular given its key role in growth (Squires 2009; Squires and Vestergaard 2013).

Method

I will use the model in Hannesson *et al.* (2010) as a starting point for my methodological discussion, in part because it was the inspiration for this work, but also because it is the only recent application of time period dummies to estimate technical change that I am aware of. Harvey *et al.* (1986) mention earlier uses in macroeconomic models.

The empirical equation in Hannesson *et al.* (2010, p. 756) can be written as follows:

$$\ln Y_t = \ln A + \ln F(X_t) + \sum_{i=1}^{T-1} \beta_i d_i + e_t \qquad (1)$$

In (1), Y_t is output and $F(X_t)$ is a Cobb-Douglas function of the vector of inputs X_t. There are T periods and $T-1$ period dummies of equal length. The intercept ($\ln A$) estimate the technology level in the residual period, and otherwise the estimate for period i is $\ln A + \beta_i$. The period dummies essentially generate an artificial panel structure in the data and the coefficients represent period averages. The period averages are independent of the order of observations within given periods, and as such (1) does not utilize the full information set. Furthermore, period shifts are decided *ad hoc* by the period length. Alternatively, I suggest considering the ensemble of all possible dummy configurations and average estimates across them.

The ensemble of all possible dummy configurations is generated as follows. Take one feasible configuration of period dummies. Shift all dummy variables in one direction or the other, for example such that the dummy variable that covered observations i through j now covers observations $i+1$ through $j+1$. The shifted dummy variables constitute a new configuration of the dummy variables. Repeat the procedure until all possible configurations are obtained (for a period length of l observations, there are l different configurations). To be specific, a feasible configuration here means the following: (i) All periods are of equal length, with exceptions for truncated periods at both ends of the time series. (ii) Every observation belong to exactly one period. (iii) Periods have no holes and cover subsequent observations.

The ensemble of different dummy variable configurations and the procedure to obtain them are perhaps best illustrated with an example. Let the period length be 3 such that each dummy variable covers 3 observations. (A period length of 3 is perhaps short for a real application, but suffices for illustration.) There are three different possible dummy configurations ($D_i,\ i = 1 \ldots 3$). In matrix representation, the different dummy variable configurations look like the following:

$$D_1 = \begin{bmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 1 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \cdots \\ 0 & 0 & 1 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ & \vdots & & \ddots \end{bmatrix} \quad D_2 = \begin{bmatrix} 1 & 0 & 0 & \\ 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 1 & 0 & \\ 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \\ 0 & 0 & 1 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & \\ & \vdots & & \ddots \end{bmatrix} \quad D_3 = \begin{bmatrix} 1 & 0 & 0 & \\ 1 & 0 & 0 & \\ 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 1 & 0 & \cdots \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 1 & \\ 0 & 0 & 1 & \\ & \vdots & & \ddots \end{bmatrix} \qquad (2)$$

D_1 has period shifts between observations 1 and 2, 4 and 5, etc., D_2 has shifts between observations 2 and 3, 5 and 6, etc., and D_3 has shifts between observations 3 and 4, 6 and 7, etc. The three configurations are exhaustive in the sense that all possible period shifts are represented. (Note that the number of necessary periods required to cover all observations depend on the period length and the number of observations. For example, if the number of observations is divisible by 3 for the configurations in (2), D_3 need one less period than D_1 and D_2 to cover all observations.) The configuration that perhaps comes natural to mind is the third configuration in (2), with the initial period of equal length as other periods, and is indeed the type of configuration used in Hannesson *et al.* (2010) (notably with a longer period length).

As explained above, the estimated dummy coefficient for a given period represent a period average. When an equation like (1) is estimated three times (in the case with period length of three), each time with a different dummy configuration, the average dummy coefficient estimate over configurations will generally differ for all observations. (Note that one dummy variable should be left out in each regression to avoid the dummy variable trap.) For observation number four, for example, the average estimate will be an average of three averages: the average for observations 2, 3, and 4 (for D_1), the average for 3, 4, and 5 (for D_2), and the average for 4, 5, and 6 (for D_3). No other average coefficient will consist of these three averages, although two of them will contribute to the two neighboring average coefficients. In a way, the average dummy index is a rolling window smoother with period length as window size. But one should note that estimates for additional parameters (in $F(X_t)$ in (1), for example) will generally differ with different dummy structures, and as such, the average dummy index is something more than a simple moving average.

More generally, the estimation problem can be written as follows:

$$Y = [X \quad D_j]\beta_j + \epsilon \tag{3}$$

where Y is a vector of n observations for the dependent variable, X is a matrix of corresponding observations of m independent variables (including a constant), D_j is a matrix of dummy variables as discussed above, β is a vector of parameters, and ϵ is a vector of random errors. Given that all the usual assumptions hold, the ordinary least squares estimate of β is relevant and given by

$$\widehat{\beta_J} = \left(\begin{bmatrix} X^T \\ D_j^T \end{bmatrix} \quad [X \quad D_j]\right)^{-1} \begin{bmatrix} X^T \\ D_j^T \end{bmatrix} Y = \begin{bmatrix} X^T X & X^T D_j \\ D_j^T X & D_j^T D_j \end{bmatrix}^{-1} \begin{bmatrix} X^T Y \\ D_j^T Y \end{bmatrix} \tag{4}$$

The inverse can be further expanded (see Lu and Shiou 2002 for a general treatment) and explicit expressions for the different elements of $\widehat{\beta_J}$ can be obtained. $\widehat{\beta_J}$ has two types of elements: m elements corresponding to X and elements corresponding to D_j (as discussed above, the number of necessary period dummies to cover all observations differ across specifications, and consequently so will the number of elements in $\widehat{\beta_J}$ corresponding to D_j). Let $\widehat{\beta_{X,J}}$ denote the first type and $\widehat{\beta_{D,J}}$ the second (see appendix for explicit expressions). Thus, $\widehat{\beta_J} = \left[\widehat{\beta_{X,J}} \quad \widehat{\beta_{D,J}}\right]^T$. For the first type of element, the average estimate over the different specifications is straight forward:

$$\widehat{\beta_X} = \frac{1}{l}\sum_{j=1}^{l} \widehat{\beta_{X,J}} \tag{5}$$

Notably, the intercept estimate in (4) correspond to the omitted dummy in D_j, and the omitted dummy correspond to different observations depending on j. Thus, the average intercept estimate – a part of (5) – is not of interest here. Intercept estimates will rather enter in the average dummy coefficients below.

The average estimates corresponding to the dummy variables need to be treated a little different because $\widehat{\beta_D}$ has one element for each observation while $\widehat{\beta_{D,J}}$ has one element for each dummy variable. One also need to take care of the omitted dummy variable. (The construction below is admittedly somewhat cumbersome. I provide a small example in the appendix that may promote an understanding of the construction.) One way to define $\widehat{\beta_D}$ is as follows: Let D_j^* be the full representation of dummy variables for specification j, that is, including the variable omitted from D_j. The relevant estimate for the omitted variable in D_j is the intercept variable in $\widehat{\beta_{X,J}}$, which I will denote $\widehat{\beta_{X,J}^*}$. Further, let $\widehat{\beta_{D,J}^*}$ be identical to $\widehat{\beta_{D,J}}$ but with a zero element at the position corresponding to the omitted dummy in D_j. For example, if the last dummy was omitted from D_j, let $\widehat{\beta_{D,J}^*} = \left[\widehat{\beta_{D,J}} \quad 0\right]^T$. That is, a zero element is added at the end of the vector $\widehat{\beta_{D,J}}$. The expression $D_j^* \times (\widehat{\beta_{D,J}^*} + \widehat{\beta_{X,J}^*} \times 1_n)$, where 1_n is a n-vector of ones, is then an n-vector with the relevant estimate for observation i in position i for dummy configuration j. The average estimate over the different configurations is now a simple mean of these vectors for different j:

$$\widehat{\beta_D} = \frac{1}{l}\sum_{j=1}^{l} D_j^* \times (\widehat{\beta_{D,J}^*} + \widehat{\beta_{X,J}^*} \times 1_n) \tag{6}$$

Asymptotics of $\widehat{\beta_J}$ carry over in the linear combinations in (5) and (6), and $\widehat{\beta_J}$ can be assumed to be normal distributed with mean $\widehat{\beta_J}$ and variance equal to the average variance.

To apply the method outlined above, one need to decide on the period length. A long period length leads to a smoother trend, while a short period length will provide a closer fit (smaller root mean squared errors and coefficient of determination). Changing period length may further influence both coefficient estimates ($\widehat{\beta_X}$) and serial correlation; Hannesson *et al.* (2010) took note of both effects, for example. One idea is to consider a criteria like the Akaike Information Criteria (AIC) for different period lengths, but statistics of fit and serial correlation should also be consulted. In all examples below, AIC and statistics of fit improve with shorter period length, while the Durbin-Watson statistic for serial correlation increase. One could perhaps entertain the idea to average across period lengths. But again, certain specifications may suffer from severe serial correlation and should likely not be included in such an average. Another idea is to set up a bootstrap-like approach where period lengths are sampled at random for a given specification. Averaging over many such specifications will make the results independent of period lengths. However, given the simple methodology above, to estimate the system for different period lengths is easy, and then consider for example the trade-off between fit and serial correlation, and decide on an appropriate period length. I demonstrate this procedure in examples below.

When compared with estimates from a single set of period dummies, the ensemble average has a number of advantages. First, there are no *ad hoc* period shifts, and the ensemble averages are fully sensitive to the order of observations; they use the full information set. The underutilization of the information set for a given, individual dummy specification justifies the repeated estimations and as such repeated usage of observations. The ensemble averages provide a trend estimate at the observation frequency, which is more intuitively appealing, and more readily interpreted, than period averages, and estimates at the observation frequency improve goodness of fit. Estimates at the observation frequency also facilitates hypothesis testing of the type: Did the event in a given year (if observations are yearly) impact the underlying trend? Period averages can generally not answer such questions. Finally, when an actual trend is represented by a mean over a number of observations, errors will be serially correlated. With estimates at the observation frequency, error serial correlation is much less likely. As one of the empirical examples below shows, using ensemble averages has much the same effect on the Durbin-Watson statistic as the Prais-Winston procedure that was used to deal with serial correlation in Hannesson *et al.* (2010).

The approach above is a simple solution to a difficult problem. Hannesson *et al.* (2010) turned to Baltagi and Griffin (1988) with a desire for model free estimates of technical change. The ingenious appropriation of period dummy variables takes one a long way, and indeed provides an appropriate description of the long run development. Description of short run dynamics is on the other hand not provided. Further, period length compromises independence from modeling. Short run development can be described by considering all possible dummy configurations, and the consequences of period length better understood. Nevertheless, more comprehensive approaches like state-space methods (Harvey 1989) or nonparametric regression may be called for. Indeed, in the absence of independent variables (X), the approach above is simply a moving average, and ultimately a special case of a locally weighted regression (Cleveland 1979) with linearly declining weights and regression polynomial of order zero. I still find the approach above worthwhile to consider because of its simplicity and its close connection to standard regressions.

Examples

To illustrate the methodology, I provide four examples: two twin experiments with simulated observations from a known process, and two empirical examples. In the first, I sample from a simple, nonlinear trend. I compare the average estimates with estimates using only one dummy specification. In the second example, I consider one of the regressions in Hannesson *et al.* (2010), again comparing the average estimates with estimates from a single dummy specification (the single specification is identical to the one used in the original analysis). In the third example, I sample from a stochastic trend. In the final example, I consider one of the estimations in Harvey *et al.* (1986).

I sample $N = 50$ observations with the nonlinear trend

$$x(t) = \left(\frac{t}{N}\right)^2 \tag{7}$$

I sample random errors $e(t)$ from a normal distribution with mean zero and standard deviation $\sigma = 0.05$ and have observations $y(t) = x(t) + e(t)$. The empirical equation is

$$y_t = \alpha + \sum_{i=1}^{T-1} \beta_i d_i + e_t \tag{8}$$

I estimate (8) for a range of period lengths and consider AIC, root mean squared error (RMSE), the difference R-squared (R_D^2, see Harvey 1984), and the Durbin-Watson (DW) statistic. Figure 1 plot these statistics for different period lengths. AIC, RMSE, and R_D^2 all improve with smaller period lengths (AIC and RMSE become unstable at very short period lengths; this behavior should likely be takes as signs of trouble, short period lengths may for example lead to problems with the degrees of freedom, and longer period lengths should be chosen). The DW-statistic has a theoretical value of 2 for a series with no serial correlation; DW is closest to 2 when the period length is $l = 10$.

Table 1 lists parameter estimates and statistics for the natural dummy configuration with all dummies of equal length ($l = 10$). Note that the dummy for the last period is omitted (here and in all subsequent examples). The negative R_D^2 means that the model gives fit worse than would a random walk with drift model and 'should not be seriously entertained' (Harvey 1984, p. 270). Further, the Durbin-Watson statistic suggest rather severe serial correlation problems.

Figure 2 (left panel) compare the predicted trend for the results in table 1 with observations y_t and the underlying trend (x_t). Figure 2 (right panel) make the same comparison for the average predicted trend across all configurations. (In the interest of space, I do not table results for all configurations, neither here nor in subsequent examples.) The root mean squared error (calculated with the average degrees of freedom across configurations) for the average predicted trend is 0.0651, while $R_D^2 = 0.530$ and the Durbin-Watson statistic is

2.05. All statistics are improved from those reported in table 1. In particular, the serial correlation problem is resolved. The average predicted trend (figure 2, right panel) fits quite well to the underlying, true trend. At the end of the time series, the fit deteriorates slightly because the last dummy variable has shorter length in most configurations and the estimate is less representative. This is less of a problem at the beginning of the time series because the underlying trend is relatively flat.

The simulation example shows that averaging over all possible dummy variable configurations improves statistics like root mean squared errors and R^2 and the predicted trend is closer to the underlying, true trend. Further, the predicted trend is resolved at the observation frequency.

As the underlying trend is known (7), I can calculate what I call the root mean squared true error (RMSX), defined as error relative to the underlying trend (rather than error relative to observations as in RMSE). I can also calculate the difference R-squared statistics relative to the underlying (true) trend. These statistics of fit relative to the underlying trend are plotted for different period lengths in figure 3. Both plots suggest that fit to the underlying trend is best with a period length of $l = 7$. This example thus illustrates that both fit and serial correlation should be considered when deciding on the period length, and that the most appropriate period length may need to compromise between fit and serial correlation.

Table 1: Coefficient estimates and statistics for (8) with a single set of dummy variables. See figure 1 for plot of observations.

	Estimate	t-stat
α	0.841	26.4
β_1	-0.795	-17.6
β_2	-0.707	-15.7
β_3	-0.561	-12.4
β_4	-0.356	-7.92
No. obs.		50
DoF		45
RMSE		0.101
R_D^2		-0.143
DW		1.172

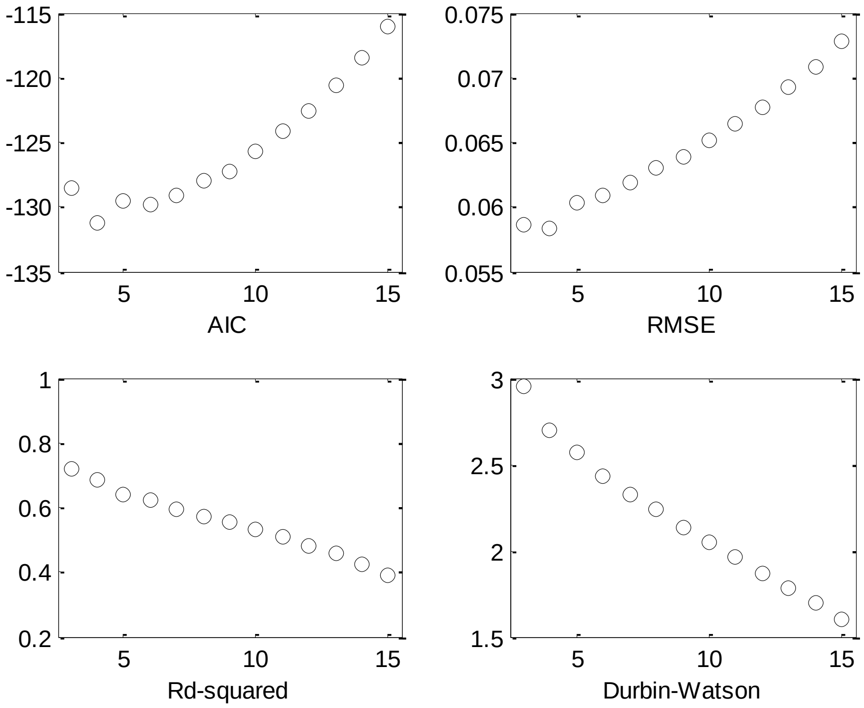

Figure 1: AIC, RMSE, R_D^2, and the Durbin-Watson statistic for different period lengths for estimates of (8).

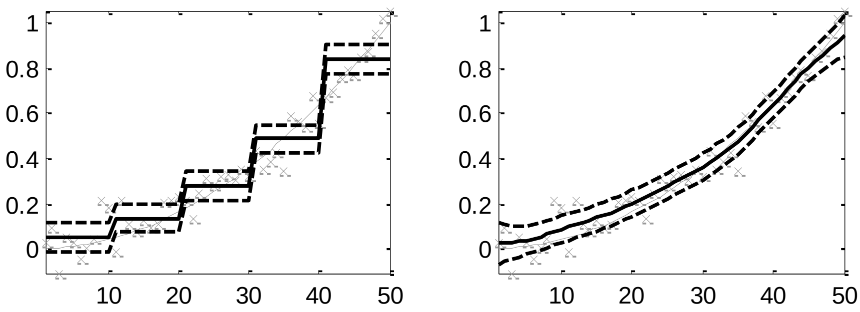

Figure 2: Simulated observations (x-marks), underlying trend (shaded curve), and predicted trend (solid curve) with prediction interval (solid dashed curve) for simulation example (8) with one specification of the dummy variables (left panel) and the average over all specifications (right panel).

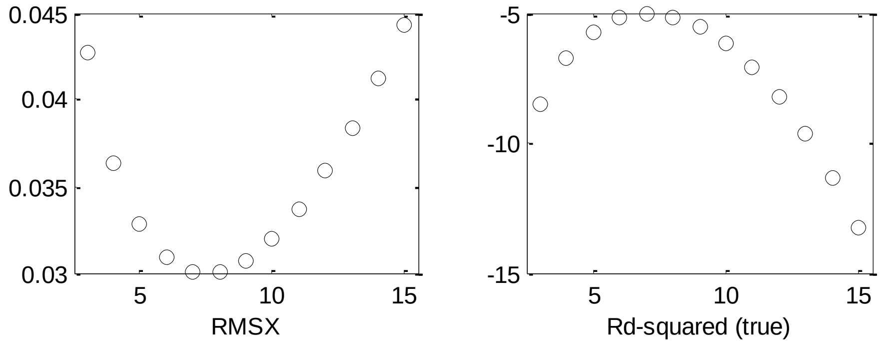

Figure 3: Fit statistics with respect to the underlying (true) trend (7) for estimates of (8) for different period lengths.

Next, I apply the method to one of the regressions[1] in Hannesson *et al.* (2010), which should be consulted for a description of the dataset and further background material. For simplicity, I ignore the Prais-Winston procedure that was applied to deal with serial correlation in the original analysis. The empirical equation is as follows:

$$y_t = \alpha + \beta_e e_t + \beta_s s_t + \sum_{i=1}^{T-1} \beta_i d_i + \epsilon_t \tag{9}$$

y_t is the logarithm of output, α is the intercept, and e_t and s_t are logarithms of inputs with elasticities β_e and β_s. The period length is six years, and the dummy for the last period is excluded from the regression. Table 2 reports results from estimating (5) with the natural dummy configuration. The estimated trend is reported in figure 4.

Figure 4 also reports the trend from averaging over all configurations. The shown standard errors (dashed curves) pertain to the dummy variables to show differences in estimates. Standard errors between the two approaches are comparable, but the average index is reported annually while the single dummy set index reports six year averages. The errors increase toward the end of the time series because the intercept, which represent the residual final period, has large standard errors in the regressions. Average elasticity estimates are, with *t*-statistics in parenthesis, $\widehat{\beta_e} = 0.9496\ (5.1283)$ and $\widehat{\beta_s} = 0.4641\ (3.6656)$. The discrepancy between these estimates and the estimates reported in table 2 explain the difference in trend levels in figure 4, and also illustrate a problem by only considering one of several possible dummy configurations. The RMSE for the average estimation is 0.284, $R_D^2 = 0.644$, and the Durbin-Watson statistic 1.862. All statistics are improved with the

[1] Regression (iii) for gear type gill nets, table 1, p. 756.

average index; the improvement in the Durbin-Watson statistic is nearly identical to the improvement that resulted from the Prais-Winston procedure in the original analysis.

Figure 5 reports AIC, RMSE, R_D^2, and the Durbin-Watson statistic for a range of period lengths. The above reports for a period length of $l = 6$ to compare directly to the original analysis. The pattern of the statistics in figure 5 is similar to the pattern seen in figure 1. The Durbin-Watson statistic suggest that a period length of 5 would eliminate all traces of serial correlation while improving the fit statistics.

Table 2: Coefficient estimates and statistics for (5) with a single set of dummy variables. Dummy variable coeffiecient subscripts denote observation years.

	Estimate	t-stat
α	0.6188	0.5287
β_e	1.0331	5.8481
β_s	0.4224	3.6346
β_{00-05}	-1.5517	-5.0093
β_{06-11}	-1.4070	-4.5665
β_{12-17}	-1.1434	-4.4246
β_{18-23}	-0.6544	-2.6538
β_{24-29}	-0.4840	-1.7397
β_{30-35}	-0.4765	-1.6541
β_{36-41}	-0.3859	-1.3992
β_{42-47}	0.0154	0.0603
β_{48-53}	-0.1520	-0.6295
β_{54-59}	-0.3535	-1.6746
β_{60-65}	-0.3597	-1.6145
β_{66-71}	0.4250	2.0742
β_{72-77}	0.1579	0.7615
β_{78-83}	0.4038	2.0471
No. obs.		89
DoF		72
RMSE		0.321
R_D^2		0.540
DW		1.677

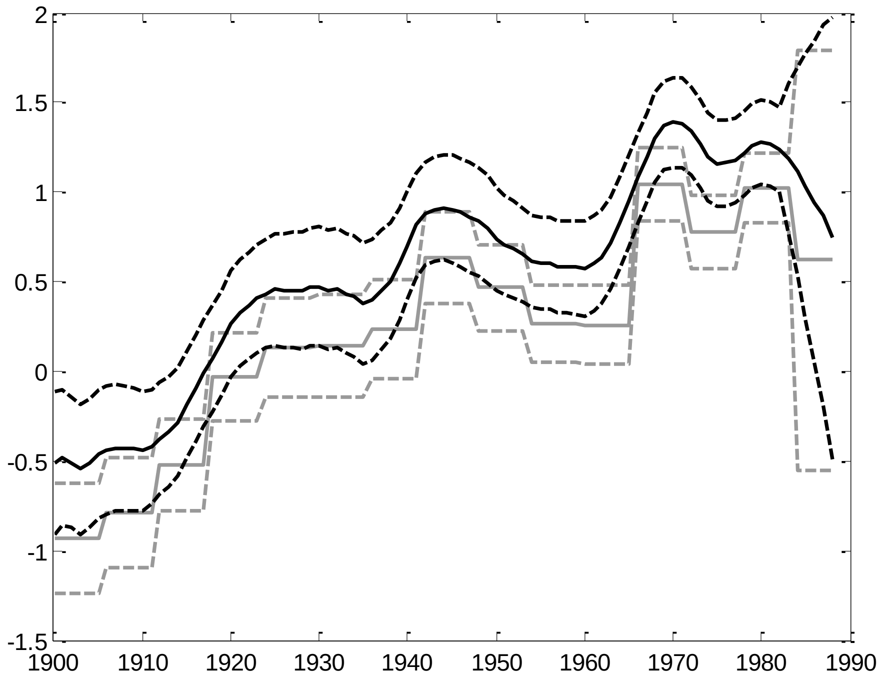

Figure 4: Technology index with single set of dummies (shaded curves) and with full set of dummies (solid curves) for estimates of (9).

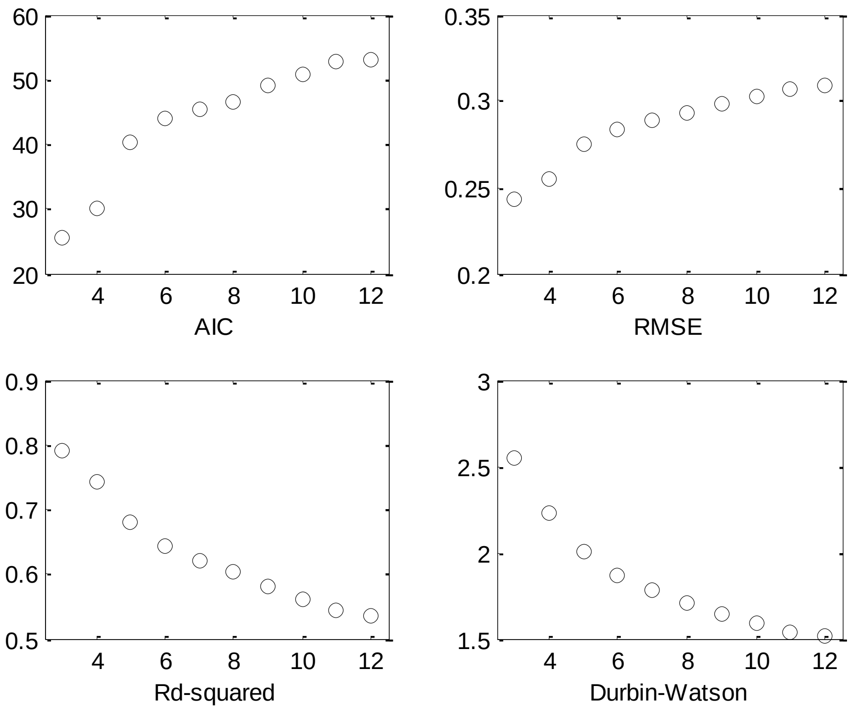

Figure 5: AIC, RMSE, R_D^2, and the Durbin-Watson statistic for different period lengths for estimates of (9).

The next example consider a stochastic trend, following Harvey *et al.* (1986):

$$y_t = x_t + \delta z_t + \epsilon_t \tag{10}$$

y_t are the observations, z_t are observed, independent variables (here, an observed random vector), δ are parameters corresponding to z_t (here $\delta = 5$), and ϵ_t are normally distributed, serially independent disturbance terms with mean zero and constant variance equal to one. x_t is the stochastic trend with slope, γ_t, which evolve slowly over time:

$$\begin{aligned} x_t &= x_{t-1} + \gamma_t + \nu_t \\ \gamma_t &= \gamma_{t-1} + \omega_t \end{aligned} \tag{11}$$

The disturbance terms ν_t and ω_t are both normal and independent with zero means and variances equal to 0.25, and all disturbance terms in the system are independent of each other at all times. Both x_1 and γ_1 are set equal to zero, and 50 observations are simulated.

Figure 6 displays the observations y_t, the underlying trend x_t, and the average estimated trend with standard errors ($l = 6$ was chosen based on the best Durbin-Watson statistic). The observations are scattered substantially away from the trend because of the random vector z_t, but the trend is well estimated because δ is well estimated (estimate is $\hat{\delta} = 4.93$, *t*-statistic is 17.3; *t*-statistic against the true value of 5 is 0.246). RMSE is 1.34, R_D^2 is 0.958, and the Durbin-Watson statistic is 1.90. Notably, the estimated trend is much more smooth than the underlying trend, something that cannot be avoided when the underlying trend (11) has more structure than the empirical equation. A shorter period length could pick up more of this structure, but serial correlation would then have to be dealt with.

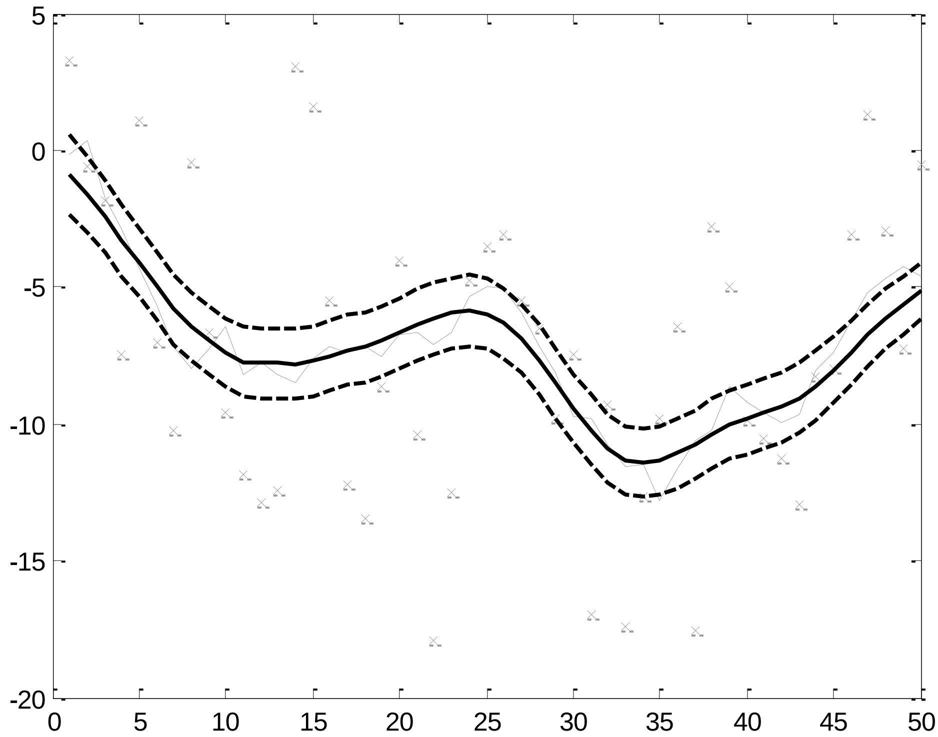

Figure 6: Simulated observations (x-marks), underlying trend (shaded curve), estimated trend (solid curve) with standard errors (dashed curves) for (10) and (11).

Figure 7 displays, similar to figure 3, goodness of fit statistics based on the underlying (true) trend (11). In figure 3, both statistics suggested the same period length as preferable. In figure 7, RMSX (left panel) is smallest at $l = 10$, while the difference R-squared relative to the underlying (true) trend peaks at $l = 4$. As RMSX is relatively flat near $l = 10$, $l = 6$, which corresponds to the best Durbin-Watson statistic, seems like a decent compromise. Plots of the various statistics was omitted, but displays similar patterns of the statistics as seen in figures 1 and 5.

As an aside, I also estimated (10) without the trend component, that is, I simply regressed y on z. The estimated coefficient was 4.92 (t-statistic 10.3), surprisingly similar to the actual value of 5, and indistinguishable from the coefficient estimated with the trend. But both RMSE (3.06) and R_D^2 (0.739) suggest the estimation with trend is better (the statistics for the model with trend was 1.34 and 0.958). Further, the Durbin-Watson statistic (0.363) suggest a substantial serial correlation problem when the trend is excluded.

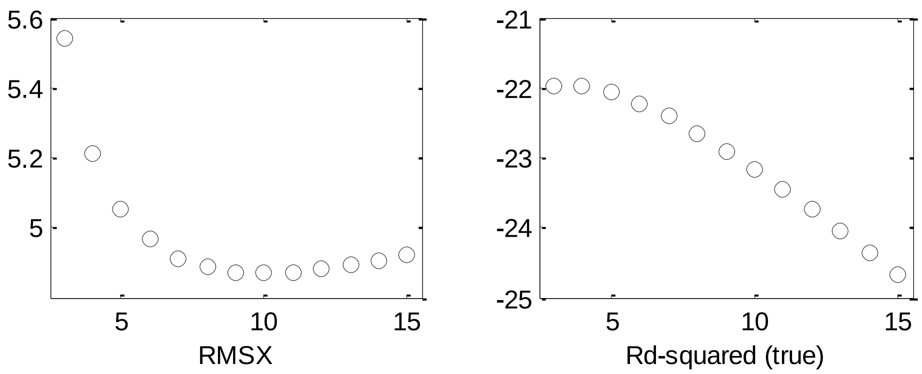

Figure 7: Fit statistics with respect to the underlying (true) trend (11) for estimates of (10) for different period lengths.

My last example considers one of the empirical equations in Harvey *et al.* (1986).[2] The original equation contains a deterministic trend component. This component is left out here and replaced with period dummy variables:

$$n_t = \alpha + \sum_{i=1}^{T-1} \beta_i d_i + \beta_{n,-1} n_{t-1} + \beta_{n,-2} n_{t-2} + \beta_q q_t + \beta_{q.-1} q_{t-1} + \beta_{q,-2} q_{t-2} + \epsilon_t \tag{12}$$

N_t are quarterly observations on employment in UK manufacturing from first quarter, 1963, to third quarter, 1983, while Q_t is an index of output (1980 = 100); variables in (12) are logarithms and denoted in lower case letters. The data was seasonally adjusted. See Harvey *et al.* (1986) for further background material and discussion of the theory behind the employment-output relationship. It should be noted that Harvey *et al.* (1986) had misgivings about the approach embodied in the original equation, in part because of the deterministic trend. Here, the deterministic trend has been replaced.

Figure 8 displays AIC, RMSE, R_D^2, and the Durbin-Watson statistic for estimates of (12) averaged over all dummy variable configurations, for a range of period lengths. The overall pattern of the statistics is similar to patterns in earlier examples. The Durbin-Watson statistic is above 2 for all period lengths, suggesting negative autocorrelation, but is similar for $l = 6$ and higher. As AIC and the goodness of fit statistics suggest smaller is better, I use $l = 6$ here. Table 3 contains results from the average estimation of (12). The reported degrees

[2] Equation (15), p. 981. Data was collected from Harvey (1989).

of freedom is averaged over the different configurations (as discussed above, some configurations require an additional dummy variable to cover all observations).

The estimated coefficients (table 3) are similar to those estimated in Harvey *et al.* (1986). In subsequent analysis, they concluded that the major problem with the deterministic trend equation was that the deterministic trend did not correspond well to the actual, underlying trend, while coefficient estimates was more or less reasonable. I am thus satisfied with the results in table 3. Goodness of fit, for example, is better here than for the estimations in the original analysis.

The estimated trend is shown in figure 9, and is similar in shape to the stochastic trend ultimately estimated by Harvey *et al.* (1986). (The level is different because of a different specification in the ultimate stochastic trend model in the original analysis.) In particular, the crucial feature of substantial changes in the trend in the late 1970's is evident, and agrees with a hypothesis of reduced rate of technical progress after the recession in 1974/5.

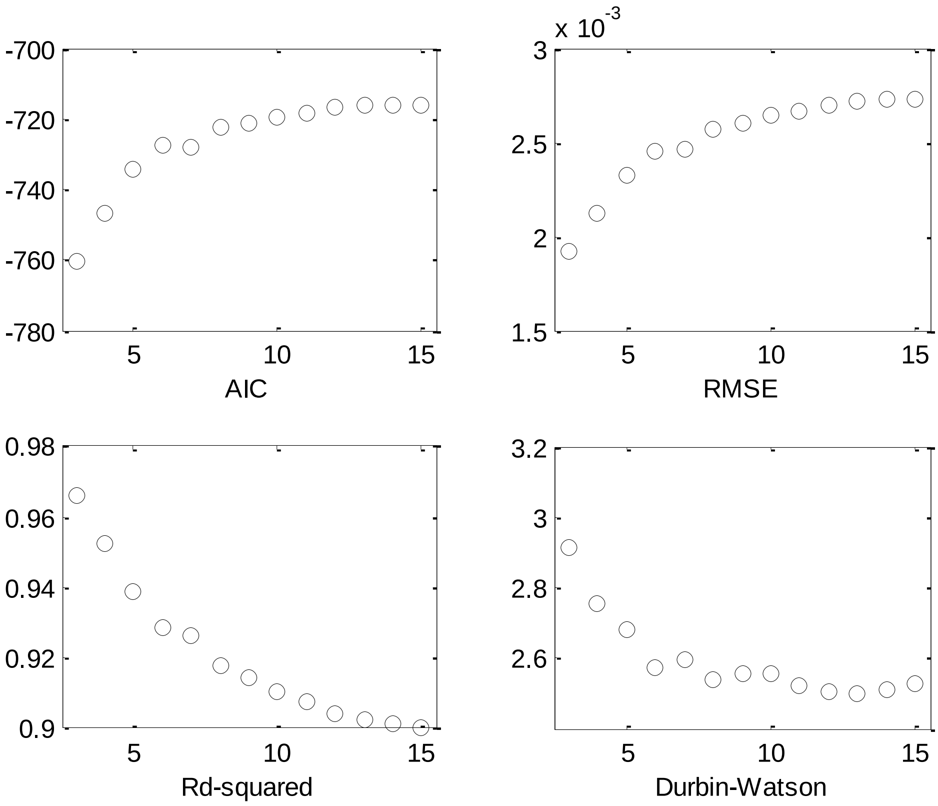

Figure 8: AIC, RMSE, R_D^2, and the Durbin-Watson statistic for different period lengths for estimates of (12).

Table 3: Averaged coefficient estimates and statistics for (12).

	Estimate	t-stat
$\beta_{n,-1}$	1.42	12.6
$\beta_{n,-2}$	-0.467	-4.39
β_{q}	0.105	5.49
$\beta_{q,-1}$	-0.0139	-0.607
$\beta_{q,-2}$	-0.0523	-2.84
No. obs.		81
DoF (avg.)		61.7
RMSE		0.00245
R_D^2		0.929
DW		2.57

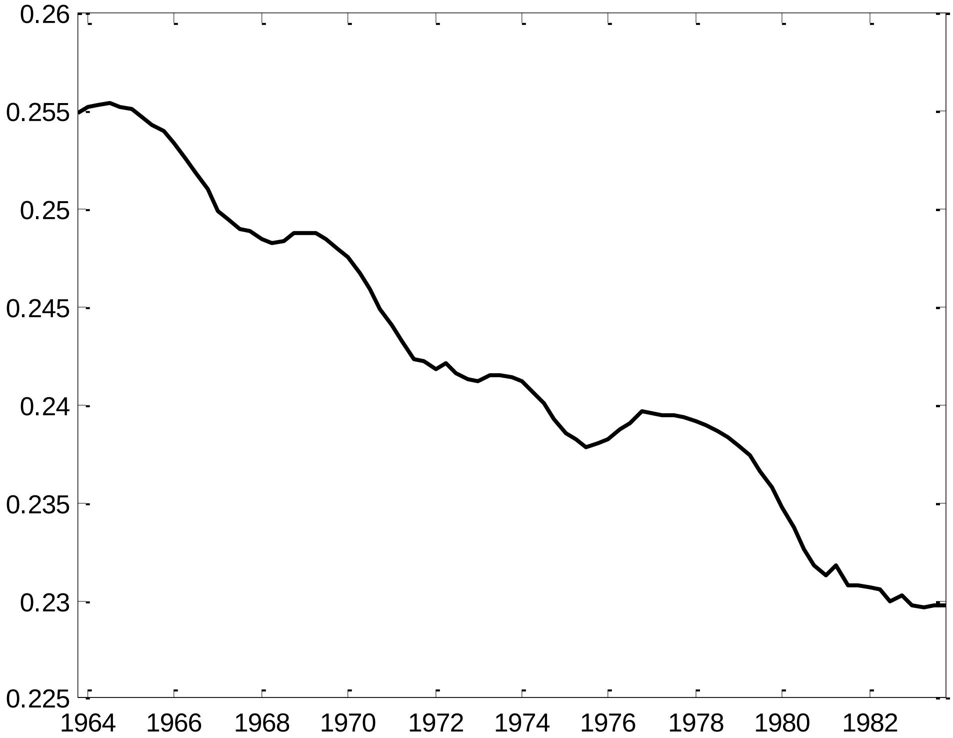

Figure 9: Underlying trend for estimates of (12).

Discussion

The average index of technical change advances the idea to use period dummy variables when aggregated data is all that is available (Hannesson *et al.* 2010), and resolves most issues related to period dummies. For the unresolved issue of period length, unresolved in the sense that one has to use one's judgement and consider the tradeoff between fit and serial correlation, I think the serial correlation problem should carry most weight. As the twin experiments above show, goodness of fit statistics increase steadily as the period length decreases, but fit with the actual, underlying process increases only up to a point. That is, for too short period lengths, noise is mistaken for signal. What constitutes too short is left for judgement, much like grid mesh size in numerical optimization procedures or bandwidth in nonparametric and kernel-based methods often are.

The average index seems to perform well in both the twin experiments and empirical settings above. A setting where it demonstrably does not perform too well is with a discontinuous trend; further unfavorable settings likely exist. In particular, structural information is not recovered with the average index, and results can for example not be used in forecasting.

The average index has here been presented as a method to estimate technical change in aggregated data settings. But the method can estimate any kind of trend without the aid of a model for the trend development. Implementation is easy, as it simply consist of regressions with all possible dummy variable configurations and then averaging across the regression results.

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Appendix

For completeness, I here provide the full expressions for the two types of elements in $\widehat{\beta_\iota}$, see (4):

$$\begin{aligned}\widehat{\beta_{X,J}} &= \left(X^T X - X^T D_j \left(D_j^T D_j\right)^{-1} D_j^T X\right)^{-1} X^T Y \\ &\quad + (X^T X)^{-1} X^T D_j \left(D_j^T X (X^T X)^{-1} X^T D_j - D_j^T D_j\right)^{-1} D_j^T Y \\ \widehat{\beta_{D,J}} &= \left(D_j^T D_j\right)^{-1} D_j^T X \left(X^T D_j \left(D_j^T D_j\right)^{-1} D_j^T X - X^T X\right)^{-1} X^T Y \\ &\quad + \left(D_j^T D_j - D_j^T X (X^T X)^{-1} X^T D_j\right)^{-1} D_j^T Y\end{aligned} \tag{A1}$$

Below, I write out expressions for the average dummy coefficient estimates in a small example with five observations and a period length of two ($n = 5$, $l = 2$). The full representation for one of the dummy variable configuration is:

$$D_1^* = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A2}$$

The last dummy is omitted from the regression. The coefficient matrix extended with a zero at the position of the omitted dummy is then:

$$\widehat{\beta_{D,1}^*} = \begin{bmatrix} \widehat{\beta_{D,1}}(1) \\ \widehat{\beta_{D,1}}(2) \\ 0 \end{bmatrix} \tag{A3}$$

The number in parenthesis simply denotes element number in the vector $\widehat{\beta_{D,1}}$, which full expression is given in (A1). The expression $D_1^* \times \left(\widehat{\beta_{D,1}^*} + \widehat{\beta_{X,1}^*} \times 1_n\right)$ is then:

$$\begin{aligned} D_1^* \times \left(\widehat{\beta_{D,1}^*} + \widehat{\beta_{X,1}^*} \times 1_n\right) &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \left(\begin{bmatrix} \widehat{\beta_{D,1}}(1) \\ \widehat{\beta_{D,1}}(2) \\ 0 \end{bmatrix} + \begin{bmatrix} \widehat{\beta_{X,1}^*} \\ \widehat{\beta_{X,1}^*} \\ \widehat{\beta_{X,1}^*} \end{bmatrix} \right) \\ &= \begin{bmatrix} \widehat{\beta_{D,1}}(1) + \widehat{\beta_{X,1}^*} \\ \widehat{\beta_{D,1}}(1) + \widehat{\beta_{X,1}^*} \\ \widehat{\beta_{D,1}}(2) + \widehat{\beta_{X,1}^*} \\ \widehat{\beta_{D,1}}(2) + \widehat{\beta_{X,1}^*} \\ \widehat{\beta_{X,1}^*} \end{bmatrix} \end{aligned} \tag{A4}$$

For the second dummy variable configuration, the corresponding expression becomes:

$$D_2^* \times \left(\widehat{\beta_{D,2}^*} + \widehat{\beta_{X,2}^*} \times 1_n\right) = \begin{bmatrix} \widehat{\beta_{D,2}}(1) + \widehat{\beta_{X,2}^*} \\ \widehat{\beta_{D,2}}(2) + \widehat{\beta_{X,2}^*} \\ \widehat{\beta_{D,2}}(2) + \widehat{\beta_{X,2}^*} \\ \widehat{\beta_{X,2}^*} \\ \widehat{\beta_{X,2}^*} \end{bmatrix} \tag{A5}$$

Ultimately, the average over the two configurations – see (6) – become:

$$\widehat{\beta_D} = {}^1\!/_2 \sum_{j=1}^{2} D_j^* \times (\widehat{\beta_{D,J}^*} + \widehat{\beta_{X,J}^*} \times 1_n)$$

$$= \begin{bmatrix} {}^1\!/_2 \left(\widehat{\beta_{D,1}}(1) + \widehat{\beta_{X,1}^*}\right) + {}^1\!/_2 \left(\widehat{\beta_{D,2}}(1) + \widehat{\beta_{X,2}^*}\right) \\ {}^1\!/_2 \left(\widehat{\beta_{D,1}}(1) + \widehat{\beta_{X,1}^*}\right) + {}^1\!/_2 \left(\widehat{\beta_{D,2}}(2) + \widehat{\beta_{X,2}^*}\right) \\ {}^1\!/_2 \left(\widehat{\beta_{D,1}}(2) + \widehat{\beta_{X,1}^*}\right) + {}^1\!/_2 \left(\widehat{\beta_{D,2}}(2) + \widehat{\beta_{X,2}^*}\right) \\ {}^1\!/_2 \left(\widehat{\beta_{D,1}}(2) + \widehat{\beta_{X,1}^*}\right) + {}^1\!/_2 \widehat{\beta_{X,2}^*} \\ {}^1\!/_2 \widehat{\beta_{X,1}^*} + {}^1\!/_2 \widehat{\beta_{X,2}^*} \end{bmatrix} \tag{A6}$$

From (A6), it is clear that in general, all elements of $\widehat{\beta_D}$ differ.

The Baltagi-Griffin general index of technical change for panel data has earlier been applied to aggregated data via the use of period dummy variables. Period dummies force modeling into estimation of the latent level of technology through choice of dummy structure. Period dummies also do not exploit the full information set because the order of observations within periods is ignored. To resolve these problems, I suggest estimating the empirical equation for all possible structures of the dummy variables. The average over the different estimates provides an index of technical change. I demonstrate the method with both simulated and real data.

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