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Optimal age-structured harvest in a dynamic model with heterogenous capital

by

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Abstract

A dynamic optimization model with four state variables and two control variables is developed in order to analyze a fishery where on fleet-segment is targeting young fish and another fleet-segment is targeting older fish of the same species. The state varibles are the biomass levels of young and old fish and the stock of capital in each fleet-segment. The control variables are investment or de-investment in each type of capital which again determine the harvest rates. Irreversible investments are represented through an asymmetric cost of investment. In addition market conditions, cost structure and technologies differ between the two fleet-segments.

The model contains both biological interaction between young and old fish (stock-recruitment, cannibalism) and economic interaction (inter-dependent market prices).

INTRODUCTION

Many fish stocks are harvested by two or more different fleet segments. In particular, in most fishing nations fishing vessels can be divided into one of two groups, namely small coastal vessels and larger ocean-going vessels. These are typically harvesting different species, but in many cases they are also competing about the same species. When the latter is the case, there is a tendency that the coastal fleet, which operates closer to land, targets larger and older fish that come near shore to spawn. whereas the ocean-going part of the fleet targets younger and smaller fish that migrate into open ocean to feed. There is also a cost difference between the two fleet segments, and typically the larger ocean-going vessels are believed to have lower operating costs than the smaller vessels. The reason for this, if it is the case, is simply some degree of economies of scale. The large vessels have usually a lower labour/capital ratio than the small vessels, and, at least in industrialised countries with high labour cost, this implies lower total costs.

From a social point of view the advantage of the coastal fleet is that it mainly harvests older fish and thus fully exploits the growth potential of the resource. The ocean vessels, on the other hand, harvest on younger fish with an unexploited growth potential. The advantage of this fleet is that it typically is more cost efficient.

There is in general both a biological and economic difference between the two vessels categories that we shall concentrate on here, and the aim of this paper is to develop a model that can be used to assess an optimal investment pattern over time when these differences are taken into account. In order to avoid unnecessary complications we assume nonvanishing equilibrium activity in both groups of vessels and we assume efficient use of the capital at all points in time; in other words there is no overcapacity. Hence, the harvest pattern emerges as a direct consequence of the investment pattern.

As far as we know, very little attention has been devoted to this question in the fisheries economics literature. Armstrong and Sumaila (2001) study the allocation rule applied to split the total allowable catch (tac) for Norwegian cod between coastal vessels and trawler vessels and the implications of implementing an ITQ system for this fishery. They find that the current allocation rule is far from optimal. Bjørndal and Gordon (2000) perform a cost analysis of the Norwegian spring-spawning herring fishery with three different vessel types, purse seiners, trawlers and coastal vessels. They find that purse seiners and trawlers are highly cost efficient compared to coastal vessels, and that the average cost of harvesting is fairly constant for the purse seiners despite large fluctuations in the fish stock.

The model applied in this paper is quite different from the models in the papers mentioned above as it is a multi-dimensional dynamic optimization model with four state variables and two control variables. The model is outlined in the next section, then a couple of applications of the model are provided and finally a summary is given.

THE MODEL

The model is stated as an optimal control problem. The state variables are the following:

x_1 = biomass of small, non-mature fish,

x_2 = biomass of spawning stock,

K_1 = capital in the fleet segment that harvests on x_1,

K_2 = capital in the fleet segment that harvests on x_2.

The control variables are:

I_1 = investment in K_1,

I_2 = investment in K_2.

The harvest rate, h, is given by the production function

$$h_i = q_i x_i K_i, \quad i = (1, 2), \tag{1}$$

where q_i are the catchability coefficients. The profit, or net revenue, is given by

$$\Pi(x, K, I) = \sum_{k=1}^{2} \Pi_k(x, K, I), \quad x = (x_1, x_2), \quad K = (K_1, K_2), \quad I = (I_1, I_2)$$

$$\Pi_k(x, K, I) = (p_k - Q_k h)h_k - c_k K_k - G_k(I_k), \quad h = \sum_{k=1}^{2} h_k,$$

where $p_k - Q_k h$ is the unit price of harvest, which depends on total harvest h and p_k and Q_k are parameters. In the special case that $Q_k = 0$ the agents are price-takers. The per unit operating cost of capital is c_k.

The functions $G(\cdot)$ represent the cost of investment. In the following we assume that there is one price for buying capacity and another price for selling capacity. The price for selling capacity is typically lower than the price for buying capacity. This is equivalent to a certain degree of irreversible investments. The price for buying capacity in fleet segment i is p_i^b, and the price for selling capacity is p_i^s. In the following we assume $p_i^b \geq p_i^s \geq 0$. With this assumption the G functions can be formulated as

$$G_i(I_i) = \max(p_i^b \cdot I_i, p_i^s \cdot I_i). \tag{2}$$

as p_i^b is in effect when $I_i > 0$ and p_i^s is in effect when $I_i < 0$.

The objective then is:

$$\max_{I_1, I_2} \int_0^{\infty} e^{-\delta t} \Pi \, dt$$

subject to the dynamic constraints given as:

$$\begin{aligned}
\dot{K}_1 &= -\beta_1 K_1 + I_1, \\
\dot{K}_2 &= -\beta_2 K_2 + I_2, \\
\dot{x}_1 &= f_1(x_1, x_2) - v x_1 - h_1 \equiv F_1(x_1, x_2) - h_1, \\
\dot{x}_2 &= f_2(x_1, x_2) + v x_1 - h_2 \equiv F_2(x_1, x_2) - h_2,
\end{aligned} \tag{3}$$

where dots are used to denote time derivatives. The parameters β_i represent the depreciation rates of capital in the two fleet segments. If $p_i^b = p_i^s$, there is no element of irreversible investments (perfectly malleable capital), and if $p_i^s = 0$ and $\beta_i = 0$ then there is perfectly irreversible investments. If $p_i^s = 0$ but $\beta_i > 0$, then the capital is quasi-malleable.

The parameter υ denotes the part of x_1 that matures and enter the spawning stock at any point in time. The functions f_i account for recruitment, individual growth and natural mortality for stock x_i. The reason why x_2 is one of the arguments in f_1 is twofold. First, the recruitment to the stock depends on the spawning stock. Secondly, there may also be cannibalism, and therefore the natural mortality in x_1 depends on x_2. Cannibalism is also the reason why x_1 is included in f_2 as individual growth for larger fish may depend upon the biomass of small fish. The f-functions will be specified as follows:

$$
\begin{aligned}
f_1(x_1, x_2) &= x_1(r_1 + s_1 x_1 + u_1 x_2), \\
f_2(x_1, x_2) &= x_2(r_2 + s_2 x_2 + u_2 x_1).
\end{aligned}
$$

If there is cannibalism, then $u_1 < 0$ and $u_2 > 0$. If both u_1 and u_2 are negative, then we have competition.

First-order conditions

The current value Hamiltonian for the above problem is

$$
\begin{aligned}
&H(x, K, I, \mu, \lambda) = \Pi(x, K, I) + \lambda_1(-\beta_1 K_1 + I_1) + \lambda_2(-\beta_2 K_2 + I_2) \\
&+\mu_1 \left[F_1(x_1, x_2) - q_1 x_1 K_1\right] + \mu_2 \left[F_2(x_1, x_2) - q_2 x_2 K_2\right], \\
&\lambda = (\lambda_1, \lambda_2), \quad \mu = (\mu_1, \mu_2),
\end{aligned}
$$

where λ_i are the costate variables associated with capital and μ_i are the costate variables associated with the fish stocks. From the maximum principle we have

$$H_{I_i} = 0 = \lambda_i - \begin{cases} p_i^b, & I_i > 0 \\ -p_i^s, & I_i < 0 \end{cases}, \quad i \in \{1, 2\}. \tag{4}$$

It is seen from (4) that the shadow price of capital is constant and positive when capital is bought and negative when capital is sold

In addition we have the adjoint equations $\dot{\lambda}_i = \delta\lambda_i - H_{K_i}$ and $\dot{\mu}_i = \delta\mu_i - H_{x_i}$, implying

$$\begin{aligned} \dot{\lambda}_i &= (\delta + \beta_i)\lambda_i + q_i x_i \mu_i + 2\widetilde{Q}_i q_i x_i K_i + \left(\widetilde{Q}_i q_j x_j + \widetilde{Q}_j q_i x_i\right) K_j - P_i, & \text{(5a)} \\ \dot{\mu}_i &= \left(\delta - \frac{\partial F_i}{\partial x_i}\right)\mu_i - \frac{\partial F_j}{\partial x_i}\mu_j + \left[q_i\mu_i + \widetilde{Q}_i' h + \widetilde{Q}_i q_i K_i + \widetilde{Q}_j q_i K_j - P_i'\right] K_i, & \text{(5b)} \end{aligned}$$

using

$$\widetilde{Q}_k(x_k) = Q_k q_k x_k, \; P_k(x_k) = p_k q_k x_k - c_k, \quad i, j, k = \{1.2\}, \; j \neq i.$$

As the λs are constant, Equation (5a) can be solved for μ:

$$q_i x_i \mu_i = \Lambda_i \equiv P_i(x_i) - (\delta + \beta_i)\lambda_i - 2\widetilde{Q}_i q_i x_i K_i - \left(\widetilde{Q}_i q_j x_j + \widetilde{Q}_j q_i x_i\right) K_j. \tag{6}$$

Equations (5b) then represent two equations to determine the relationships between the four state variables and the two control variables. The structure of the equations ensures that these two equations can be solved with respect to the controls. This can be seen by the fact that the time derivatives on the left-hand side are expressions that contain time derivatives of the state variables. These time derivatives are known from the dynamic equations in (3). Hence we have two ordinary algebraic equations (not DE) to determine the two control variables as functions of the four state variables. Thus we have formally found a feedback policy, that is $I_i = g_i(x, K)$, for the true dynamic problem. In this article, however, we will concentrate on the analysis of possible steady states.

Steady state

It is obvious that capital cannot be sold forever, hence from (4) $I_i > 0$ and $\lambda_i = p_i^b$ in a permanent steady state. Equation (6) is also valid in the dynamic setting (as the λs are constant), and it gives us the shadow prices of capital as functions of the state variables, x and K. Further,

$$K_i = \frac{F_i}{q_i x_i}, \tag{7}$$

from steady state with respect to the stock levels. Equations (6) and (7) inserted into (5b) yields two equations for the two stocks:

$$\left(\delta + \frac{F_i}{x_i}\right)\Lambda_i + \widetilde{Q}_i' \cdot F_i \cdot (F_i + F_j) + \frac{\widetilde{Q}_i}{x_i} \cdot F_i^2 = \tag{8}$$

$$\frac{\partial F_i}{\partial x_i} \cdot \Lambda_i + \frac{q_i x_i}{q_j x_j} \cdot \frac{\partial F_j}{\partial x_i} \cdot \Lambda_j + \frac{\widetilde{Q}_j}{x_j} \cdot F_i \cdot F_j + F_i \cdot p_i \cdot q_i, \quad j \neq i,$$

The two equations given in (8) are highly non-linear in the stock. They can, however, easily be solved numerically, and thereby all values in steady state are determined. These equations therefore represent a generalization of "the Golden Rule".

In the special case that the fishermen are price-takers we have $\widetilde{Q}_i = 0$, and the equations are simplified as follows:

$$\begin{aligned} \left(\delta + \frac{F_i}{x_i}\right)\Lambda_i &= \frac{\partial F_i}{\partial x_i} \cdot \Lambda_i + \frac{q_i x_i}{q_j x_j} \cdot \frac{\partial F_j}{\partial x_i} \cdot \Lambda_j + F_i \cdot p_i \cdot q_i, \\ \Lambda_i &\equiv P_i - (\delta + \beta_i)\, p_i^b. \end{aligned}$$

Further, if the stocks in addition do not depend on each other, we get the following equation for each of the stocks:

$$\left(\delta + \frac{F}{x}\right) \cdot \Lambda = \frac{\partial F}{\partial x} \cdot \Lambda + F \cdot p \cdot q.$$

Note that with quadratic F-functions, the problem given in (8) are two fourth-degree equations, which is a practicable problem to solve.

EMPIRICAL RESULTS

In this section we use data for the Norwegian fishery of Arctic cod. For this we need economic data for prices, operating costs and investment costs and some biological data for the stock dynamics. The economic data are taken from profitability analysis performed by the Directorate of Fisheries Norway (2001) and the biological data are taken from International Council for Exploration of the Seas (ICES, 2001).

Numerical specification

The numerical specification of the model is given in Table 1.

Table 1. Numerical specification

p_1	15.98	p_1^b	18.3
p_2	14.96	p_2^b	11.7
c_1	11100	p_1^s	0
c_2	3500	p_2^s	0
Q_1	0.002	r_1	0.55
Q_2	0.002	s_1	- 0.000096
β_1	0.03	u_1	0
β_2	0.03	r_2	0.95
q_1	1	s_2	- 0.00066
q_2	1	u_2	- 0.00045
δ	0.05	υ	0.106

The economic parameters in Table 1 are derived as follows. The coastal vessels are represented by the average of groups 1 - 10, and the ocean going vessels by group 14 (factory trawlers). Prices are first-hand prices for cod. The slope parameters of the demand curves, Q_i, are chosen somewhat arbitrarily to indicate a weak price-quantity relationship or, in other words, an elastic demand. The intercept parameters p_i are then calibrated using price and quantity data for 2000 such that the prices in 2000 corresponds with the quantities in 2000. The same value has been used both for Q_1 and Q_2 to make the demand curves parallel.

The cost parameters c_i are calibrated using variable costs adjusted for the average time devoted to cod by each vessel group and applying the production function (1). The prices for buying capital, p_i^b, are calculated using the average depreciation for each vessel group multiplied by the expected life time for the vessels. The prices for selling capital, p_i^s, will then be set to zero or 50 percent of the buying price depending on our assumption about irreversibility.

The biological parameters are estimated using data from ICES (2001). The results from the estimations are summarized in Table 2. Total spawning biomass is defined as x_2; and x_1 is defined as the difference between total biomass and x_2. Data are annual, and therefore the time derivatives are approximated as follows:

$$\dot{x}_2 = x_2(r_2 + s_2 \cdot x_2 + u_2 \cdot x_1) + v \cdot x_1 - h_2.$$

The results of this estimation are summarised in Table 2

Table 2.

parameter	value	t-statistic	other
r_2	0.95	4.55	$R^2 = 0.12$
s_2	- 0.00066	- 1.83	DW = 1.35
u_2	- 0.00045	- 2.36	F = 2.5
v	0.106	2.63	

The reported value for R^2 is fairly low, but this is typical for this kind of model. All parameters are significant at an 8 % significance level. This also holds for the model as such as indicated by the F-value. The DW-statistic reported here does not indicate autocorrelation.

The value found for v is now used as input when we estimate:

$$\dot{x}_1 = x_1(r_1 + s_1 \cdot x_1 + u_1 \cdot x_2) - 0.106 \cdot x_1 - h_1.$$

From the first run it is evident that the parameter u_1 is not significantly different from zero, and therefore the model is re-estimated with this parameter set equal to zero. The results from this estimation are reported in Table 3.

Table 3.

parameter	value	t-statistic	other
r_1	0.55	7.22	$R^2 = 0.46$
s_1	- 9.6E-5	- 2.67	DW = 1.63
u_1	0	-	F = 29.3

The estimation reported in Table 3 is actually very good for this kind of model. All parameters are significant at a 2 % significance level. This also holds for the model as such as indicated by F. Further, the reported R^2 is very high for this kind of estimation, and the DW-statistic does not indicate autocorrelation. The result that u_1 equals zero indicates either that there is no significant cannibalism or spawning-recruitment relationship, or that the two more or less cancel each other. The result that $u_2 < 0$ indicates that there is no significant cannibalism but rather some degree of competition for the food.

We will now proceed with these estimations as basis when analyzing optimal steady states. As there is a high degree of uncertainty associated with many of the parameters, sensitivity analysis will be crucial.

Basic model

The results regarding the optimal steady state from the basic model described above are summarised in Table 4.

Table 4. Results from the basic model

	Year 2000	Optimum	Change
x_1	820	2462	+ 200 %
x_2	223	385	+ 73 %
x	1043	2847	+ 173 %
h_1	289	512	+ 77 %
h_2	124	103	- 17 %
h	414	614	+ 48 %
K_1	0.353	0.208	- 41 %
K_2	0.557	0.267	-52 %

Several interesting things can be noted from Table 4. First of all, the optimal total biomass should be 2.7 times higher than the present. It is, however, the stock of young fish that ought to increase most. In fact, the stock of young fish ought to increase three times whereas the spawning stock biomass ought to increase by a bit more than 70 percent.

After this build-up of the stock has taken place, total harvest can increase by almost 50 percent. However, it is interesting to note that whereas the harvest of young fish can be increased by almost 80 percent after an optimal build-up, the harvest from the spawning stock should actually be decreased about 17 percent compared to the present harvest. This result may come as a bit of a surprise as intuition tells us that mature fish has already used most of its growth potential and therefore can be harvested at a higher rate than young fish. A closer look at the figures tells us that the harvest rate actually is higher for mature fish than for young fish. The problem is that the rate of overharvesting is even higher for mature fish than for young fish. Therefore the capacity in the fleet segment harvesting mature fish ought to be decreased by more than 50 percent whereas the fleet segment harvesting young fish should be decreased by about 40 percent.

All in all, these results indicate that there is huge overcapacity in both fleet segments. Further, the actual overcapacity is even higher than the results here indicate, as present capacity has been calculated only on biological terms; that is as the capacity needed to take the present harvest when the fishery is performed efficiently. This approach ignores the economic overcapacity due to inefficiency in the fishery. Such inefficiencies are not a topic in this article, but there are strong evidence that they are present and therefore the actual need for fleet reduction is even higher than reported here. On the other hand, this assumes that maximizing economic returns is the only objective. If, for example, labour employment is of any concern, than the result may be different. At least the way the optimal steady state is approached in the short run is affected by this. In this article, however, we mainly look at steady states.

Sensitivity analysis

The empirical model presented above is rather stylized, and therefore it is of interest to see how sensitive the main results presented above are to some of the more uncertain parameters.

First we start by varying the output price level, that is the intercept of the demand curve. As it is the relative price relationship between the two fleet segments that counts, we only vary the price for fleet segment 1, p_1, and keep all other parameters constant. The price p_1 is varied by +/- 25 percent, and the result is reported in Table

5.

Table 5. Results when p_1 is varied by +/- 25 %

	$p_1 = 11.98$	$p_1 = 19.98$
x_1	2639	2365
x_2	373	391
x	3012	2756
h_1	503	513
h_2	99	105
h	602	618
K_1	0.191	0.217
K_2	0.265	0.269

As seen from Table 5, the results are not very sensitive to changes in the output price, but there are some interesting points to note. The stock x_1 decreases with a higher price as expected, but at the same time the stock x_2 increases. The overall stock effect of an increasing price p_1 is a smaller stock. It is not obvious why the stock x_2 should increase when p_1 increases, but it has to do with the fact that the biological parameter u_2 is negative. In other words, the direct decrease in x_1 has an indirect positive effect on x_2.

It can be noted that the same effects are observed when p_2 is varied, but in this case both stocks decrease when the price increase. This has to do with the fact that $u_1 = 0$. Further, we can note that the harvest from both stocks increase when the price of stock one increases, but by a rather insignificant amount. As a consequence the capacity in fleet 1 must increase, and also the capacity of fleet 2 increases, but very little.

All in all, however, a quite significant variation in the output price only results in moderate adjustments in the stock sizes, the harvest levels as well as in catching capacities. Hence we can conclude that the results are robust with respect to the level of the output price.

The next question is how sensitive the results are to variations in the slopes of the demand curves represented by the parameters Q_1 and Q_2. This is reported in Table 6.

Table 6. Results when the slope parameters Q_1 and Q_2 are varied

	$Q_1 = Q_2 = 0$	$Q_1 = 0.008$ $Q_2 = 0.002$	$Q_1 = 0.002$ $Q_2 = 0.008$	$Q_1 = 0.008$ $Q_2 = 0.008$
x_1	2410	2844	2509	2708
x_2	369	408	447	446
x	2779	3252	2956	3154
h_1	513	487	509	498
h_2	116	56	54	36
h	629	543	563	534
K_1	0.213	0.171	0.203	0.184
K_2	0.315	0.139	0.121	0.081

In the case where $Q_1 = Q_2 = 0$ the price is set equal to the actual price in 2002, namely $p_1 = 14.98$ and $p_2 = 13.96$. In the other cases the intercept of the demand curves are the same as in Table 1. This means that in the three rightmost columns the actual price is in effect reduced compared to the basic model. There are of, course, numerous combinations that could be investigated, but the ones shown in Table 6 clearly indicate that the results are somewhat, but not very, sensitive to these two parameters.

Next we test for sensitivity with respect to costs, and again it is the relative cost relationship that counts. As the cost parameter is a much more uncertain parameter than the output price, the parameter c_1 is now varied by +/- 50 percent. The results are presented in Table 7.

Table 7. Results when c_1 is varied by +/- 50 %

	$c_1 = 5550$	$c_1 = 22200$
x_1	2195	2945
x_2	403	357
x	2598	3302
h_1	512	475
h_2	110	94
h	622	569
K_1	0.233	0.161
K_2	0.272	0.263

As expected the consequences of a higher cost c_1 are the opposite of a higher price, namely a higher stock x_1, a lower stock x_2 but a higher total stock. The harvest from both stocks decrease and so do the capacities in both fleet segments. The results are not very sensitive to changes in the cost parameter either, taking into account that it has been varied by +/- 50 percent.

The optimal steady states are even less sensitive to the parameters associated with investment, that is the prices of buying and selling capacity and the depreciation rate of capital. In fact, the optimal steady states are hardly affected by changes in these variables at all.

Therefore we turn to the biological parameters to see how sensitive the results are to these. First we look at the biological parameters representing biological links such as stock-recruitment relationship, cannibalism or competition for food, namely u_1 and u_2. The parameter values suggested by the data and used in the basic model are $u_1 = 0$ and $u_2 = -0.00045$. This indicates that the stock of young fish is not affected by the size of the stock of old fish, for example because the stock-recruitment relationship is counteracted by cannibalism and/or competition for food. Numerous combinations of these parameters are possible, many of which yield no meaningful results at all. A couple of examples, however, demonstrate that the results are highly dependent on these parameters. The first example used here is when the values are changed such that $u_1 = -0.00045$ and $u_2 = 0$. This indicates that young fish is affected by old fish through competition or cannibalism whereas the affect of young fish on old fish is insignificant. The results of this is reported in Table 8. In this case the stock of young fish is reduced and the spawning stock is significantly increased. In addition almost all harvest takes place from the spawning stock and hardly anything from the young stock. Consequently capacity in fleet segment 2 must be increased significantly and capacity in fleet segment 1 must be decreased accordingly.

The other example is when there is mutual competition and possibly cannibalism such that both stocks affect each other negatively. In this example we have $u_1 = u_2 = -0.00045$. The result is that the optimal size of both stocks are reduced compared to the basic model. The harvest of young fish is reduced by about 50 % whereas the harvest of old fish is increased. Also in this case the capacity in fleet segment 1 is decreased and the capacity in fleet segment 2 is increased.

We also look at how sensitive the results are to changes in the intrinsic growth rates r_1 and r_2. If r_1 is increased by 20 % this calls for an increase in the stock of young fish and a small decrease in the stock of old fish, an increase in the harvest of young fish and a decrease in the harvest of old fish and also a similar change in the fleet segments associated with the two stocks. When r_2 is increased, this calls for a small reduction in stock 1 and an increase in stock 2, almost unchanged harvest of stock 1 and increased harvest from stock 2. Accordingly there will be a small decrease in the capacity in fleet segment 1 and an increase in fleet segment 2.

All in all, the results are quite sensitive to changes in the biological parameters and the productivity of the stocks. This implies that it is more important to put effort into estimating the correct biological parameters than the economic parameters, and, especially, the parameters related to investment turn out not to affect the steady states very much.

Table 8. Sensitivity analysis with respect to biological parameters

	$u_1 = -0.00045$ $u_2 = 0$	$u_1 = -0.00045$ $u_2 = -0.00045$	r_1 $+20\%$	r_2 $+20\%$
x_1	1263	1863	3094	2390
x_2	710	289	369	448
x	1973	2152	3463	2838
h_1	4	252	795	513
h_2	475	175	75	149
h	479	427	870	662
K_1	0.003	0.135	0.257	0.215
K_2	0.67	0.605	0.204	0.333

Finally, we will also take a look at how sensitive the steady states are to a change in the discount rate. This is done by setting the discount rate to zero instead of five percent. This is a quite significant change in the discount rate, and the results are reported in Table 9. First of all, we note that the change in any of the variables is not higher than 9 percent, indicating that the results are not very sensitive to discounting. Secondly, we note that all changes are as expected except, namely that a lower discount rate implies a higher standing stock and lower harvest and effort; except that the stock of old fish is slightly reduced instead of increased. This obviously has to do with the biological interaction between the stocks. As the parameter u_2 is negative there is a negative effect of x_1 on x_2, and therefore the required increase in x_1 indirectly implies a small decrease in x_2. This is associated with the fact that x_1 is more important economically than x_2 in this model.

Table 9. Changed discounting

	$\delta = 0.05$	$\delta = 0$	Change
x_1	2462	2642	+ 7 %
x_2	385	376	- 2 %
x	2847	3018	+ 6 %
h_1	512	503	- 2 %
h_2	103	97	- 6 %
h	614	600	- 2 %
K_1	0.208	0.190	- 9 %
K_2	0.267	0.257	- 4 %

CONCLUSIONS AND SUMMARY

A model with two fleet segments and a fish stock divided in two cohorts has been presented. The fish stock is divided in young fish and old fish, and there is a biological relationship between the two, for example stock-recruitment relationship, cannibalism, competition for food, etc. In addition, at each time period a certain proportion of the stock goes from the cohort of young fish into the cohort of old fish. Each fleet segment catches exclusively either young fish or old fish. The fleet segments are characterized by different costs structures, they get different prices for the products, they may have different depreciation rates and they pay different prices for investment in new capital. In addition to the biological interaction there is also an economic interaction as the price each fleet segment gets depends not only on own harvest but also on the harvest of the other fleet segment. It is assumed that all capital available is used in each time period such that harvest is regulated through investment or disinvestment in capital. In other words, there is no excess capital. The model can vary from completely malleable capital through semi-malleable capital to completely irreversible investments by determining the buying and selling price of capital and the depreciation rate.

The model has been applied to the Norwegian cod fishery where large trawlers typically harvest young fish whereas smaller vessels harvest from the spawning stock. The trawlers get a higher price but also have higher costs. The biological data suggest a negative effect from young fish on old fish (competition) and zero effect from old fish on young fish. The result from the basic model is that the total stock ought to be build up and increase by almost 200 percent. The stock of young fish ought to increase much more than the spawning stock. The harvest of young fish could in the optimal steady state be increased by almost 80 % whereas the harvest from the spawning stock ought to be decreased. The capacity in the trawler fleet should be decreased by more than 40 % whereas the capacity in the fleet of smaller vessels ought to be decreased by more than 50 %.

The sensitivity analysis shows that the results are a bit, but not very, sensitive to changes in the economic parameters such as the slope and intercept of the demand curve and unit costs of harvest. The results are hardly sensitive at all to changes in the parameters related to investment such as the buying or selling price of capital or the depreciation rate. On the other hand, the results are quite sensitive to changes in the biological parameters, that is the intrinsic growth rate and the parameters representing the interaction between the two stocks. This indicates that future effort is best spent on estimating the biological submodel, but of course all parts of the total model must be reasonably correct in order to draw robust conclusions. Finally, the results are not very sensitive to a reasonable change in the discount rate either.

Other areas for future research within the framework of this model is to find the feedback rule, that is harvest and investment as functions of the four state variables, the stock levels and the capital levels. Another possibility is to let both fleets harvest on both stocks, and a third possibility is to open for excess capital in each period such that in addition to determining the capital level we also determine the utilization level of capital. This will add to the number of control variables and make the model larger. It will have no effect on the optimal steady states as in steady state it will never be optimal to have excess capital.

REFERENCES

[1] Armstrong, C. and Sumaila, U.R., 2001, Optimal allocation of TAC and the implications of implementing an ITQ management system for the North-East Arctic cod, *Land Economics* 77: 350 - 359.

[2] Bjørndal, T. and Gordon, D.V., 2000, The Economic Structure of Harvesting for Three Vessel Types in the Norwegian Spring-Spawning Herring Fishery, *Marine Resource Economics* 15: 281 - 292.

[3] Directorate of Fisheries Norway, 2001, "Lønnsomhetsundersøkelser for helårsdrevne fiskefartøyer 8 m.l.l. og over", Bergen.

[4] ICES, 2001, "Arctic Fisheries Working Group: Cod", International Council for the Exploration of the Sea, Copenhagen.