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**Cost Allocation and Pricing in
a Supply Chain
An Application for Aumann-Shapley Prices**

by

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Cost Allocation and Pricing in a Supply Chain

An Application of Aumann-Shapley Prices

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We consider the problem of choosing among different distribution channels for combinations of different products, and how to price, or share the costs under the various alternatives, in an efficient and fair way. The problem could also be interpreted in terms of producing different products in a joint production process, and choosing between technologies with different costs and cost structures. More specifically, we consider technologies with combinations of fixed and variable costs. The variable costs are assumed to be linear and separable in the products, i.e. for a given technology and product type, we have constant marginal costs. The optimal choice of distribution channel / production technology will depend on the total production plan, or demand. That is, both the level of the total quantities demanded, and also the relative shares of the demands for the different products influence what is the best solution.

In a marginal cost pricing regime, this would lead to prices changing according to production level and product mix. The price changes would be abrupt, depending on the boundaries between the areas where the different production technologies dominate. As a function of output, the marginal cost prices may show large increments or decrements depending on which production or distribution technology is the best for the given product mix. In this setting we will consider cost sharing rules using game theoretic concepts. More specifically, we consider Aumann-Shapley prices, which can be interpreted as a natural extension of average cost prices to the case of joint production of several goods. Throughout, we illustrate the pricing rules in a small example, with two products, and several technologies to produce or distribute them.

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1. Introduction

In microeconomic theory and welfare economics the matter of interest is the optimal allocation of resources. In the process of finding the optimal social surplus or profit maximum for a price-taking firm, we are familiar with the marginal cost pricing principle. Moreover, in order to attain Pareto optimum through a competitive equilibrium in the economy, the usefulness of the competitive prices to evaluate the desirability of certain products or activities is well known (see for instance Intriligator (1971)). The standard analyses assume strictly convex preference and production possibility sets, and result in uniquely defined linear prices. The pricing problems however, become much more complex when there are non-convexities in the economy, like for instance fixed charges, start-up costs etc. The problems this poses for economic analyses and the optimality of decentralized decision making are described in Scarf (1994). In general, non-convexities may require non-linear pricing schemes.

In spite of the theoretical successfulness of the marginal cost pricing principle, some variant of full cost based prices is often adopted in practical pricing situations (see for instance Zimmermann (1979), or more recently, Balakrishnan and Sivaramakrishnan (2002)). It is observed that companies do allocate fixed costs to products, as a product's full cost typically consists of its variable production cost plus an allocated amount of the company's fixed overhead. There may be several reasons for this practice (Banker et al. (2002)), including opportunity costs, managerial incentives and capacity choices.

When studying the literature on pricing, a distinction may be drawn between the demand side oriented pricing rules often encountered in the economics literature, taking into account the demand elasticity and setting marginal revenue equal to marginal cost, and the cost oriented pricing procedures that use the product costing methodology and terminology from accounting. In this paper we will be focusing on different forms of cost based prices, including marginal cost prices, full cost prices, and solution concepts from cooperative game theory. The demand side is represented by assuming some fixed quantities to be consumed.

In microeconomic theory and optimization, prices are used for coordination, in the sense that they may be utilized as a communication device in order to arrive at the system optimal allocation. However, prices may also serve other purposes. The prices charged or costs allocated also imply an allocation of revenues or profits, i.e. the pricing mechanism is also a way of allocating costs and benefits between participants in a market or an arrangement. If for instance demand is totally inelastic, any price will result in the same quantity demanded, and thus the same level of efficiency (at least in a static partial

equilibrium sense). However, different price levels will represent different distributions of surplus among the market participants on the production and consumption side.

If we turn to supply chain management (SCM) we may analyze the pricing and cost allocation issue from the perspective that several firms or divisions are to cooperate to yield profits that are greater than the sum of the individual profits. This is at the core of what SCM is really about, and the pricing issue is therefore very important, not only as a coordination device but also as a distribution mechanism. More specifically, it may be profitable to centralize decision-making even in an inter-firm sense, i.e. coordinate the activities of several firms, and not only the activities of divisions or departments within firms, and market complications like non-convexities can make such coordination impossible to implement in a decentralized market based manner, with only a linear price. Even if it is possible, the decision of participating or not in an SCM partnership is not a marginal decision, but rather a discrete one. Thus, the total costs and benefits must be distributed such that it is mutual beneficial for the parties involved, and the pricing and cost allocation procedures must take into account the infra-marginal nature of these issues.

From this point of view, the links to regulation are obvious. Cost allocation is also a major issue in regulated or partly deregulated industries, especially industries that are based on capital-intensive infrastructures like electricity and telecommunications. These industries are often considered to include functions that are natural monopolies, i.e. the average cost per unit of providing a service falls with increases in output, for instance due to large fixed costs. Moreover, these industries are often characterized by irreversible investments, implying problems with stranded costs as well.

The paper has the following structure. In section 2 we refer to an example from Mirman et al. (1985) illustrating that the marginal cost pricing approach may imply cross-subsidies among products. We also provide an example from the Norwegian regulation of Telenor that illustrates the difficulties involved when allocating common fixed cost. Section 3 describes the Aumann-Shapley prices in the case of variable costs only, while section 4 discusses pricing when there are fixed costs as well. In section 5 we compare different pricing rules for an example involving different cost functions with different combinations of variable and fixed costs. Section 6 provides concluding remarks and suggestions for future research.

2. Marginal Cost Pricing and Cross Subsidies

The following example from Mirman et al. (1985) illustrates the possible allocation problems when using marginal cost pricing or direct cost pricing when the cost function is the result of the solution of an optimization prob-

lem, in the example, a linear programming problem. The difficulty arises from that fact that the resulting marginal cost prices may involve a kind of cross-subsidy among products.

Consider products A and B that are produced and sold in quantities x_A and x_B , respectively. The products are processed on two types of machines, machine type 1 that can be operated for 400 minutes, and machine type 2 with a total working capacity of 800 minutes. The production function is such that product A requires 1 minute on machine 1 and 10 minutes on machine 2, while product B needs 2 minutes on machine 1 and 30 minutes on machine 2. The processing costs are for product A 1 \$ on machine 1 and 10 \$ on machine 2, and for product B 3 \$ on machine 1 and 25 \$ on machine 2. If we let y_{ij} be the quantity of product i processed on machine j , the cost function F , can be stated as:

$$F(x_A, x_B) = \min y_{A1} + 10y_{A2} + 3y_{B1} + 25y_{B2} \quad (1)$$

$$\text{s.t } y_{A1} + y_{A2} \geq x_A \quad (2)$$

$$y_{B1} + y_{B2} \geq x_B \quad (3)$$

$$y_{A1} + 2y_{B1} \leq 400 \quad (4)$$

$$10y_{A2} + 30y_{B2} \leq 800 \quad (5)$$

$$y_{ij} \geq 0 \quad (6)$$

The processing costs are minimized under the constraints that the quantity produced of products A and B must be at least equal to the quantities sold ((2) and (3)), and the working time restrictions on machine types 1 and 2 ((4) and (5)) must be fulfilled. If we consider $x_A = 40$ and $x_B = 200$, the optimal solution to the min cost problem is $y_{A2} = 40$, $y_{B1} = 200$ and $y_{A1} = y_{B2} = 0$. The minimal cost is $F(40,200) = 40 \cdot 10 + 200 \cdot 3 = 1000$. This suggests a marginal or direct cost of 10 \$ for each A (total 400) and 3 \$ for each B (total 600). Does this reflect the real contributions of products A and B?

Observe that product A can be processed faster and cheaper on any machine than product B. However, the penalty of not assigning product B to machine 1 is much higher than the penalty of not assigning product A to machine 1. This means that it is the existence of the joint product B that makes it more expensive to produce product A, and the allocation of (10, 3) as product cost, though it is the direct cost of producing the products in the present production plan, seems highly unreasonable. An alternative, which we will pursue further in this paper, is the Aumann-Shapley prices, which would give the price vector $(20/11, 51/11) \approx (1.818, 4.636)$, and a total allocation of costs to the products equal to (72.7, 927.3). These prices result from considering all product costs of the form $F(t40, t200)$ for $t \in [0,1]$.

The other example is from the Norwegian regulation of the telecommunication sector. The pricing procedures in regulated industries vary, however, in the telecommunication industry a certain degree of standardization is due to the EU directive (Directive 2002/19/EC) on access to and interconnection of electronic communications networks and services. The directive emphasizes cost oriented prices in order to promote efficiency, sustainable competition and maximum benefits to end-users, and activity based costing (ABC) is recommended by EU and implemented in the Norwegian regulation of the former monopolist, Telenor ASA. Bjørnenak and Fjell (2004) argue that ABC may not be appropriate as a basis for pricing leased lines and other services provided by the existing infrastructure. ABC is based on assumptions of separability, linearity and homogeneity, and especially the separability requirement seems to be a major problem. This implies that by using a fully distributed cost principle for product costing, there will be some arbitrary allocation of common cost at the product level.

In 1991, the private entrant NetCom GSM AS was licensed to build and operate a GSM mobile phone network, linked to the ground based network of Telenor, especially through the use of leased lines. The pricing of the regulated services, including leased lines, were based on estimated costs and volumes (ex ante), however, volumes increased rapidly in the period 1993-1996, leaving the regulated services very profitable (ex post). In 1998, NetCom sued Telenor for overpricing of digital leased lines, demanding a repayment of NOK 97 mill. An interesting issue in this case was that Telenor claimed that the volume increase was mainly due to other Telenor-products and not the leased lines. The question raised to the court was then really how to distribute the benefits of the volume increase among the different products, both those experiencing the volume increase and those with unchanged volume. NetCom was awarded NOK 51 mill in Oslo City Court and settled for NOK 35 mill before the appeal.

3. Aumann-Shapley Prices

The Aumann-Shapley prices can be interpreted as a generalized average cost. In Billera et al. (1981) and Mirman and Tauman (1981) an axiomatic approach is adopted, where prices are required to fulfil a number of axioms, and it is shown that the resulting price vector must be the Aumann-Shapley prices.

In order to give some intuition, consider first the case of variable costs only, i.e. $F(0, 0, \dots, 0) = 0$. The purpose of the axiomatic approach is to define the average cost of each output in the general case where F is not necessarily separable. This means that we want to find for each non-negative output vec-

for \mathbf{x} , a price vector $AC(F, \mathbf{x}) = [AC_1(F, \mathbf{x}), \dots, AC_m(F, \mathbf{x})]$ with the following requirements:

1. *Cost sharing.*

For each output vector \mathbf{x} , the average cost covers production costs, i.e. $\mathbf{x}_1 AC_1(F, \mathbf{x}) + \mathbf{x}_2 AC_2(F, \mathbf{x}) + \dots + \mathbf{x}_m AC_m(F, \mathbf{x}) = F(\mathbf{x})$.

2. *Additivity.*

If $F(\mathbf{x}) = F_1(\mathbf{x}) + F_2(\mathbf{x})$, then $AC_j(F, \mathbf{x}) = AC_j(F_1, \mathbf{x}) + AC_j(F_2, \mathbf{x})$.

3. *Positivity.*

If increasing the production increases costs, then $AC_j(F, \mathbf{x}) \geq 0 \forall j$.

4. *Rescaling.*

The average cost is independent of the units of measurement, i.e. if $G(\mathbf{x}) = F(\lambda_1 \mathbf{x}_1, \lambda_2 \mathbf{x}_2, \dots, \lambda_m \mathbf{x}_m)$, then $AC_j(G, \mathbf{x}) = \lambda_j AC_j[F, (\lambda_1 \mathbf{x}_1, \lambda_2 \mathbf{x}_2, \dots, \lambda_m \mathbf{x}_m)] \forall j$.

5. *Consistency.*

If all goods are the same, they should have the same price, i.e. if there is a cost function C such that $F(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_m) = C(\mathbf{x}_1 + \mathbf{x}_2 + \dots + \mathbf{x}_m)$, then $AC_1(F, \mathbf{x}) = AC_2(F, \mathbf{x}) = \dots = AC_m(F, \mathbf{x}) = AC(C, \mathbf{x}_1 + \mathbf{x}_2 + \dots + \mathbf{x}_m)$.

It is shown that the expression

$$AC_j(F, \mathbf{x}) = \int_0^1 \frac{\partial F(t\mathbf{x}_1, t\mathbf{x}_2, \dots, t\mathbf{x}_m)}{\partial \mathbf{x}_j} dt \quad (7)$$

gives the only price vector that satisfies the requirements for all continuously differentiable cost functions F with $F(0) = 0$. In the special case of separable cost, the Aumann-Shapley prices are equal to the standard average cost, and with constant returns to scale, they coincide with the marginal cost.

Cooperative game theory provides a number of other solution concepts for cost allocation games, like for instance the Shapley value, the nucleolus or τ -value. However, we have chosen to focus on the Aumann-Shapley prices due to the interpretation as average cost prices, and the widespread use of full cost pricing in practice. In the literature, the Aumann-Shapley values are used as solutions to cost and risk capital allocation problems and also interpreted in the context of equilibria in production economies.

For electricity networks, Wu and Varaiya (1995) suggest a method for pricing marginal losses that bears resemblance with the Aumann-Shapley prices. Marginal losses increase with transmitted energy, so if all trades are priced as marginal ones, i.e. as if they were the last ones added to the load of the grid, there will be a collection of revenue that exceeds the cost of the total losses. In some systems (for instance where the agents are allowed to pay in kind) this is considered undesirable. In the Coordinated Multilateral Trade model suggested by Wu and Varaiya, the trades representing the total load are added to the grid sequentially, and marginal losses are paid for in relation

to total accumulated output in every step. This way, only total losses will be paid for, and moreover, a new trade will face the marginal losses relating to the total present load, so the marginal signals are correct.

4. Pricing with Fixed Costs

The distinction between fixed and variable cost is not a straightforward one, and depends, among other factors, upon the time frame considered. In the long run all or most costs are variable, however at a given point in time, with a specific technology or infrastructure in place, only a small fraction of total cost may be variable, and dependant on the activity level. In regulation this poses a problem together with the fact that there is technological and other developments that make a given investment non-optimal under ex post conditions. This may lead to problems with stranded costs, and the question of how this is to be compensated.

The trade off in regulation is between on the one hand giving the right incentives for operation and investment decisions, and on the other hand providing sufficient revenues for the regulated companies. In this setting the use of long run incremental costs (LRIC) have been considered, for instance in the telecommunication sector (see for instance Bromwich and Hong (2000)). This will compensate the regulated firms for more than the short term variable cost, but will take into account the technological and economical development as the firms are not automatically compensated for historical costs.

In this part of the paper we will focus on the cost allocation problem when there are fixed costs, and investigate how the Aumann-Shapley average costs can be used. First, we will focus on the case where the long run cost function $H(\mathbf{x})$, that is the minimum cost over all possible technologies, do not have a fixed part, while the implied short run technology $G(\mathbf{x})$, i.e. the technology that is used to produce a given output vector has (refer figure 1). So $G(\mathbf{x})$ may be written as $F(\mathbf{x}) + C$, where C is the fixed cost. There may be plausible explanations for such a setting, and one is for instance given in Billera et al. (1981): “However, in many instances, situations which appear to have fixed setup costs in fact do not. Usually, small quantities of goods can be obtained through alternative sources at a low cost. In the situation considered by Billera, Heath and Raanan [1978], for example, the computer-controlled switching device for a WATS line system may seem to reflect the setup cost, but in fact if very little long distance telephone services were required, one could avoid the WATS lines and purchase ordinary long distance (DDD) service instead.” However, though similar alternatives can be relevant in other cases, there may also be long run cost functions with truly fixed parts.

When the long run cost function does not include a fixed part, we can use the Aumann-Shapley prices on the cost function $H(\mathbf{x})$ in order to allocate

costs. If we assume that it is the optimal short run technology that provides the output vector, \mathbf{x}' , considered, we have that $H(\mathbf{x}') = G(\mathbf{x}') = F(\mathbf{x}') + C$ in point \mathbf{x}' , and possibly in a neighbourhood around \mathbf{x}' . In that case, the Aumann-Shapley prices, that allocate the long run cost, also provide an implicit allocation of the fixed and variable costs of the short run cost function $G(\mathbf{x})$. If we let

$$p_j = \int_0^1 \frac{\partial H(tx_1, tx_2, \dots, tx_m)}{\partial x_j} dt \quad (8)$$

and

$$v_j = \int_0^1 \frac{\partial F(tx_1, tx_2, \dots, tx_m)}{\partial x_j} dt \quad (9)$$

then $p_j - v_j$ will be the part of the price p_j which may be thought of as covering the fixed cost C .

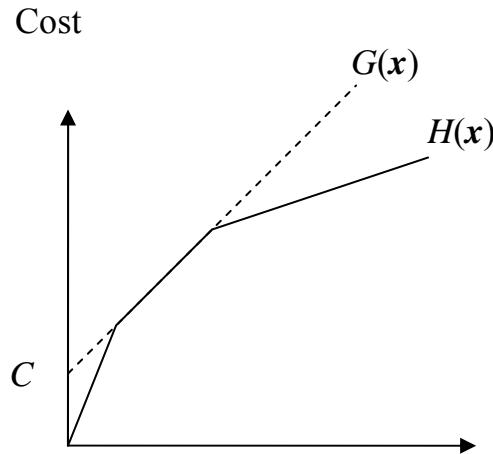


Figure 1. Cost functions

The allocation scheme given by (8) and (9) does not work if 1) the long run cost function has a fixed part, or 2) the optimal short run technology cannot be used for the output quantity considered. According to Mirman and Tauman (1985) it can be shown that every price mechanism that allocates the fixed cost independently of the variable cost will violate either the rescaling or consistency requirement. However, it turns out that it is possible to modify the additivity requirement and to find a cost allocation scheme that satisfies the adjusted set of requirements.

Suppose the cost function is given by $G(\mathbf{x}) = F(\mathbf{x}) + C$, where F is the variable part and C is the fixed part. If the variable part can be written $F(\mathbf{x}) = F_1(\mathbf{x}) + F_2(\mathbf{x})$, then the modified additivity requirement specifies that it is possible to split the fixed cost into two parts C_1 and C_2 such that $C_1 + C_2 = C$,

with the largest portion being attributed to the larger variable cost component, i.e.

$$C_2 \geq C_1 \quad \text{if} \quad F_2(\mathbf{x}) \geq F_1(\mathbf{x}) \quad (10)$$

and such that the average cost of good j should be the sum of the average cost of the parts, i.e.

$$AC_j(F + C, \mathbf{x}) = AC_j(F_1 + C_1, \mathbf{x}) + AC_j(F_2 + C_2, \mathbf{x}) \quad (11)$$

It is shown by Mirman et al. (1983) that the allocation given by

$$AC_j[F + C, \mathbf{x}] = \left[1 + \frac{C}{F(\mathbf{x})} \right] \int_0^1 \frac{\partial F(t\mathbf{x}_1, t\mathbf{x}_2, \dots, t\mathbf{x}_m)}{\partial \mathbf{x}_j} dt \quad (12)$$

is the only one satisfying the adjusted requirement set for all continuously differentiable functions of the form $F(\mathbf{x}) + C$.

5. Pricing in an Example with Fixed Costs

In this section, where we will study the characteristics of different accounting based cost concepts in relation to the Aumann-Shapley prices, we will concentrate on an example where the long run cost function has no fixed part, and where we assume that it is in fact possible to choose the optimal short run technology for the output vector considered. The example can be given different interpretations, for instance the one of choosing among different distribution channels with different cost structures, i.e. combinations of fixed and variable costs. An alternative interpretation is that of producing different products in a joint production process, and choosing between technologies with different costs and cost structures. In both cases the total costs of the joint distribution or production plan are to be allocated to the different products.

We assume that we have two products that are produced in quantities x_1 and x_2 . There are four different distribution channels (or alternatively production technologies), and the cost structures for the four alternatives are given by the following cost functions:

- I: $3x_1 + 2x_2$
- II: $x_1 + x_2 + 9$
- III: $2x_1 + x_2 + 5$
- IV: $x_1 + 2x_2 + 5$

i.e. the alternative cost functions show different combinations of fixed and variable costs.

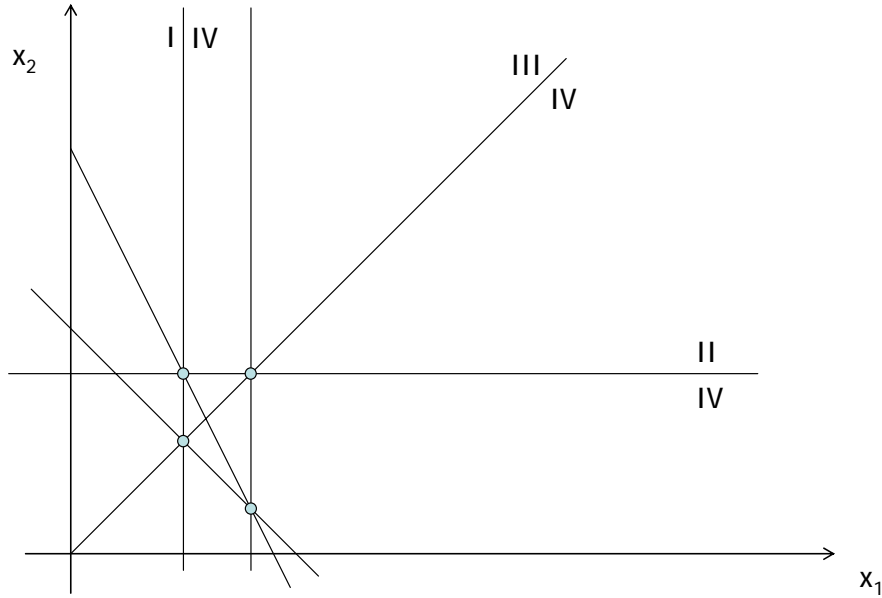


Figure 2. Comparing Cost Functions

In figure 2 we have depicted the hyperplanes following from comparing cost functions two by two. For instance all comparisons relating to cost function IV is marked in the figure. The border between III and IV is for instance found by

$$\text{III} \succ \text{IV} \quad \text{if} \quad 2x_1 + x_2 + 5 \leq x_1 + 2x_2 + 5 \Rightarrow x_1 \leq x_2 \quad (13)$$

so that the hyperplane separating the best performance areas of cost functions III and IV (when comparing only the two of them) is given by $x_1 - x_2 = 0$. Similar borders can be found for all other comparisons of cost functions by twos, and the resulting separating lines are given in figure 2.

The result is not easy to read, however we may colour the different parts of the positive orthant according to which cost function that is preferred for the different combinations of outputs 1 and 2. This is shown in figure 3, where we notice that the areas of best performance for the different cost functions form convex sets.

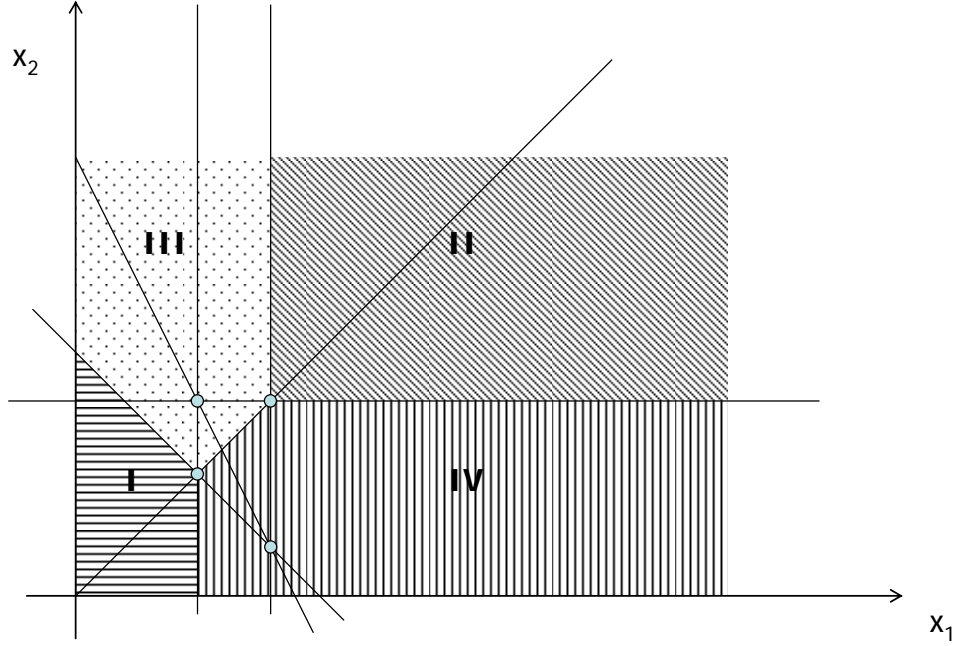


Figure 3. Areas of Best Cost Functions

This is a general result for linear cost functions (with possibly a fixed cost part). In order to see why, we focus on one of the alternatives, for instance alternative IV. The area where alternative IV is the better one, is defined by the set of linear inequalities

$$IV \succ I \Rightarrow x_1 + 2x_2 + 5 \leq 3x_1 + 2x_2 \Rightarrow x_1 \geq 2.5 \quad (14)$$

$$IV \succ II \Rightarrow x_1 + 2x_2 + 5 \leq x_1 + x_2 + 9 \Rightarrow x_2 \leq 4 \quad (15)$$

$$IV \succ III \Rightarrow x_1 + 2x_2 + 5 \leq 2x_1 + x_2 + 5 \Rightarrow x_1 \geq x_2 \quad (16)$$

This means that the area where a given cost function is best, is given by the intersection of a number of closed half spaces (equal to the number of different cost functions less 1), and therefore the area forms a polyhedral convex set if it is non-empty. Moreover, these convex sets exhaust the whole positive orthant since in every point there is a best cost function (though possibly with ties).

In figure 3 it is clearly illustrated that the optimal choice of distribution channel (or production technology) will depend on the total production plan, i.e. the level of the total quantities demanded and the relative shares of the demands for the different products. For instance, if products 1 and 2 are produced in fixed proportions, such that the quantity produced of product 2 constitutes a fraction of the number of products of type 1, the best technology will shift from I to IV and then to II, depending on the total production plan. With high quantities of the products, cost function II is the better one. If the production mix is such that the number of products of type 2 is higher than

the number of products of type 1, the best cost function varies from I to II, via alternative III for medium production plans.

Introducing a new technology will impose a new convex area in the diagram if the new cost function is the better one for some product mixes. Consider for instance

$$V: 2x_1 + 2x_2 + 2$$

The area where cost function V is the best one is given by the set of inequalities

$$x_1 \geq 2 \quad (\text{V better than I})$$

$$x_1 + x_2 \leq 7 \quad (\text{V better than II})$$

$$x_2 \leq 3 \quad (\text{V better than III})$$

$$x_1 \leq 3 \quad (\text{V better than IV})$$

In figure 4 we show the hyperplanes resulting from comparisons of cost functions two by two, and figure 5 shows the areas where the different cost functions are best. Figure 5 is found by imposing the best area for the new cost function V onto figure 3.

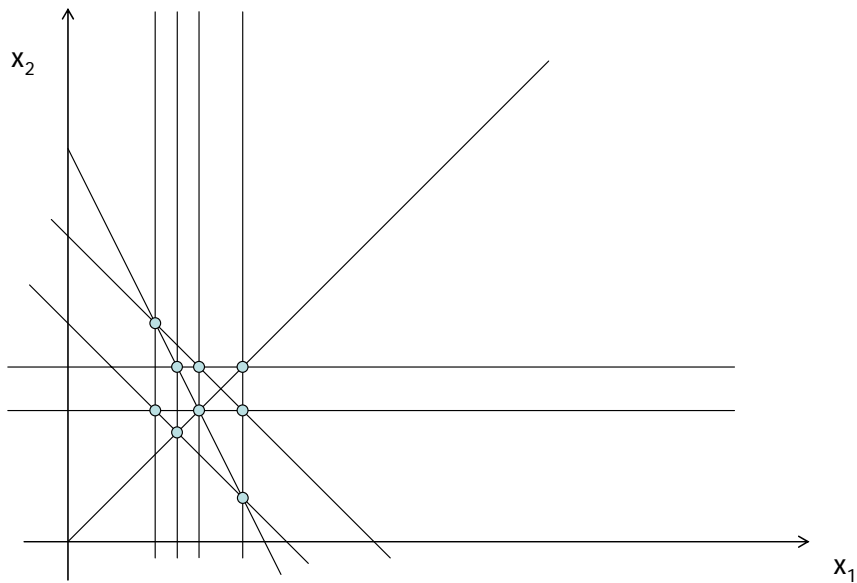


Figure 4. Comparisons with New Cost Functions

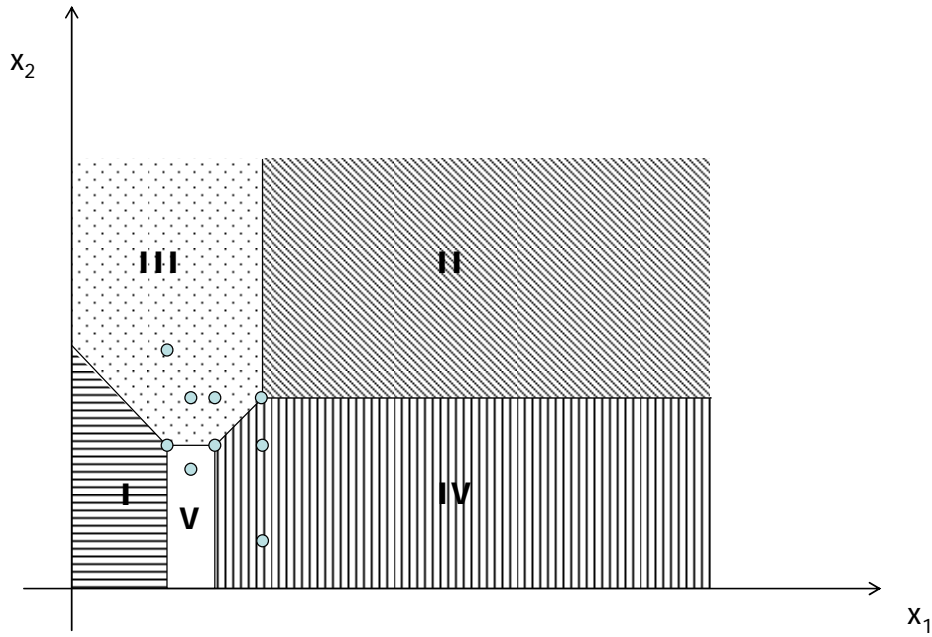


Figure 5. Best Areas with New Cost Function

In the following, we consider different pricing or cost allocation schemes for this problem. We focus on product prices resulting from accounting based procedures, i.e. 1) allocation of direct costs only, here corresponding to setting prices at marginal costs, and 2) allocation of full costs. We assume that the cost allocation base for the fixed costs is the total number of products produced, i.e. every product unit in a given production plan is allocated the same share of the fixed costs. Other alternatives could have been chosen, though. We will compare these accounting based prices with the results obtained through the Aumann-Shapley pricing method.

When computing the product prices for the different alternatives; marginal cost prices (MC), full cost prices (FC) and Aumann-Shapley prices (AS), for different product mixes, we focus on how the unit-prices develop when we increase the production level along a ray from the origin. Below, product mixes such that $x_1 = 2x_2$ are studied. The most interesting points are those where we move from one best technology to another, i.e. where there is a shift in the best cost function. That means points (2.5, 1.25) and (8, 4) when cost function V is not eligible, and points (2, 1), (3, 1.5) and (8, 4) when it is. These are depicted in figure 6.

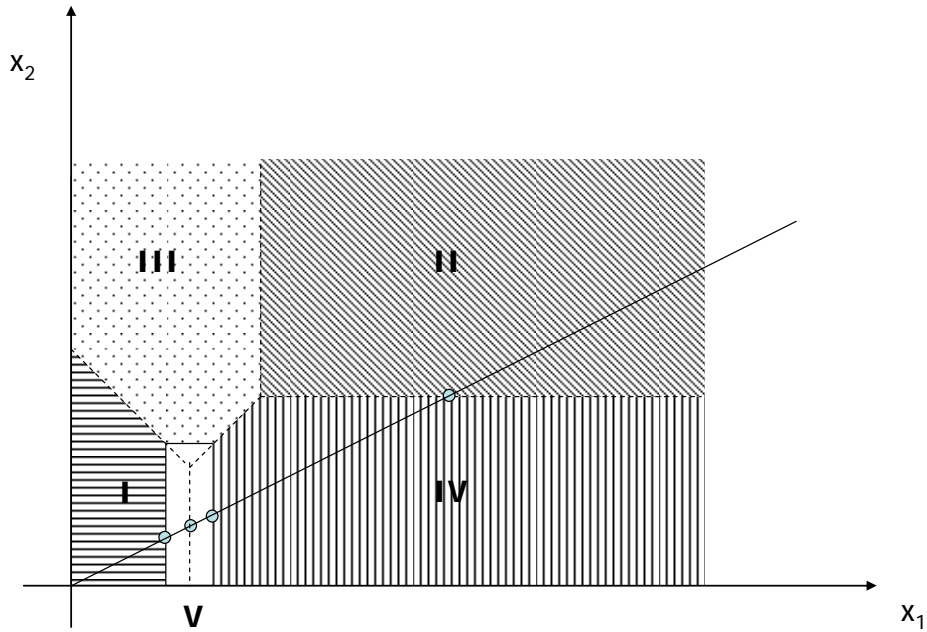


Figure 6. Ray $x_1 = 2x_2$ and Its Critical Points

In table 1 the different prices are exhibited for some product combinations along $x_1 = 2x_2$, and we show two variants, i.e. prices with and without cost function V. For instance, the different prices for product combination (6, 3) are found in the following way:

MC: The lowest cost is achieved with cost function IV: $x_1 + 2x_2 + 5$, in which the marginal cost for product 1 is equal to 1 and for product 2 it is equal to 2.

FC: We have assumed that the full cost consists of the direct variable cost and a share of the fixed cost using the total number of products produced as the cost allocation base. That means that for product mix (6, 3), the full cost of product 1 is equal to $1 + 5/9 \approx 1.56$, and for product 2 it is equal to $2 + 5/9 \approx 2.56$.

AS: The Aumann-Shapley prices are calculated according to (8), and for product combination (6, 3) it means that all output vectors of the form $(t6, t3)$, where $0 \leq t \leq 1$ contribute in the computation of the prices. Since the cost functions considered are separable and linear in the products, it is fairly easy to compute the AS-prices, when we know the t -values for which there is a shift of best cost function. In the example, this occurs at $t = 5/12$ when only I – IV are considered, and at $t = 1/3$ and $t = 1/2$ when V is also possible to choose. In the first case, we get $p_1 = (5/12-0) \cdot 3 + (1-5/12) \cdot 1 \approx 1.83$, and $p_2 = (5/12-0) \cdot 2 + (1-5/12) \cdot 2 \approx 2$,

while in the second case, we get $p_1 = (1/3-0) \cdot 3 + (1/2-1/3) \cdot 2 + (1-1/2) \cdot 1 \approx 1.83$ and $p_2 = (1/3-0) \cdot 2 + (1/2-1/3) \cdot 2 + (1-1/2) \cdot 2 = 2$.

In table 1 we have found the different prices for some points between the origin and point (10, 5). It is easily seen that the prices are quite different, and especially, the development when the production level increases is very different for the various alternatives considered. In the points where there are ties, i.e. where there is a shift in the least cost cost-function, we have alternative prices for the MC and FC cases. For instance for product combination (2, 1), the best cost function shifts from I to V (assuming V can be chosen), and depending on which cost function we use, the MC price vector is equal to (3, 2) (cost function I) or (2, 2) (cost function V). So moving along the ray where $x_1 = 2x_2$, the marginal cost prices vary erratically, depending on the production level and which is the best cost function. Moreover, with the simple cost allocation base we have chosen for the fixed cost part, this characteristic also carries over to the FC based price vectors. On the other hand, the AS prices vary with production level, but they vary smoothly as one would expect from average cost prices.

Table 1. Example Prices

(x_1, x_2)	Type of unit-price	Product	(2, 1)	(2.5, 1.25)	(4, 2)	(6, 3)	(8, 4)	(10, 5)
Best of functions I-IV			I	IV	IV	IV	IV	II
	MC	1	3	3	1	1	1	1
		2	2	2	2	2	2	1
	FC	1	3	3	2.33	1.83	1.56	1.42
		2	2	2	3.33	2.83	2.56	2.42
	AS	1	3	3	2.25	1.83	1.625	1.5
		2	2	2	2	2	2	2
Best of functions I-V			I	V	IV	IV	IV	II
	MC	1	3	2	2	1	1	1
		2	2	2	2	2	2	1
	FC	1	3	2.67	2.53	1.83	1.56	1.42
		2	2	2.67	2.53	2.83	2.56	2.42
	AS	1	3	3	2.8	2.25	1.83	1.625
		2	2	2	2	2	2	2

6. Conclusions and Future Research

In this paper we have shown that prices based on marginal cost may result in unreasonable allocations of cost and revenue compared to the actual contributions of the different products in a total production plan. We have seen from the example with fixed costs that the development of the marginal cost for the products in a product mix can vary erratically based on the total production level, and that this characteristic may carry over to the full cost prices when the fixed costs are allocated independently of the variable costs. As we may tend to think about the full cost prices as some average cost, this characteristic seems unreasonable and non intuitive. As an alternative, we have shown how the Aumann-Shapley prices evolve when changing the production level, and we suggest that this method could be considered in the case of joint distribution or production of several products, both in regulation and for pricing within a supply chain, where different units may be responsible for the profits of the various products. The Aumann-Shapley prices imply a different allocation of revenue among the product types, than do the marginal cost pricing and the more accounting based procedures, and attaining a good and fair allocation mechanism is an essential part for decisions on whether to cooperate or not, and in order to establish the best possible alliances in the production and distribution processes.

In the paper we have considered linear prices. It could also be interesting to compare the pricing mechanisms with non-linear price mechanisms as described in Bjørndal and Jørnsten (2004). It could also be interesting to refine the assumptions of variability in order to study the relationship with activity based costing practices. Another interesting research direction is how the different price mechanisms can be used in relation to the issue of cross-subsidies. This is a very important topic in partly deregulated industries, and proves quite hard to define in practice, as illustrated in Fjell (2001).

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